Mr. *Wingate's* *Arithmetick*;

CONTAINING

A PLAIN and FAMILIAR

METHOD

For Attaining the

Knowledge and *Practice*

OF

COMMON ARITHMETICK.

The Ninth Edition, very much Enlarged.

First Composed by *Edmund Wingate* late of *Grays-Inn*, Esquire.

Afterwards, upon Mr. *Wingate*'s Request, Enlarged in his Life-time: Also since his Decease Carefully Revised, and much Improved, as will appear by the *Preface*, and *Table* of *Contents*,

By *JOHN KERSEY*, Teacher of the *Mathematicks*, at the Sign of the *Globe* in *Shandois-street* in *Covent-Garden*.

Boetius Arith. lib. 1. cap. 2.

Omnia quæcunque à primævâ rerum naturâ constructa sunt, Numerorum videntur ratione formata: Hoc enim fuit principale in animo Conditoris Exemplar.

LONDON,

Printed for *John Williams*, and are to be Sold by the Booksellers of *London* and *Westminster*. 1694.

This Book is one of the best of its kind, yt ever was wrote: It has undergone 15 Editions from 1629 to 1726 —
The latest Editions have a Supplement added to ym by Mr George Shelley —: but in other respects don't differ from this 9th Edition — excepting that they're corrected a little by Jn Kersey ye famous Kersey's son. —

TO THE
RIGHT HONOURABLE
THOMAS
Earl of *Arundel* and *Surrey*,
Earl Marshal of
ENGLAND, &c.

Right Honourable,

T*He good Affection you bear to all kind of* Learning, *and in particular to the* Mathematicks, *makes me adventure to present your Lordship with this Tractate of* Arithmetick, *because that Art, compared with other Mathematical Sciences, is as the* Primum Mobile, *in respect of the other*

other inferior Orbs: For as the Poets used in times past to say of Venus, Sine Cerere & Baccho Friget Venus, *so may I also confidently averr of them, without* Arithmetick *they are Poor, and without* Motion. *Presuming therefore that your Lordship, loving the Art, cannot disaffect the Artist, nor his intention to do good in that kind, I am bold to shelter this Treatise under your Lordship's protection, humbly intreating your gracious Acceptation, and earnestly desiring for ever to remain*

Your Honours, in all

Service affectionately

devoted,

EDM. WINGATE

THE PREFACE OF *JOHN KERSEY.*

ABout the year 1629. our Learned Country-man *Edmund Wingate* Esquire, published a Treatise of *Arithmetick* divided into two *Books*, the one entituled *Natural Arithmetick*, the other *Artificial Arithmetick*; and in regard his principal design in that Treatise was, to remove the difficulties which ordinarily arise in the practice of *Common-Arithmetick*, by the help of artificial, or borrowed Numbers, called *Logarithms*, (whose proper work is to perform *Multiplication* by *Addition*; *Division* by *Subtraction*, &c.

 he)

he did then in his ſaid firſt Book omit divers pieces of *Common* or *Practical Arithmetick*, which, for the perfect and univerſal underſtanding thereof, were neceſſary to have been inſerted: But after the firſt impreſſion of both thoſe Books was ſpent, our ſaid Author being importuned to take care of the ſecond Edition, he promiſed his aſſiſtance therein; yet his other neceſſary Employments not permitting him to purſue his ſaid purpoſe, he was pleaſed to impart his thoughts concerning the ſame unto me, together with his requeſt, that I would purſue the ſaid firſt Book, and ſupply it with ſuch pieces of *Practical Arithmetick*, which for the reaſons aforeſaid were wanting in the firſt Edition.

In purſuance of which requeſt, I have contributed my Talent towards perfecting this Tractate, upon our Author's foundation; partly in his life time, to his good liking, and partly ſince his deceaſe, in ſeveral Editions committed to my Care to be prepared for the Preſs: wherein I have uſed my beſt endeavours, as well to preſerve this Book as a Monument of our ſaid Author's worth, as alſo to make it a compleat Store-houſe of *Common Arithmetick*; from

from whence the ingenious may be furniſh'd with the excellencies of that Art, in reference both to common Affairs, as alſo to the Practical parts of the Mathematicks. And in order to thoſe ends I have made theſe following Alterations and Additions; namely,

Firſt, For the Eaſe and Benefit of ſuch Learners, who deſire only ſo much Skill in Arithmetick, as is uſeful in Accompts, Trade, and ſuch like ordinary Employments; the Doctrine of whole Numbers, (which in the firſt Edition was intermingled with Definitions and Rules concerning broken Numbers, commonly called Fractions) is now entirely handled apart. And to the end the full knowledge of *Practical Arithmetick* in whole Numbers might more clearly appear, I have explained divers of the old Rules in the firſt five Chapters, and framed anew the Rules of *Diviſion*, *Reduction*, and the *Golden Rule*, in the ſixth, ſeventh, eighth, and ninth Chapters; ſo that now Arithmetick in whole Numbers is plainly and fully handled before any entrance be made into the craggy paths or Fractions, at the ſight whereof ſome Learners are ſo diſ-

couraged,

courraged, that they make a stand, and cry out, *non plus ultra*, there's no progress farther.

Secondly, To assist such young Students as desire to a lay a good Foundation for the attaining of a general Knowledge in the Mathematicks, I have in a familiar Method delivered the entire Doctrine of Fractions, both Vulgar and Decimal, which was omitted in the first Edition; and have also newly framed the Extraction of the Square and Cube Roots, in a method which by Experience is found to be much easier than that commonly used heretofore, and is exactly suitable to the Construction or Composition of Square and Cube numbers.

Lastly, I have added an *Appendix*, which is furnished with variety of choice and delightful knowledge in Numbers, both Practical and Theoretical. In all which performances, I have earnestly aimed at Truth, Perspicuity, and exact Correction, both of the Text and Numbers; so that I hope this Book is now supplied with all things necessary to the full Knowledge and Practice of *Common Arithmetick*, the usefulness whereof is so gegenerally known, that there will be no need of Arguments to excite any one that desires

sires his own or the Publick Good, to be acquainted with so excellent an Art.

But if the more curious Artist, after he is well exercis'd in vulgar Arithmetick, desires farther inspection into the Mysteries of Numbers, his best Guide is the admirable Art called *Algebra*; the Elements whereof I have expounded at large in a Treatise lately Publish'd.

JOHN KERSEY.

THE

The Table of Contents.

Where those Chapters of Mr. *Wingate's*, that have been altered and framed anew by *John Kersey*, are distinguished by this mark ☞, and those Chapters that have been entirely Composed by the said *J. K.* may be discovered by this Asterisk *.

The Doctrine of whole Numbers is contained in the first 15 Chapters, the Titles whereof are these following.

The

The Doctrine of Fractions both Vulgar & Decimal, is contained in the 16th Chapter next following.

Concerning Vulgar Fractions.

Concerning Decimal Fractions.

In Vulgar and Decimal Fractions.

The Extraction of Roots is contained in the two Chapters next following.

The Relation of Numbers in Quantity and Quality is contained in the two following Chapters

The

The Contents of the Appendix.

A

A TREATISE OF *Common Arithmetick*,

The First Book.

CHAP. I.

Concerning Notation of Numbers.

I. A*RITHMETICK* is the Art of Accompting by Number. As Magnitude or Greatness is the Subject of *Geometry*, so Multitude or Number is that of *Arithmetick*.

II. Number is that by which every thing is numbred; or that which answers *Number.*

swers this question, how many? (unless it be answered by nothing:) So if it be asked how many days are in a week, the answer is seven, which is called number.

The Characters by which number is expressed.

III. The Notes or Characters, by which Number is ordinarily expressed, are these; 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, o nothing.

IV. These Notes or Characters are either significant figures, or a Cypher.

V. The significant figures are the first nine; *viz.* 1,2,3,4,5,6,7,8,9. The first whereof is more particularly called an Unit, or Unity, and the rest are said to be composed of Unities: so 2 is composed of two Unities, 3 of three Unities, *&c.*

VI. The Cypher is the last, which though of it self it signifies nothing, yet being annexed after any of the rest, it encreaseth their value: As will appear in the following Rules.

VII. Arithmetick hath two parts, Notation and Numeration.

VIII. Notation teacheth how to express, read, or declare, the signification or value of any number written; and also to write down any number propounded, with proper Characters in their due places.

The Places or Degree of any number.

IX. A number is said to have so many places or degrees, as there are Characters in the number; *viz.* when divers figures, whether they be intermixt with a Cypher or Cyphers or not, are placed together like letters in a word, without any point, comma, line, or other note of distinction interposed

posed, all those Characters make but one number, which consists of so many places as there are Characters so placed to one another: so this number 205 consists of 3 places, and this 30600 of five places, &c.

X. Notation consists in the knowledge of two things; viz. the Order of places, and the Value of every place in any number.

The Order of places in any number.

XI. The Order of the places is from the right hand towards the left: So in this number 465, the figure 5 standeth in the first place, 6 in the second, and 4 in the third; likewise in this number 7560, a Cypher stands in the first place, 6 in the second, 5 in the third, and 7 in the fourth.

The Value of places in any number.

XII. The first place of a Number, (which as before is the outermost towards the right hand) is called the place of Units or Unities; in which place any figure signifieth its own simple value: so in this number 465, the figure 5 standing in the first place signifieth five Unities, or five.

XIII. The second place of a number is called the place of Tens; in which place any figure signifieth so many Tens as the figure containeth Unities: so in this number 465, the figure 5 in the first place signifieth simply five, but the figure 6 in the second place signifieth six tens, or sixty.

XIV. The third place of a number is called the place of Hundreds: in which place any figure signifieth so many Hundreds as there are Unities contain'd in the figure: So in this number 465, the figure 4 in the third place signifieth four Hundreds: wherefore if it be required to read or pronounce this number 465, you are to begin on the left hand, and

and according to the aforesaid rules to pronounce it thus, four hundred sixty five likewise this number 315 is to be pronounced thus, three hundred and fifteen: and this number 205, two hundred and five; also this number 500, five hundred. Whence it is manifest, that although a Cypher of it self signifies nothing, yet being placed on the right hand of a figure it increaseth the value thereof, by advancing such figure to an higher place than that wherein it would be seated, if the Cypher were absent.

The true reading or pronouncing the value of any number written, as also the writing down any number propounded, depends principally upon a right understanding of the three first places beforementioned, and therefore I shall advise the Learner to be well exercis'd therein, before he proceeds to the following Rules.

XV. The fourth place of a number is called the place of Thousands (that is, any number of Thousands under ten thousand;) the fifth place tens of thousands; the sixth place hundreds of thousands; the seventh place Millions (a Million being ten hundred thousand;) the eighth place tens of Millions; the ninth place hundreds of Millions; the tenth place thousands of Millions; the eleventh place tens of thousands of Millions; the twelfth place hundreds of thousands of Millions: And in that order you may conceive places to be continued infinitely from the right hand towards the left, each following place being ten times the value of the next preceeding place; but to give names to them would be both a troublesome and an unnecessary task.

XVI.

XVI. *From the rules aforegoing, an easie way may be collected to read or express the value of a Number propounded, Viz.* Let it be required to read or pronounce this number 521426341. *First*, Distinguish by a Comma, or Point, every three places, beginning at the right hand, and proceeding towards the left, so will the aforesaid number be distinguished into parts, which may be called *Periods*, and stand thus 521, 426, 341. where you may note the *first period* towards the right hand to consist of these figures 341, the *second* of these 426, and the *third* of these 521. *Secondly*, read or pronounce the figures in every *Period* as if they stood apart from the rest, so will the first *Period* be pronounced three hundred forty one, the second four hundred twenty six: and the third five hundred twenty one. *Thirdly*, to every *Period* except the first towards the right hand, a peculiar *denomination* or *sirname* is to be applyed, *Viz.* the *sirname* of the second *Period* is *Thousands*; of the third, *Millions*; of the fourth, *Thousands of Millions, &c.* Therefore beginning to pronounce at the highest *Period*, which in this Example is the third, and giving every *Period* its due *sirname*, the said number will be pronounced thus, *Five hundred twenty one Millions, four hundred twenty six Thousands, three hundred forty one.*

A brief way of Notation.

A Period.

Note, When a number is distinguished into *Periods*, as before, the highest *Period* will not always compleatly consist of three places, but sometimes of one place, and sometimes of two, nevertheless after such *Period* is pronounced as if it stood apart, the due *sirname* is to be annexed; so this

B num-

number 3204689, after it is divided into *Periods* will stand thus, 3, 204, 689. and to be pronounced thus, *Three Millions, two hundred and four thousands, six hundred eighty nine.*

XVII. The aforesaid Rules for the right pronouncing or reading of a Number which is written down, being well understood, will sufficiently inform the Reader how to write down any number propounded to be written.

The Table of Notation.

The order of places.			*The values of Places.*
	&c.		*&c.*
Fourth Period,	*Twelfth place*	3	*Hundreds of Thousand Millions.*
	Eleventh place	2	*Tens of Thousand Millions.*
	Tenth place	1	*Thousand Millions.*
Third Period,	*Ninth place*	9	*Hundreds of Millions.*
	Eighth place	8	*Tens of Millions.*
	Seventh place	7	*Millions.*
Second Period,	*Sixth place*	6	*Hundreds of Thousands.*
	Fifth place	5	*Tens of Thousands.*
	Fourth place	4	*Thousands.*
First Period,	*Third place*	3	*Hundreds.*
	Second place	2	*Tens.*
	First place	1	*Units.*

Notation of Numbers by Latin Letters.

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	IIII. or thus IV.	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIIII. or thus LIX.
7	VII.	60	LX.
8	VIII. or thus IIX.,	89	LXXXIX.
9	VIIII. or thus IX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII. or thus IIXX.	500	D. or thus IƆ.
19	XVIIII. or thus XIX.	600	DC. or thus IƆC.
20	XX.	700	DCC. or thus IƆCC.

10000	CIƆ. or thus M.
20000	CIƆ. CIƆ.
30000	CIƆ. CIƆ. CIƆ.
50000	IƆƆ.
100000	CCIƆƆ.

50000	IƆƆƆ.
100000	CCCIƆƆƆ. or thus CM,
500000	IƆƆƆƆ.
1,000,000	CCCCI. ƆƆƆƆ.
1677	CIƆDCLXXVII. or MDCLXXVII.

CHAP. II.

Concerning English Moneys, Weights, Measures, &c.

I. THe *things expressed by Numbers are principally Money, Weight, Measure, Time, and things accompted by the dozen:* Of the three first of these, there are infinite kinds and varieties according to the diversity of the several *Common-wealths* in which they are used, all which here to produce were both endless and needless: wherefore we intend here to treat only of such *Moneys, Weights, Measures,* &c. as are used in this *Nation,* being indeed only necessary for our present purpose.

II. *The least piece of Money used in* England *is a Farthing, from whence this following Table is produced.* Of English Moneys.

1 *Farthing*	make	1 *Farthing.*
4 *Farthings*	make	1 *Peny.*
12 *Pence*	make	1 *Shilling.*
20 *Shillings*	make	1 *Pound.*

English (or sterling) Money is ordinarily written down with Figures after this manner.

l.	s.	d.	f.
34	13	05	2
09	05	10	1
69	00	06	3
00	12	11	0
00	00	07	2

The

The first Rank of the said Numbers signifies thirty four pounds, thirteen shillings, five pence, two farthings: the second Rank expresseth nine pounds, five shillings, ten pence, one farthing: the third Rank, six pounds, no shillings, six pence, three farthings, *&c.*

III. The smallest Weight used in *England* is a grain, that is, the weight of a grain of Wheat well dried and gathered out of the middle of the ear, whereof thirty two make another weight, called a Peny-weight, and twenty Peny-weight, make an Ounce Troy. *Vide Stat. de compositione ponderum* 51 *Hen.* 3.

Here observe, That by the *Statutes* quoted in the Margent, the weight of two and thirty grains of Wheat make a peny-weight, which weight being once discovered by two and thirty such grains, the said peny-weight (being the twentieth part of an ounce Troy) is usually subdivided into four and twenty parts only, called also *Grains,* as appears by the ensuing Table. 31 *Ed.* 1 *v. Rast. weights.* 7 & 8. 12 *Hen.* 7. 5.

A Table of Troy Weights. *Troy weight.*

24 *Grains of Wheat*	make	24 *Artificial Grains.*
24 *Grains*	make	1 *Peny weight.*
20 *Peny Weight*	make	1 *Ounce.*
12 *Ounces*	make	1 *Pound Troy.*

Troy Weight is ordinarily written down with Figures after this manner.

lb.	*oz.*	*p.w.*	*gr.*
17	05	13	13
00	11	07	06
00	00	05	20

 The

The first rank of the said numbers expresseth seventeen pounds, five ounces, thirteen peny weight, thirteen Grains of *Troy weight*: the second rank, no pounds, eleven ounces, seven peny weight, six grains: and the third, no pounds, no ounces, five peny weight, and twenty grains.

Now this *Troy weight* serveth only to weigh Bread, Gold, Silver, and Electuaries. And here observe also by the way, that *Troy weight* regulateth and prescribeth a form how to keep the Money of *England* at a certain *Standard*.

Malynes lex Mercat. p. 49.

Malynes ib. *pag.* 252.

For about two hundred years before the Conquest, *Osbright* a Saxon, being then King of *England*, caused an ounce *Troy* of Silver to be divided into 20 pieces, at the same time called Pence; and so an Ounce of Silver at that time was worth no more than twenty pence, or one shilling eight pence, which continued at the same value until the time of *Henry* the sixth, (who in regard of the enhancing of Moneys in Foreign parts) valued the same at thirty pence, so that then there were accordingly thirty pieces made out of the Ounce, and the old pieces went then for three half-pence, until the time of *Edward* the Fourth, who valued the Ounce at forty pence, and then the old pieces went for two pence apiece. After this, *Henry* the Eight valued the Ounce of sterling Silver at forty five pence, which value continued until Queen *Elizabeth*'s time, who valued the same Old pence at Three-pence the piece, so that all Three-pences coined by the same Queen weighed but a peny weight, and every Six pence two peny weight; and so in like manner the Shilling and other pieces accordingly; which made the ounce *Troy* of Silver, to be valued at sixty pence or five shillings, as it now remains at this day without alteration.

IV. The weights used by Apothecaries are derived from a pound *Troy*, which is subdivided as in the following Table:

Apothecaries weights.

A Table of Apothecaries Weights.

℔	*A Pound Troy*	is equal unto	12	*Ounces.*
℥	*An Ounce*		8	*Drams.*
ʒ	*A Dram*		3	*Scruples.*
℈	*A Scruple*		20	*Grains.*

So that if you were to express in Figures 12 pounds, 10 ounces, five drams, two Scruples, and 16 grains: also three pounds, five ounces, seven drams, one scruple, and two grains, the ordinary way to write them down is briefly thus:

℔	℥	ʒ	℈	*gr.*
12—	10—	5—	2—	16
03—	05—	7—	1—	02

V. Besides *Troy* weight before mentioned, there is another kind of weight used in *England*, called Averdupois weight, a pound whereof is equal unto 14 Ounces, twelve peny weight *Troy*. This Averdupois weight serveth to weigh all kind of Grocery-ware, as also Butter, Cheese, Flesh, Tallow, Wax and every other thing, which beareth the name of *Garbel*, and whereof issueth a refuse or waste.

Malynes ib. pag. 49.

VI. Averdupois weight is either greater or less.

VII. The greater is, when one hundred and twelve pounds Averdupois are considered as one intire weight com-

Averdupois greater weight.

commonly called an hundred weight, and then such hundred weight is subdivided first into four quarters, and each quarter into eight and twenty pounds: again, each pound into four quarters, or (if you will be more exact) into 16 Ounces, and if you please each Ounce into four quarters. But ordinarily a pound is the least quantity that is taken notice of in Averdupois gross weights.

A Table of Averdupois greater weight.

28 *pounds*	make	*a quater of* 112 *lb.*
4 *quarters*	make	*an hundred weight, or* 112 *lb.*

So that if you were to express by Figures eight hundred, three quarters, and five pounds; likewise, seven hundred, one quarter, and seventeen pounds; the ordinary way to write them down is briefly thus,

C.	q.	lb.
8	3	5
7	1	17.

Averdupois lesser weight. *VIII.* The lesser Averdupois weight is, when a pound is the highest name or Integer, each pound being subdivided into sixteen ounces, and each ounce again into 16 drams, and if you please each dram into 4 quarters, as by the subsequent Table is manifest.

A Table of Averdupois lesser Weight.

4 *Quarters of a Dram*	make	1 *Dram.*
16 *Drams*	make	1 *Ounce.*
16 *Ounces*	make	1 *Pound.*

So

So that if you were to express by figures eighteen pounds, twelve ounces, five drams, and three quarters of a dram; likewise five pounds, no ounces, twelve drams, and one quarter of a dram, the ordinary way to write them down is briefly thus,

lb.	*oz.*	*dr.*	*q.*
18	12	05	3
5	00	12	1

IX. The measures used in *England* are either of Capacity or Length.

X. The measures of Capacity are those which are produced from Weight, and they are either Liquid or Dry.

XI. The Liquid measures are those, in which all kind of Liquid substances are measured, and they are expressed in the Table following. *Liquid Measure.*

A Table of Liquid Measures.

1 *Pound of Wheat Troy weight*	makes	1 *Pint.*
2 *Pints*	makes	1 *Quart.*
2 *Quarts*	makes	1 *Pottle.*
2 *Pottles*	makes	1 *Gallon.*
8 *Gallons*	makes	1 *Firkin of Ale, Soap, Herring,*
9 *Gallons*	makes	1 *Firkin of Beer.*
10 *Gallons and an half*	makes	1 *Firkin of Salmon or Eels.*
2 *Firkins*	makes	1 *Kilderkin.*
2 *Kilderkins*	makes	1 *Barrel.*
42 *Gallons*	makes	1 *Tierce of Wine.*
63 *Gallons*	makes	1 *Hogshead.*
2 *Hogsheads*	makes	1 *Pipe or But.*
2 *Pipes or Buts*	makes	1 *Tun of Wine.*

XII. Dry

Dry Measures.

XII. Dry measures are those, in which all kind of dry substances are meted, as Grain, Sea-coal, Salt, and the like; their Table is this that follows:

A Table of Dry Measures.

1 *Pint*	make	1 *Pint.*
2 *Pints*		1 *Quart.*
2 *Quarts*		1 *Pottle.*
2 *Pottles*		1 *Gallon.*
2 *Gallons*		1 *Peck.*
4 *Pecks*		1 *Bushel land-measure.*
5 *Pecks*		1 *Bushel water-measure.*
8 *Bushels*		1 *Quarter.*
4 *Quarters*		1 *Chalder.*
5 *Quarters*		1 *Wey.*

Long Measures.

XIII. Long Measures are express'd in this Table following.

3 *Barly Corns in length*	make	1 *Inch.*
12 *Inches*		1 *Foot.*
3 *Foot*		1 *Yard.*
3 *Foot nine Inches*		1 *Ell.*
6 *Foot*		1 *Fathom.*
5 *Yards and an half*		1 *Pole or Perch.*
40 *Poles or perches*		1 *Furlong.*
8 *Furlongs*		1 *English Mile.*

Note, That a Yard, as also an Ell, is usually subdivided into four Quarters, and each Quarter into four Nails.

XIV. Super-

Land Measures.

XIV. Superficial or square Measures of Land, are such as are express'd in the Table following.

9 feet ma
1 yard.
30 yards $\frac{1}{2}$
one pole

40 *Square Poles or Perches*	make	1 *Rood or quarter of an Acre.*
4 *Roods*		1 *Acre.*

So that if you would express by Figures these quantities of Land, *viz.* Thirty six Acres, three Roods, twenty Perches: also seven Acres, no Roods, thirty two Perches; the ordinary way to write them down is thus,

A.	*R.*	*P.*
36	3	20
7	0	32

Time.

XV. A Table of time is this that follows.

1 *Minute*	make	1 *Minute.*
60 *Minutes*		1 *Hour.*
24 *Hours*		1 *Day natural.*
7 *Days*		1 *Week.*
4 *Weeks*		1 *Month of twenty eight days.*
13 *Months* 1 *Day, and* 6 *Hours*		1 *Year very near*

But in ordinary computations of time, the whole year consisting of three hundred sixty five days, is divided either into twelve equal parts or months; each month then containing thirty days and ten hours: or else into twelve unequal *Kalendar* months, according to the ancient Verse:

Thirty days hath September, April, June, and November:
February hath twenty eight alone, and each of the rest thirty one.

Note

Note, That every *Leap-year* (which happeneth once in four years) containeth three hundred sixty six days, and in such year *February* containeth twenty nine days.

XVI. Of things accounted by the dozen, a Gross is the Integer consisting of twelve dozen, each dozen containing again twelve particulars: so that if you would express in Figures, seven Gross, four dozen, and five particulars; also four dozen and eight particulars, they may be briefly written thus.

Of things accounted by the dozen.

G.	*D.*	*P.*
7	04	05
0	04	08

CHAP. III.

Addition of whole Numbers.

I. Concerning Notation of Numbers, and how thereby the quantities of things are usually exprest, a full Declaration hath been made in the preceeding Chapters: Numeration ensueth, which comprehends all manner of operations by Numbers.

II. In Numeration, the four primary or fundamental operations (commonly called Species) are these, Addition, Substraction, Multiplication, and Division.

III. Addition is that by which divers Numbers are added together, to the end that their sum aggregate, or total, may be discovered.

IV. In Addition, place the Numbers given, one

above another in such sort, that like places or degrees in each number may stand in the same rank: that is Units above Units, Tens above Tens, Hundreds above Hundreds, &c. So these numbers 1213 and 462 being given to be added together, you are to order them as you see in the margent.

Addition of numbers of one denomination.

```
1213
 462
```

V. Having thus placed the Numbers, and drawn a line under them, add them together, beginning with the Units first, and saying thus, 2 and 3 make 5, which write under the Rank of Units, then proceed to the second Rank and say, 6 and 1 make 7, which write under the second Rank (being the place of tens) again 4 and 2 make 6, which write under the third Rank. Lastly, write down 1 being all that stands in the fourth Rank, so the sum of the said given Numbers is found to be 1675, and the operation will stand as in the Margent.

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1213
 462
----
1675
```

In like manner the numbers 2315, 7423, and 141, being given to be added together, their sum will be found to be 9879, and the operation thereof will stand as you see in the Example.

```
2315
7423
 141
----
9879
```

VI. When the sum of the Figures of any of the Ranks amounts unto ten, or any number of tens without any excess, write down a Cypher under that Rank; but when the sum of any Rank exceeds ten, or any number of tens, write down the excess under such Rank, and for every ten contained in the sum of any Rank, reserve an Unit or 1 in your mind, and add such Unit or Units to the Figures

gures of the next Rank towards the left hand, so the numbers 4937, 9878, and 394 being given to be added together, the operation will be thus, *viz.* beginning with the rank of Units, I say 4, 8 and 7 make 19, wherefore I write down 9, the excess above 10, and carrying 1 in mind instead of the ten contained in the said 19, I say 1 and 9 (9 being the lowermost figure of the second rank) make 10, which added to 7 and 3, the other figures of the same rank, the whole sum of them is 20, wherefore setting down a Cypher under the line in that rank (because the excess above the two tens is nothing) I carry 2 to the third rank, and say 2 and 3 (3 being the lowermost figure of the third rank) make 5, which being added to 8 and 9 (the other figures of the same rank) the sum of them is 22, wherefore writing down 2 (being the excess above the two tens) under the line in the third rank, I carry 2 in mind (because there were two tens in 22) to the fourth rank, and say 2 and 9 make 11, which added to 4 makes 15, this 15 because it is the sum of the last rank I write totally down under the line, on the left hand of the Figures before subscribed; so the sum of the three Numbers given is found to be 15209, as in the Example.

4937
9878
394
15209

Addition of Numbers of divers Denominations.

VII. When numbers given to be added, do express things of divers Denominations; first write them down orderly (according to the Examples in Chap. 2.) then after a line is drawn under them all, begin to add the numbers of

of the least Denomination, and if the sum of them amounts to one Integer, or many Integers of the next greater Denomination, with some excess of the less Denomination, write down that excess, or a Cypher when there is no excess, under the line, so as it may stand under the least Denomination, and keep the said Integer or Integers in mind, to be added to those of the next greater Denomination, on the left hand: But when the sum of the numbers of the least Denomination amounts not to one Integer of the next greater Denomination, set down the sum it self under the line; then add the Integer or Integers kept in mind (when any happens) to the numbers of the next greater Denomination on the left hand, and proceed to add them, as also those of every greater Denomination, in like manner as above is directed, until you come to the numbers of the greatest (or highest) Denomination, which are to be added according to the foregoing Rules *V.* and *VI.* of this Chapter. So these several sums 24*l.*—13*s.*—5*d.*—3*f.* Also 12*l.*—0*s.*—8*d.* and 5*l.*—18*s.*—2*f.* being propounded to be addded, their total sum is 42*l.*—12*s.*—2*d.*—1*f.* For having written them down orderly according to the second Rule of the Second Chapter, and drawn a line underneath, I begin with the Farthings first, and say, two Farthings and three Farthings make five Farthings, that is, one Peny with a Farthing over and above; wherefore setting down 1 under the Denomination of Farthings, I carry

l.	*s.*	*d.*	*f.*
24	13	05	3
12	00	08	0
05	18	00	2
42	12	02	1

carry one Penny to the denomination of Pence, then I say, 1, 8, and five pence make 14 Pence, which contain one Shilling and two pence, wherefore writing two under the denomination of Pence, I likewise carry 1 shilling to the denomination of shillings: Then adding the said 1 shilling unto 18 shillings and 13 shillings, the sum will be found 1 pound and 12 shillings, wherefore setting down 12 under the denomination of shillings, I carry 1 pound in mind unto the denomination of pounds saying, 1 pound in mind, together with 5, 2, and 4 pounds which stand in the first Rank of pounds, make 12 pounds, wherefore (according to the sixth Rule of this Chapter) I write 2, the excess above 10, underneath the said first rank of pounds, and carry 1 in mind for the said 10 to the second Rank of pounds, then saying in like manner, 1 in mind, together with 1 and 2 which stand in the second Rank of pounds make 4, which I write underneath the line, that done, I find the total of the three sums propounded to be 42 *l*.--- 12. *s*.—*and*—1 *f*.

In like manner 3 *lb*.-- 05 *oz*.--19 *p. w.* 15 *gr*. Also 2 *lb*.—0 *oz*. —3 *p. w*.—7 *gr*. Also 0 *lb*. — 10 *oz*.— 6 *p. w*.--- 0 *gr*. And 0 *lb*.— 9 *oz*.— 0 *p.w*. -- 17 *gr*. being given to be added together, their sum will be found 7 *lb*.—— 10*z*.—— 9 *p. w*.— 15 *gr*. and the work will stand thus,

lb.	*oz.*	*pw.*	*gr.*
03	05	19	15
02	00	03	07
00	10	06	00
00	09	00	17
07	01	09	15

Note,

Note, In adding together the Numbers in the last Example, it must be remembred that 24 grains make one Penny weight; 20 Penny weight, one ounce; and 12 ounces one pound *Troy* (as before declared in the third Rule of the second Chapter;) And then you are to proceed according to Rule VII. of this Chap.

More Examples of Rule VII. are these following, which presuppose the Learner to be well exercis'd in the Tables of Chap. 2. that he may readily know what Integers are to be carried from every lesser Denomination to the next greater.

Addition of English Money.

lb.	*s.*	*d.*	*f.*	*l.*	*s.*	*d.*
230	17	10	1	0	13	05
175	12	11	3	0	17	08
052	05	06	0	0	00	10
009	00	08	1	0	10	03
506	13	00	2	0	15	06
974	10	00	3	2	17	08

Addition of Troy Weight.

lb.	*oz.*	*pw.*	*gr.*	*oz.*	*pw.*	*gr.*
23	07	16	13	536	13	16
17	10	15	07	208	11	10
325	06	19	20	063	10	05
49	11	07	12	099	00	12
417	00	19	04	907	15	19

Addition of Averdupois Weight.

C.	*q.*	*lb.*	*lb.*	*oz.*	*dr.*
235	3	13	14	13	12
576	1	17	05	10	14
628	2	15	12	00	06
412	0	10	06	09	05
1852	3	27	39	02	05

Addition of Measures of Length.

yards.	*q.*	*nails.*	*Ells.*	*q.*	*nails.*
26	3	2	15	3	2
13	1	3	16	1	3
12	0	1	09	0	1
29	1	1	12	2	1
81	2	3	53	3	3

Addition of Superficial Measures of Land.

Acres.	*Roods.*	*Per.*	*A.*	*R.*	*P.*
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	10
1379	3	09	996	3	36

CHAP.

CHAP. IV.

Subtraction of whole Numbers.

I. SUbtraction is that by which one number is taken out of another, to the end that the remainder or difference, between the two numbers given may be known.

II. The number out of which the Subtraction is to be made, must be greater, or at least, equal with the other. As you may Subtract, 4347 or 9478 out of 9478, so can you not subtract 9478 out of 4347.

Subtraction of numbers of one denomination.

III. In subtraction rank the two given numbers one under the other as in Addition, with this caution, that the number placed uppermost may exceed, or at least be equal unto the other: So if the number 4347 be given to be subtracted from 9478, I order them as in the Margent: then proceeding to the Subtraction, I say 7 taken out of 8, there remains one, which I place in the same rank under the line. In like manner 4 being taken out of 7, the remainder is 3, which likewise I set under the line in the next rank; again taking 3 from 4, the remainder is 1, which I likewise place under the third rank; lastly subtracting 4 from 9, there will remain 5, which I subscribe under the fourth rank; so the whole operation being finished, I find, that if 4347 be taken out of 9478, the remainder is 5131, or (which is the same) the difference between the numbers 9478 and 4347 is 5131, as in the Example.

$$\begin{array}{r} 9478 \\ \underline{4347} \\ 5131 \end{array}$$

In

In like manner if 106 be subtracted from 2856 the remainder will be found 2750; for after the numbers are orderly ranked, I begin at the place of Units, and say, 6 from 6, there remains nothing; wherefore I subscribe 0. then proceeding to the second rank I say, if 0 (or nothing) be taken from 5, there will remain 5, which I also subscribe under the line; again 1 from 8, there remains 7: lastly 0 from 2, there remains 2. See the work in the Margent.

2856
106
2750

IV. When any of the figures of the number given to be subtracted is greater than the upper figure out of which it is to be subtracted, you must borrow 10 of the next rank towards the left hand, and add the said 10 to the said upper figure, then from the sum of such Addition subtract the lower figure and set down the remainder: In this case the figure of the next rank which is to be subtracted; must be esteemed an unite greater than it is; wherefore keeping one in your mind add it to the next figure of the number given to be subtracted, and deducting all out of the figure above it, proceed in like sort till you have finished the whole operation. *Example*, Let it be required to subtract 374 out of 8023. Having ranked them as before, I say four out of 3, that cannot be, wherefore borrowing ten of the next rank, and adding the same to the said 3, I say 4 out of thirteen, there remains 9; then writing 9 under the line, and carrying 1 in my mind, I say 1 and 7 make 8, 8 out of two that cannot be, but 8 out of 12 (12, because 10 being borrowed, and added to 2, makes 12) there remains 4, which I subscribe under the line;

8023
374
7649

line; again 1 in my mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6, which I likewise subscribe under the line; lastly 1 in mind being take out of 8 there remains 7. Thus you see that the remainder after 374 is subtracted from 8023 is 7649. Note diligently, that as often as 10 is borrowed, 1 must be kept in mind to be added to the figure standing in the next place of the lower number, and the sum of such Addition must be subtracted from the upper place; but if it happen that there is no figure in the next place of the lower number, then the 1 in mind must be subtracted from the upper place, (as in the last rank of the last Example.) *Another Example.* Let it be required to subtract 92 from 62801. Having placed the greater number uppermost, and the lesser orderly underneath, I begin at the place of units, and say, 2 from 1 I cannot take, but borrowing 10, and adding it to the said 1, I say 2 from 11 there remains 9, which I subscribe under the line; then I proceed and say, 1 in mind with 9, makes 10, 10 out of 0 I cannot take, but borrowing 10, I say 10 out of 10 and there remains 0, wherefore I subscribe 0 under the line; again, 1 in mind out of 8, there remains 7; then because there are no more Figures in the lower number, I say 0 out of 2 there remains 2; lastly, 0 out of 6 there remains 6; therefore I conclude that 62801 exceeds 92 by 62709.

62801
92
62709

Subtraction of numbers of divers denominations.

V. If the numbers propounded have divers denominations, place them as before, and beginning with the

the leaſt denomination firſt, ſubtract the lower number from the upper when it may be ſubtracted, and place the remainder underneath; but if it happen that the lower number cannot be taken out of the upper, you muſt borrow an integer of the next greater denomination on the left hand; which integer, after it is converted into the ſame denomination with the ſaid upper number, muſt be added to it: then from the ſum of ſuch Addition, you are to ſubtract the lower number, and write down the remainder, keeping 1 (that is the integer borrowed) in your mind, to be added to the next place of the number given to be ſubtracted, as before: ſo *90l.* — *14s.* — *10d.* — *3f.* being ſubtracted from *124l.* — *11s.* — *7d.* — *1f.* the remainder is *33l.* — *16s.* — *8d.* — *2f.* For beginning with the farthings, I ſay, 3 farthings out of 1 farthing I cannot take, wherefore borrowing 1 Penny (that is an integer of the next greater denomination) and having converted this penny into four farthings, I add them to the aforeſaid 1 farthing; ſo the ſum is five farthings, out of which ſubtracting 3 farthings, there remains 2 farthings, which I place underneath the denomination of farthings; then I proceed to the next denomination, and ſay 1 penny which I borrowed and *10d.* make *11d.* this *11d.* out of *7d.* I cannot take, wherefore borrowing 1 ſhilling or 12 *d.* and adding 12*d* to the ſaid *7d.* the ſum is *19d.* from which I ſubtract the ſaid *11d.* ſo there remains *8d.* which I ſubſcribe under the denomination of pence; again 1 ſhilling which I borrowed being added to *14s.* makes

l.	*s.*	*d.*	*f.*
124	11	07	1
90	14	10	3
33	16	08	2

makes 15 *s.* which I cannot ſubtract out of 11 *s.* and therefore I borrow 1 pound or 20 *s.* which being added to the ſaid 11*s*, makes 31*s*, from which ſubtracting 15*s.* there remains 16*s.* which I ſubſcribe under the denomination of ſhillings; then carrying 1 pound which I borrowed to the lower place of pounds, I ſay 1 in mind with 0 makes 1, which taken out of 4, there remains 3; again 9 out of 2, I cannot take, but 9 out of 12 (10 being borrowed and added to the ſaid 2, according to the fourth Rule of this Chapter) and there remains 3. Laſtly, 1 (for the 10 that was borrowed) being taken out of 1, there remains nothing; and ſo at laſt I find, that if A. being indebted to B. in 124 — 11*s.* — *7d.* — *1f.* hath paid in part thereof *90l.* — *14s.* — *10d.* — *3f.* there remains yet undiſcharged *33l.* — *16s.* — *8d.* — *2f.*

Subtraction of many numbers from one number.

VI. When many numbers are given to be ſubtracted from a number propounded, you muſt firſt add thoſe numbers together, according to the rules of the third Chapter, and then the ſum found is to be ſubtracted from the number firſt propounded. *Example*, A. being indebted unto B. in 3240*l.* paid thereof at one time 700*l.* at a ſecond payment 1236. and at a third 305*l.* the queſtion is how much of the debt remained undiſcharged? Firſt, I add together the ſeveral ſums paid, and find the total to be 2241 *l.* this I ſubtract from 3240*l* ſo there remains 999*l.* undiſcharged as you ſee by the operation in the Margent.

l.	
3240	*The Debt.*
700	
1236	*Payments*
305	
2241	*Total paid*
999	*reſt unpaid*

	l.	*s.*	*d.*
The Debt	500	00	00
Payments	340	12	06
	13	18	03
	17	16	10
Paid in all	372	07	07
Rest unpaid	127	12	05

Another Example of like nature. A. being indebted to B. in 500*l.* paid in part thereof at one payment. 340*l.* —— 12*s.* —— 06*d.* at a second payment 13*l.* —— 18*s.* — 3*d.* at a third 17*l.* — 16*s.* — 10 *d.* the question is how much was in arrear? Here if the operation be prosecuted as before it will appear that there was 127*l.* —— 12*s.* —— 05. unpaid: see the work in the Margent.

The Proof of Addition and Subtraction.

VII. Addition is proved by Subtraction, and Subtraction by Addition. For having added divers numbers together, if you subtract one of them out of the sum, the remainder must be equal to all the rest, as you may observe by the Example following, *viz.* supposing these 4 numbers are given to be added *viz.* 236, 452, 29, 217. and that their sum is found to be 934 (by the Rules of the third Chapter) it is required to prove whether the said sum be true or not; to perform this, I draw a line under the uppermost number 236, to separate it from the rest, and seek the sum of all the numbers given, except that uppermost, which sum I find to be 698. Then I subtract the said uppermost number 236 from 934, (the total sum of all the numbers first found) and because the remainder 698 is the same with

236	
452	934
29	236
217	698
934	
698	

with the sum of all the numbers, excluding the uppermost, I conclude that the sum of all the numbers first found was truly computed.

In like manner is Subtraction proved by Addition, for if you add the remainder, and the number given to be subtracted together, the sum must be equal to the number out of which the Subtraction is made, so if 4347 be subtracted from 9478 the remainder is 5131, for if 5131 be added to 4347, the sum is 9478, which is the same with the number out of which the Subtraction was made. Again, if a Servant receive 24*l.* —— 13*s.* —— 7*d.* and lay out or disburse 19*l.* —— 15*s.* —— 08*d.* there must remain in his hands —— 4*l.* —— 17*s.* —— 11*d.* for this being added to 19*l.* —— 15*s.* —— 08*d.* which was the Money he expended, the sum will be equal to 24*l.* —— 13*s.* —— 07*d.* (being the Money wherewith he was first charged.)

Example 1.		*Example* 2. *l.*	*s.*	*d.*
out of	9478	24	13	07
subtr.	4347	19	15	08
Rest	5131	04	17	11
Proof	9478	24	13	07

More Examples of Subtraction are these that follow,

Subtraction of English Money.

	l.	*s.*	*d.*	*f.*	*l.*	*s.*	*d.*	*f.*
Rec.	3090	10	07	1	24	00	00	0
paid	0099	14	08	3	05	17	11	3
rest	2990	15	10	2	18	02	00	1
proof	3090	10	07	1	24	00	00	0

Sub-

Subtraction of Troy Weight.

	lb.	oz.	pw.	gr.	oz.	pw.	gr.
Bought	352	10	13	15	205	13	19
Sold	019	11	16	18	118	16	20
Rest	332	10	16	21	86	16	23
Proof	352	10	13	15	205	13	19

Subtraction of Averdupois Weight.

	C.	q.	lb.	lb.	oz.	dr.
Bought	256	2	23	25	13	12
Sold	079	3	26	00	14	13
Rest	176	2	25	24	14	15
Proof	256	2	23	25	13	12

Subtraction of Superficial Measures of Land.

	Acres,	Roods,	Perches.	A.	R.	P.
Bought	780	2	35	2040	1	20
Sold	090	3	36	919	3	30
Rest	689	2	39	1120	1	30
Proof	780	2	35	2040	1	20

Questions to exercise Addition and Subtraction.

Qu. 1. Two persons, A. and B. owe several debts the lesser debt being that of A. is 3045*l.* the difference of their debts is 104*l.* what is the debt of B? *Ans.* 3149*l.*

Quest.

Quest. 2. Two persons A. and B. are of several ages, the age of the elder, being that of A. is 70, the difference of their ages is 19, what is the age of B? *Answer*, 51.

Quest. 3. What number is that which being added to 168 maketh the sum to be 205? *Ans.* 37.

Quest. 4. The sum of two numbers is 517, the lesser is 40, what is the greater? *Ans.* 477.

Quest. 5. A certain person born in the year of our Lord 1616, desired to know his age in the year 1676, what was his age? *Ans.* 60.

Quest. 6. The greater of two numbers is 130, their difference is 49, what is the lesser number? *Ans.* 81.

CHAP. V.

Multiplication of whole Numbers.

I. Multiplication teacheth how by two numbers given to find a third, which shall contain either of the numbers given so many times as the other contains 1 or unity.

II. Of the two numbers given in Multiplication, one (which you will) is called the Multiplicand and the other the Multiplicator, (or both are called factors.)

III. The number sought, or arising by the multiplication of the two numbers given, is called the product, the Fact, or the Rectangle: so if 5 be given

given to be multiplyed by 3 or 3 by 5, the product is 15, that is 3 times 5, or 5 times 3 makes 15: and here 5 may be called the Multiplicand, and 3 the Multiplicator, or 3 may be called the Multiplicand, and 5 the Multiplicator; and as 3 (one of the two numbers given) containeth 1 or unity thrice, so 15 the product containeth 5 (the other given number thrice; likewise as 5 (one of the given numbers) contains unity 5 times, so 15 (the product) contains 3 (the other given number) five times.

IV. Multiplication is either single or compound.

Single Multiplication.

V. Single Multiplication, is, when the Multiplicand and Multiplicator consist each of them of one onely figure as in the last Example. In like manner if you multiply 9 by 5, the product is 45, this is likewise single multiplication: now the several varieties of single multiplication are well express'd in the Table following, usually called *Pythagoras his Table.*

The Table of Multiplication.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the *Table* is this, having one figure given

given to be multiplied by another to know the product of them, find the multiplicand in the top of the Table, and the multiplicator in the first Column thereof towards the left hand; this done, in the angle of position just against those two figures you shall find the product. So 9 being given to be multiplied by 5. I find 9 in the top of the table, and 5 in the first column towards the left hand, then carrying my eye from 5 in a right line equidistant to the upper side or top line of the Table, until I come to that square which is directly under 9, I find 45, which is the product required. The particular varieties of this *Table* ought to be learned by heart, (that is, a man must be able to give the Product of any single multiplication, without the least pause or stay) before he can readily work compound multiplication, as will further appear hereafter.

Compound Multiplication.

VI. Compound Multiplication, is when the multiplicator and multiplicand, either one or both, consist of more figures than one.

VII. In compound Multiplication, when the numbers given do end with significant figures, place them as in Addition and Subtraction. So 134 being given to be multiplied by 2, place them thus: then proceeding to the multiplication say thus: two times 4 is 8, which write under the line in the rank of your multiplying figure; again, say two times 3 is 6, which likewise write under the line in the next rank: Lastly, two times 1 is 2, which being likewise written down under the line in the next rank, the product is discovered to be 268, and the work will stand as in the Margent.

$$\begin{array}{r} 134 \\ 2 \\ \hline 268 \end{array}$$

VIII. When

VIII. When the Multiplicator conſiſts of more figures than one, as many figures as it hath, ſo many ſeveral products muſt be ſubſcribed under the line, which at laſt being added into one ſum, gives you the total product of all. So 1232 being given to be multiplied by 23, the operation thereof will ſtand thus, for 1232 being multiplied by 3, (according to the laſt rule) the product is 3696. Again, 1232 being multiplied by 2, the product is 2464, which ſeveral products, after they are placed in their due order, (that is, the firſt figure ariſing in each product under his reſpective multiplying figure) and added together, produce 28336, the product required: In like manner 1321 being given to be multiplied by 123, the product is 162483, and the operation will ſtand as you ſee in the Margent.

```
 1232
   23
 ----
 3696
2464
-----
28336

  1321
   123
  ----
  3963
 2642
1321
------
162483
```

IX. When the product of any of the particular figures exceeds ten, place the exceſs under the line as before, and for every ten that it ſo exceeds, keep one in mind to be added to the next Rank.

Example, 3084 being given to be multiplied by 36, the work will ſtand thus; for 6 times 4 being 24, I write 4 under the line, and reſerve 2 in mind for the two tens; then I ſay 6 times 8 is 48, unto which if I add 2 kept in mind, the whole is 50, wherefore ſubſcribing 0 in the next rank under the line 0 (becauſe there is no exceſs of 50 above 5 tens) I reſerve 5 in mind for the 5 tens; again, I ſay 6 times nothing

```
  3084
    36
 -----
 18504
 9252
------
111024
```


nothing is nothing, to which adding 5 that I kept in mind, the whole will be but 5, which I likewiſe ſubſcribe under the line in the next rank; again 6 times 3 is 18, which (in regard 3 is the laſt figure of the multiplicand) I write wholly down; ſo that the particular product ariſing from the multiplying figure 6 is 18504: in like manner proceeding with the multiplying figure 3, the particular product ariſing will be 9252. Laſtly, theſe ſeveral products being placed in due order, and added together (after the manner of the 8th Rule of this Chapter) will give 111024, which is the total product ariſing from the multiplication of 3084 by 36, and the operation will ſtand as in the Margent. After the ſame manner if 5073 be given to be multiplied by 256, the product will be found to be 1298688, and the operation will ſtand as you ſee in the example.

```
   5073
    256
  -----
  30438
 25365
10146
-------
1298688
```

X. When the two numbers given to be multiplied, do one or both of them end with a Cypher or Cyphers towards the right hand, multiply the ſignificant figures in both numbers, one by the other, neglecting ſuch Cyphers, and when the multiplication of the ſignificant figures is finiſhed, annex on the right hand of the number produced by the multiplication; the Cypher or Cyphers with which one or both of the numbers firſt given did end ſo will the whole give you the true product demanded: *Example*, 43100 being given to be multiplied by 15000 the product will be found for omitting the Cyphers which ſtand in

```
    43100
    15000
---------
 2155
431
---------
646500000
646500000
```

in the laſt places towards the right hand as well in the multiplicand as the multiplicator, I multiply the ſignificant figures 431, but the figures 15 (according to the former rules,) ſo there will ariſe 6465, to which annexing on the right hand all the Cyphers before omitted, the true product will be 646500000: more Examples hereof are theſe following.

```
   43125          5108000
    1500              125
 -------         --------
  215625            25540
 43125             10216
 -------          5108
64687500         --------
                638500000
```

XI. When in the multiplicator Cyphers are included between ſignificant figures, multiply by the ſaid ſignificant figures, neglecting ſuch Cyphers or Cypher, but obſerve diligently to ſet the particular products of the ſignificant figures in their due places according to the 8th Rule of this Chapter. So if 56324 be given to be multiplied by 20006, I firſt multiply the whole multiplicand 56324 by 6, and place the product orderly underneath the line, then paſſing over the three Cyphers, I multiply 56324 by 2 and place 8 (which is the firſt exceſs of this particular product) directly under the multiplying figure 2, and the reſt in their order, ſo at laſt the true product) will be found 1126817944, and the work will ſtand as you ſee in the Example.

```
     56324
     20006
----------
    337944
112948
----------
1126817944
```

More

More Examples hereof are theſe that follow.

```
  3094          23765
   104          10302
------      ---------
 12376          47530
3094          71295
------       23765
321776      ---------
            244827030
```

Note, That one of the principal *cautions* to be obſerved in Multiplication, is the due placing of the Particular products ariſing by each multiplying figure; and that may be performed either by taking care to place the firſt figure or Cypher which ariſeth in each Product under the reſpective multiplying figure; or at leaſt the firſt place ariſing in the ſecond Product muſt ſtand under the ſecond place of the firſt Product, and the firſt place of the third particular Product under the third place of the firſt, *&c.*

XII. When a number is given to be multiplied by a number that conſiſts of 1 (or an unit) in the firſt place towards the left hand, and a Cypher or Cyphers on the right hand of ſuch an unit (ſuch are 10, 100, 1000, 10000, &c. the multiplication is performed by annexing the Cypher or Cyphers of the multiplicator at the end (to wit on the right hand) of the multiplicand; ſo if 326 be given to be multiplied by 10, the Product is 3260; if by 100, the Product is 32600; if by 1000, the Product is 326000; in like manner if 170 be multiplied by 10, the Product is 1700; if by 100, 17000, &c.

XIII. When more numbers than two are given to be multiplied one by the other, that kind of Multiplication *Continual Multiplication.*

is

is called *Continual*, and is thus performed, *viz.* first multiply any two of the numbers given one by the other, then multiply the Product by another of the numbers given, and this product by the fourth number given (if there be so many) and in that order till every one of the given numbers hath been made a Multiplicator, so the last product is the true product required. *Example*, If 4, 18, and 22 were given to be multiplied *continually*, first 18 multiplied by 4 produceth 72, which multiplied by 22 (the third number) produceth 1584, the last product or number required. See the work in the Margent. The proof of Multiplication is by Division as will appear by the next Chapter.

```
 18
  4
----
 72 Prod. 1.
 22
----
 144
144
----
1584 Prod. 2.
```

CHAP. VI.

Division by whole Numbers.

I. Division is that by which we discover, how often one number is contained in another; or (which is the same) it sheweth how to divide a number propounded into as many equal parts as you please.

II. In Division there are always three remarkable numbers which are commonly called by these names, the *Dividend*, the *Divisor*, and the *Quotient*.

III. The *Dividend* is the number given to be divided into equal parts.

IV. The Divisor is the number by which the Dividend is to be divided; that is, it is the number which declareth into how many equal parts the dividend must be divided.

V. The Quotient is the number arising from the division, and sheweth one of the equal parts required; so if 15 were given to be divided by 5, or into 5 equal parts, the number arising, or one of the equal parts will be 3, for 5 is found three times in 15: And here 15 is the *Dividend*, 5 the *Divisor*, and 3 the *Quotient*.

Division by a single figure.

VI. Division being the hardest lesson in Arithmetick, must be heedfully intended by the Learner, for whose ease I shall use my utmost endeavours to make the way smooth by Rules and Examples, beginning with the easiest first, which will be in that case when the Divisor consists of one figure only; for example, Let it be required to divide 192, by 8, or 192 pounds into 8 equal parts or shares; here 192 is the *Dividend*, 8 is the *Divisor*, and the *Quotient* or one of the equal parts is sought.

What the dividual is.

VII. Place a crooked line at each end of the *Dividend*, that on the left hand serving for the place of the *Divisor*, and that on the right for the *Quotient*; then if the *Divisor* be a single figure, subscribe a point under the first figure of the *Dividend* towards the left hand, if such first figure be either equal unto, or greater than the *Divisor*, but if such first figure be less than the *Divisor*, put a point under the next place of the *Dividend*; which number so distinguished by the point may be called the *Dividual*; so in the example given

8) 192 (

given in the 6th Rule, 192 being the *Dividend* and 8 the *Diviſor*, I ſubſcribe a point under 9, not under 1, becauſe it is leſs than the *Diviſor*. This done the *Dividual*, or number whereof the queſtion muſt be asked, is 19.

VIII. Having thus prepared the numbers, ask how often the *Diviſor* is contained in the *Dividual*, and write the number which anſwers the queſtion in the *Quotient*; then multiply the *Diviſor* by the number placed in the *Quotient*, and ſubſcribe the product underneath the *Dividual*. Laſtly, having drawn a line under the product, ſubtract it from the *Dividual*, and ſubſcribe the remainder orderly underneath the line. So demanding how many times the *Diviſor* 8 is found in the *Dividual* 19, the anſwer is two times, wherefore I write 2 in the *Quotient*; then multiplying the *Diviſor* 8 by 2 (the number placed in the *Quotient*) the product is 16, which I ſubſcribe orderly under the *Dividual* 19; and after a line is drawn underneath the product 16, I ſubtract it from the *Dividual* 19, and place the remainder 3 underneath the line.

```
8) 192 (2
    16
   ----
     3
```

IX. Put another point under the next place of the *dividend* towards the right hand, and bring down the Figure or Cypher ſtanding in that place to the remainder; that is, ſet it next after it, ſo the whole will be a new *Dividual*: Thus a point being placed under 2 which ſtands in the next place of the *Dividend*, I write 2 next after (to wit on the right hand of) the remainder 3, ſo is 32 a new *Dividual*, or number whereof the ſecond queſtion muſt be asked, and the work will ſtand as you ſee in the Example.

```
8) 192 (2
    16
   ----
    32
```

X. A

X. A new *Dividual* being ſet apart, renew the queſtion and proceed according to the 8th Rule of this Chapter. Thus demanding how often the *Diviſor* 8 is found in the *dividual* 32, the anſwer is four times; wherefore I write 4 in the *Quotient*, then multiplying the *Diviſor* 8 by four (the figure laſt placed in the *Quotient*) the product is 32, which I ſubſcribe under the *Dividual* 32, and after a line is drawn underneath, I ſubtract the product 32 from the *Dividual* 32, & there being no remainder, I ſubſcribe o under the line, ſo the whole work being finiſht, the *Quotient* is found to be 24, and the operation ſtands as you ſee in the Example; wherefore I conclude, if 192 pounds be equally divided amongſt 8 perſons, the ſhare of each perſon will be 24 pounds.

```
8(192(24
  16
  ----
  32
  32
  ----
   o
```

A ſecond Example. Let it be required to divide 936 pounds into 9 equal parts; having diſtinguiſhed the firſt *Dividual* by a point, (according to the 7th Rule of this Chapter) I demand how often the *Diviſor* 9 is found in the *Dividual* 9, and finding it once contained in it, I write 1 in the *Quotient*; then multiplying the *Diviſor* 9 by 1, the product is 9, which I ſubſcribe under the *Dividual* 9; after this, a line being drawn under the product 9, I ſubtract it from the *Dividual* 9, and there being no remainder, I place a o underneath the line, as you ſee in the Example.

```
9) 936 (1
   9
   ----
   o
```

Again placing a point under 3 which ſtands in the next place of the *Dividend*, I tranſcribe the ſaid 3 next after the remainder o for a new *dividual*, then asking

```
9)936(10
  9
  ----
  o3
```


how

how often the *Divisor* 9 is contained in the *Dividual* 3, and not finding it once contained therein, I write o in the *Quotient*, and now because the product which ought to arise from the Multiplication of the *Divisor* by o (the Cypher last placed in the *Quotient*) amounts to o, the *Dividual* 3, out of which that product should have been subtracted, remains the same without alteration; wherefore after a point is subscribed under six, the next place of the *Dividend*, I annex 6 to the *Dividual* 3, so there will be a new *Dividual*, to wit 36, then demanding how often the *Divisor* 9 is found in the *Dividual* 36, the answer will be 4 times; wherefore I place 4 in the *Quotient*, and multiplying the *Divisor* 9 by 4, the product is 36, which I subscribe under, and subtract from the *dividual* 36, so the remainder is o, thus the whole work being finisht, the *Quotient* is found to be 104, as you see in the *Example*; wherefore I conclude, if 936*l.* be divided equally amongst 9 persons, the share of each will be 104*l.* In like manner if 296163 be divided by 7, the *Quotient* will be 42309.

```
9) 936 (104
    ...
    9
    ---
    036
     36
    ---
      0
```

The substance of division by what method soever.

The whole work of Division is briefly contained in this following Verse.

Dic quot, multiplica, subduc, transferque secundum.

Or thus,

First you must ask how oft, in Quotient answer make;
Then multiply, subtract, a new Dividual take.

A Compendious way of dividing by a single figure.

XI. When in the Division the *Divisor* consists of a single Figure only, the *Quotient* may be written down,

down, and all the operation performed in mind, without writing down any part thereof; so 82506 being given to be halfed or divided into two equal parts, the work will be thus, The *Divisor* 2 is found in 8 four times; in 2 once; in 5 twice; and there will remain 1, which one being supposed to stand before (to wit, on the left hand of) the Cypher, makes 10, then I say 2 is found in 10 five times; and last of all in 6 three times; so that the true *Quotient* or one half of the given number 82506 is found to be 41253.

2) 82506 (41253

In like manner if 82506 be given to be divided by 3, or into 3 equal parts, the work will be thus, the *Divisor* 3 is found in 8 twice, and there will remain 2, which 2 being supposed to stand before (to wit, on the left hand of) the following 2, makes 22; then I say 3 is found in 22, 7 times; in 15, 5 times, in o not at all, and lastly in 6 twice; so that the true *Quotient* or one of the 3 equal parts required is 27502. After the same manner may division be wrought by any single figure, without much charge to the memory.

3) 82506 (27502

A note, concerning the remainder after the Division is ended, if any happen.

Note, here the *Learner* may ask what shall be done with the last remainder, if any happen, when the *Division* is finished? For a full answer to this, I referr the *Reader* to the Note in the fifth *Rule* of the seventh *Chapter*; yet I shall here propound an example where the said case happens, *viz.* Let it be required to divide 351 by 8, or 351 *pounds* equally amongst 8 *persons*; now if the operation be prosecuted according to the former rules, the *Quotient* will be found to be 43, and after the *Division* is

8) 351 (43

is finisht, there will remain 7, that is, each person must have 43 pounds, and there will be an overplus of 7 pounds, which must be also divided equally among the 8 persons, but that cannot be done till the 7 pounds be reduced into shillings, and then those shillings must be divided by 8 to give every person his due share of the shillings contained in the said 7 pounds; again, if there yet remain any surplusage of shillings, they must be reduced to pence, which must also be divided by 8, to give every person his due share of pence: so that when this question is fully answered each persons share will appear to be 43*l.*—17*s.*—6*d.* But how the before-mentioned *Reduction* is performed, will be made manifest in the fifth Rule of the next Chapter.

Division by two or more figures, the first and easiest Method.

XII. When the divisor consists of two, three or how many places soever, the operation is more difficult than the former; but depends upon the same grounds, and therefore the Learner being well vers'd in the preceeding method of dividing by a single figure, will the more readily understand these that follow, which are two, whereof the first is the easier, but the latter more expeditious, and that which indeed is principally to be aimed at: For an example of the former, let it be required to divide 4112772 by 708, or (which is the same) to divide 4112772 into 708 equal parts.

First, a *Table* is to be made to shew at first sight any *Multiple* or product of the *Divisor*, it being taken twice, thrice, or any number of times under ten, so having first written down the *Divisor* it self 708, and drawn a line on the right hand thereof, I place 1 on the right hand of the line directly against

~~If ye multiplicand after operation exceed ye~~ [illegible]
[illegible] you ~~may conclude that~~ [illegible]
[illegible] of the multiplicand after operation exceed ye greater
of the multiplicand after operation you may thence conclude that ye
number of ye table [illegible] is contained nine times in the ninth

against the *Divisor*; then underneath the *Divisor* 708 I subscribe the double thereof, which is 1416, and place the figure 2 directly against the said double, to wit, on the other side of the line. Again, adding 1416 (to wit the double, to the *Divisor*) to the *Divisor* it self 708, the sum is 2124 for the triple of the *Divisor*, this triple I subscribe under the double, and place 3 on the other side of the line right against the triple. Again adding 2124 (the triple of the *Divisor*) to the *Divisor* 708, I find 2832 for the quadruple of the *Divisor*, which quadruple I subscribe under the triple, and proceeding in like manner, at last the table is finisht, which readily shews the *Divisor*, with the *duple*, *triple*, *quadruple*, *quintuple*, *sextuple*, *septuple*, *octuple*, and *noncuple* of the *Divisor*.

The Devisor.	708	1
Multiples of the Divisor.	1416	2
	2124	3
	2832	4
	3540	5
	4248	6
	4956	7
	5664	8
	6372	9

Now for a proof of the said Table, adding the last number thereof, to wit, 6372 (which was found to be nine times the *Divisor*) to the *Divisor* 708, I find the sum to be 7080, which (by the 12th Rule of the fifth Chap.) is evident ten times the *Divisor*; wherefore I conclude that the Table is true, in regard that the last number thereof is derived from all the superior numbers.

The Table of Multiples or Products of the *Divisor* being thus prepared, write down the *dividend* on the right hand of the *Divisor*; then distinguish by a point so many of the foremost places of the *Dividend* towards the left hand, as are either equal in value (being consider'd apart) to the *Divisor*, or which

708	1	) 4112772 (5809
1416	2	....
2124	3	3540
2832	4	5727
3540	5	...
4248	6	5664
4956	7	6372
5664	8	6372
6372	9	0

which being greater, yet come neareſt to the value thereof, thus I ſubſcribe a point under 2, thereby ſetting apart 4112, being the feweſt of the foremoſt places which will contain the *Diviſor* 708, ſo is 4112 the *dividual* (or number whereof the firſt queſtion muſt be asked;) then demanding how often the *Diviſor* 708 is contained in the *dividual* 4112, the anſwer will be found by the *Table* to be five times, for looking in the *Table* I cannot find the *dividual* exactly, but I ſee that 6 times the *Diviſor* is the next greater than the *dividual* 4112, and five times is the next leſſer; wherefore I write 5 in the *Quotient*, and the number in the *Table* which ſtands againſt 5, to wit, 3540 I ſubſcribe under the *dividual* 4112, then having drawn a line underneath, I ſubtract 3540 (which is five times the *Diviſor*) from the *dividual* 4112, and ſubſcribe the remainder 572 underneath the line; that done, I put a point under the next place of the *dividend* towards the right hand, and becauſe the figure 7 ſtands in that place, I tranſcribe 7 next after the remainder 572, ſo there is 5727 for a new *dividual*.

Then demanding how often the *Diviſor* 708 is contained in the *dividual* 5727, the anſwer will be found by the *Table* to be 8 times, for looking in the *Table* I find that 9 times the *Diviſor* is the next greater, but 8 times is the next leſſer than the *dividual*, wherefore I write 8 in the *Quotient*, and the

the

the number in the *Table*, which ſtands againſt 8, to wit, 5664 I ſubſcribe under, and ſubtract from the *dividual* 5727, placing the remainder 63 underneath the line.

Again, I put a point under the next place of the *dividend*, where I find the figure 7, and therefore tranſcribing 7 next after the remainder 63, the new *dividual* will be 637, then demanding how often the *Diviſor* 708 is contained in the *dividual* 637, and not finding it once contain'd therein, I write 0 in the *Quotient*, and ſince in this caſe (that is, when a Cypher anſwers the queſtion) the *dividual* remains the ſame without alteration, the figure or Cypher ſtanding in the next place of the *dividend* is to be tranſcribed after the *dividual* for a new *dividual*, ſo writing 2 next after 637, the new *dividual* is 6372, wherefore demanding how often the *Diviſor* 708 is contain'd in 6372, I find by the *Table* it is contain'd in it 9 times, wherefore writing 9 in the *Quotient*, and placing the number which ſtands againſt 9 in the *Table*, to wit, 6372 under the *dividual* 6372, and ſubtracting it from the *dividual* there will remain 0. Wherefore I conclude if 4112772 be divided by 708, or into 708 equal parts, the true *Quotient* or one of the equal parts required is 5809.

In like manner if 20304 be divided by 188, that is into 188 equal parts, the *quotient* ariſing, or one of thoſe equal parts will be 108, and the operation will ſtand as you ſee.

Diviſor.	188	1	)
Multiples of the Diviſor.	376	2	20304 (108
	564	3	...
	752	4	188
	940	5	
	1128	6	1504
	1316	7	1504
	1504	8	0
	1692	9	

The

The preceeding method of *Division* by the help of a *Table* of the *Multiples* or products of the *Divisor*, as it is most easie, so in some Cases (namely, where the *Divisor* is great, and a *Quotient* of many places is required, as in calculating *Tables* of *Interest*, *Astronomical Tables*, and such like) it excels all other ways of *Division*, both in respect of certainty and expedition, but for common practice it is too tedious, and therefore I shall proceed to the choicest practical method.

The latter and choicest practical Method of Division, when the Divisor consists of many places.

XIII. *I now come to the last and principal method of Division, when the Divisor consists of many places, which to such as have the Table of Multiplication by heart will not be difficult*; For example, let 56304 be a number given to be divided by 184, that is, into 184 equal parts and the *Quotient* or one of the equal parts is required.

First, distinguish by a point (as before) so many of the foremost places of the *dividend* towards the left hand, as are either equal in value (when they are consider'd apart) to the *Divisor*, or else, which being greater, yet come nearest unto it, thus I subscribe a point under the figure 3, thereby setting apart 563, being the fewest of the foremost places which will contain the *Divisor*; so is 563 the *dividual*, or number whereof the first question must be asked.

184) 56304 (

Having thus prepar'd the numbers, I demand how often the *Divisor* 184 is contained in the *dividual* 563; and since to answer this question and such like, there is a necessity of trial, it will be requisite to shew how this trial may fitly be made; first therefore

fore compare the number of places in the *dividual* with the number of places in the *Divisor*, and when the number of places is the same in both, let it be asked how often the first or extream figure of the *Divisor* towards the left hand is contained in the first figure of the *dividual* towards the same hand; so here demanding how often 1 is contained in 5, the answer is 5 times; whence I inferr that the *Divisor* 184 is not contained oftener than 5 times in the *dividual* 563 (for 6 times 184 is manifestly greater than 563) but whether it be contained 5 times in it or not, examination must be made either by multiplying (in some by place) the *Divisor* 184 by the said 5, and comparing the product with the *dividual*, 563; or else thus, saying 5 times 1 (to wit the 1 in the *Divisor*) is contained in 5, to wit, the first figure of the *dividual* 563, 5 times, but then 8 the following figure of the *Divisor*, cannot be found 5 times in 6, the following figure of the *dividend*, and consequently the *Divisor* 184 is not contained 5 times in the *dividual* 563; wherefore I make another trial to see whether it may be contained 4 times in it or not, saying 4 times 1 is 4 which is found in 5, and there will remain 1, but then 4 times 8, which is 32, cannot be had in 16 (for the 1 before remaining being supposed to stand on the left hand of 6 maketh 16) hence I conclude again, that the *Divisor* 184 is not contained 4 times in the *dividual* 563; wherefore I make another trial to see whether it may be contained 3 times in it or not, saying 3 times 1 is 3, which is found in 5, and there will remain 2, again, 3 times 8 is 24, which is found in 26 (for the 2 before remaining being supposed to stand before the 6 in the

the *dividual* makes 26) and there will remain 2: Lastly, three times 4 is 12, which is likewise found in 23, (for the 2 remaining being supposed to stand before the 3 in the *dividual* makes 23) whereby I see that the *Divisor* 184 is contained 3 times in the *dividual* 563, wherefore I write 3 in the *Quotient*, and proceeding according to the 8th *Rule* of this *Chapter*, I multiply the *Divisor* 184 by 3 (the figure placed in the *Quotient*) so the *Product*, is 552, which I subscribe orderly underneath the *dividual* 563, then having drawn a line underneath the said *Product*, I subtract it from the *dividual*, and subscribe the remainder which is 11 under the line.

```
184) 56304 (3
     552
     ———
      11
```

Again according to the 9th *Rule* of this *Chapter*, I bring down 0 which stands in the next Place of the *dividend*, to the remainder 11, so there is 110 for a new *dividual*, then demanding how often the *Divisor* 184 is found in the *dividual* 110, and not finding it once contained in it, I write 0 in the *Quotient* (which is to be done as often as the question is answered by nothing;) now because the *Product* arising from the multiplication of the *Divisor* by 0 (the Cypher last placed in the *Quotient*) amounts to 0; the *dividual* 110 out of which that *Product* should be subtracted, remains the same without alteration; wherefore after a point is subscribed under 4 the following place of the *dividend*, I annex 4 to the last *dividual* 110, for there will be a new *dividual*, to wit, 1104; and here the question at large is to know how often 184 is found in 1104: but to lessen the

```
184) 56304 (306
       ...
     552
     ————
      1104
      1104
         0
```


the trial, because the *dividual* consists of one place more than is in the *Divisor*, it must be asked how often the first figure of the *Divisor* on the left hand is contained in the two foremost places of the *dividual* towards the left hand, *viz.* I demand how often 1 is contained in 11, and although it may be had 11 times, yet I need never begin the trial above 9 times, therefore I make trial with 9, saying 9 times 1 is 9, which is found in 11, and there will remain 2; but then 9 times 8 which is 72 cannot be found in 20 (20 because the 2 remaining being supposed to stand before 0 in the *dividual* makes 20) therefore I make trial with 8 saying 8 times 1 is 8, which is found in 11, and there will remain 3, but then 8 times 8 cannot be had in 30 (30 because the 3 remaining being supposed to stand before the 0 or Cypher makes 30) therefore I make trial with 7, saying 7 times 1 is 7, which is found in 11, and there will remain 4; but then 7 times 8 cannot be had in 40, therefore I make trial with 6, saying 6 times 1 is 6, which is found in 11, and there will remain 5; also 6 times 8 is 48, which is found in 50, and there will remain 2; lastly, 6 times 4 is 24, which is found in 24, whereby at length I see that the *Divisor* 184 is contained 6 times in the *Dividual* 1104, wherefore I write 6 in the *Quotient*, and proceeding according to the 8th *Rule* of this *Chapter*, I multiply the *Divisor* 184 by 6 (the figure last placed in the *Quotient*) so the *Product* is 1104, which being subscribed under and subtracted from the *dividual* 1104, the Remainder is 0, so at last I conclude that the *Quotient* sought is 306.

Note, If the figure assumed for the *Quotient* holds

holds good upon trial, as aforesaid, by two or three of the foremost places of the *dividual*, it will for the most part hold throughout the *dividual*; but this must be a perpetual Rule, that whensoever the *Product* of the multiplication of the *Divisor* by the figure placed in the *Quotient* happens to be greater than the *dividual*, from which it ought to be subtracted, such *Product* must be struck out of the work, and a lesser figure is to be placed in the Quotient.

For a second *Example*, let it be required to divide 15114220 by 2987, or into 2987 equal parts.

First, the *Divisor* 2987 being greater than 1511, (to wit, the four foremost places of the Dividend) I set a point under 4, thereby setting apart 15114 for a *Dividual*; then because the *Dividual* consists of one place more than the *Divisor*, I ask how often 2 (the first figure of the *Divisor* towards the left hand) is contained in 15 the two foremost places of the *dividual*) and finding the answer to be 7 times, I inferr thence that the *Divisor* 2987 cannot be contained more than 7 times in the *dividual* 15114; but whether it will be contained 7 times in it or not, examination must be made, either by multiplying 2987 by 7 (in some by-place) and comparing the Product with the *dividual* 15114, or else by the manner of trial before delivered in the last Example: so at length it will be discovered that the *Divisor* 2987 will not be found above 5 times in the *dividual* 15114; wherefore (according to the 8th *Rule* of this *Chapter*) writing 5 in the Quotient, and multiplying 2987 by 5, I sub-

```
2987) 15114220 (5
      14935
      -----
        179
```


subscribe the product of that multiplication, which is 14935, under the *dividual* 15114, then drawing a line underneath the said product, and subtracting it from the *dividual* 15114, I subscribe the remainder 179 under the line.

Again (according to the 9th *Rule* of this *Chapter*) I bring down 2, the next place of the Dividend, to the said Remainder 179, so the new Dividual will be 1792; that done, asking how often the *Divisor* 2987 is contained in the *dividual* 1792, and not finding it once contained in it, I write 0 in the Quotient; and here because the question is answered by 0, the next place of the *dividend*, to wit 2, is to be brought down to the *dividual* 1792, so the new *dividual* is 17922. Then renewing the question, and proceeding as before, at length the Division being finisht, the Quotient will be found 5060 exactly, without any Remainder; but if any Remainder had hapned after the subtraction of the last Product it must have been prosecuted according to the note before given in the example at the latter end of the 11th *Rule* of this *Chapter*.

```
2987) 15114220 (50
      14935
      -----
       1792
```

```
2987) 15114220 (5060
      14935
      -----
       17922
       17922
       -----
          00
```

In like manner if 120893950 be divided by 19999, or into 19999 equal parts, the *quotient*, or one of those equal parts, will be found 60450, and the work will stand as here you see.

This

```
19999) 1208939550 (60450
              .....
       119994
       ------
        89995
        79996
       ------
        99995
        99995
       ------
           00
```

This latter method of Division is to be preferr'd before any of the common ways of dividing by dashing out of figures, where the steps of the Division are so confounded (besides the burden upon the memory, by a promiscuous Multiplication and Division) that if any errour happen, it can hardly be corrected without beginning the work anew; But in the way before explained, the particular Multiplications, Subtractions, and Remainders, which belong to every figure of the Quotient, are so distinctly and clearly exprest, that if an errour happen, the work may easily be reformed.

How the number of places in the Quotient may be discovered.

XIV. So often as the question is repeated in Division, so many places there must be in the quotient (which may be discovered by the number of Points placed under the *dividend*) and so many times is one and the same kind of operation repeated, the substance whereof is contained in the Verse before-mentioned at the end of the tenth *Rule* of this *Chapter*.

A Compendious way of dividing by 10, 100, 1000, *&c.*

XV. When the *Divisor* consists of 1 or an unit in the extream place towards the left hand, and nothing but Cyphers towards the right, the division is performed by cutting off with a line so many places of the *Dividend* towards the right hand as the *Divisor* hath Cyphers; so the figures which

which stand on the left hand of the line, give the Quotient, and those cut off to the right (if they be significant figures) are to be proceeded with as a surplusage or overplus remaining, according to the Note at the end of the eleventh *Rule* of this *Chapter*. So if 4720*l.* were given to be divided equally amongst 10 persons, the share of each would be 472*l.* also if the said 4720*l.* were to be divided equally amongst 100 persons, the share of each would be 47 *l.* and there would be a surplusage or remainder of 20*l.* to be also subdivided amongst them, after the said 20*l.* are converted into shillings according to the fifth *Rule* of the next *Chapter*. Lastly, if the said 4720*l.* were to be divided amongst 1000 persons, the share of each would be 4*l.* and there would be a remainder of 720*l.* to be also divided as aforesaid. See the form of the Work in the *Margent*.

```
10)   472|0   (472
100)  47|20   (47
1000) 4|720   (4
```

Another Compendium in Division.

XVI. When the *Divisor* consists of any significant figure or figures in the first or foremost place or places towards the left hand, and nothing but a Cypher or Cyphers towards the right, cut off by a line so many places of the *Dividend* towards the right hand as the *Divisor* hath Cyphers towards the right; then divide the figures of the *Dividend*, which stand on the left hand of the line, by the figures in the *Divisor* which remain, when the said Cypher or Cyphers are omitted, remembring after the division is finished, to write down next after the last remainder the places of the *Dividend* which were first cut off: So if 36732 were given to be divided

divided by 20, the Quotient will be 1836, and there will remain 12, *viz.* if you cut off one place from the *Dividend* towards the right hand (because the *Divisor* ends with one Cypher) and then divide the rest, to wit, 3673 by 2 (according to the 11th *Rule* of this *Chapter*) there will arise in the Quotient 1836, and the last remainder after such division is finisht, will be 1, unto which if 2 (the figure first cut off from the *Dividend*) be annexed, the total remainder is 12.

```
      1
2|0) 3673|2 (1836
```

In like manner if 7456787, were given to be divided by 304000, the Quotient will be 24, and there will remain 160787; *viz.* If you cut off 3 places from the *Dividend* towards the right hand (3 places, because the *Divisor* ends with 3 Cyphers) and then divide 7456 by 304, there will arise in the Quotient 24, and the last remainder, after such division is finisht, will be 160, unto which if 787 (the places first cut off from the *Dividend*) be annexed, the total remainder or surplusage is 160787, which is to be proceeded with, as is directed in the Note at the latter end of the eleventh *Rule* of this *Chapter*.

```
304|000) 7456|787 (24
         608
     ----------------
         1376
         1216
     ----------------
         160787
```

XVII. Division and Multiplication do interchangeably prove one another; for in Division if you multiply the *Divisor* by the Quotient, the Product will be equal to the *Dividend*: So in the *Example* of the 13th *Rule* of this *Chapter*; if 184 the

The proof of Multiplication and Division.

the *Divisor* be multiplied by 306 the *Quotient*, the Product is 56304, which is the same with the *Dividend*; but when, after the whole Division is finished, any figures remain of the last Subtraction, add them likewise to the Product: So in the last *Example* of the 16th *Rule* of this *Chapter*, the *Divisor* 304000 being multiplied by the *Quotient* 24, produceth 7296000, unto which if you add the number remaining, to wit, 160787, the sum is 7456787, which is the same with the *Dividend*. Again in Multiplication, if the product be divided by the Multiplicator, the *Quotient* will give you the Multiplicand, or if the product be divided by the Multiplicand, the *Quotient* will give you the Multiplicator: So in the first Example of the 9th *Rule* of the last Chapter, if the product 111024 be divided by the Multiplicand 3084, the *Quotient* gives the Multiplicator 36.

There is also of Multiplication a *Common proof* argued from the Multiplicand, the Multiplicator and the Product by casting away nines, but by that way of Proof (though rightly used) *a false* Product will be affirmed to be true: Example, if 3462 be multiplied by 786, the true Product is 2721132; but if I say 4953132 or 3153132 is the Product (or many others which may be given) the proof by nines will confirm them to be true Products, though they are false, as will be evident to such as know the *Rule*, which I mention here only to set a brand upon it, that it may be avoided by all lovers of Truth.

CHAP. VII.

Reduction.

I. Forasmuch as in *Money*, there are diversities of kinds, *viz.* in *England*, *Pounds*, *Shillings*, *Pence*, and *Farthings*; also divers kinds of *Weights*, *Measures*, &c. as hath been fully declared in the second Chapter; and because it is often times required to find how many pieces of one kind of *Money* are equal in value to a given number of another (and so likewise of *Weights*, *Measures*, &c.) it will be convenient in this place to shew how that is performed, since thereby the *Rules* of *Multiplication* and *Division* before delivered will be exercis'd. This kind of operation is called *Reduction.*

II. Reduction is either descending or ascending.

III. Reduction descending is, when some Integers of a Number of greater denomination being given, it is required to find how many Integers of a lesser denomination are equal in value to that given number of the greater: As when it is required to find how many *shillings* are contained in 30*l.* Likewise how many *pence* in 320*s.* or how many *hours* in 365 *days*, &c.

IV. Reduction ascending is, when some Integers of a number of lesser denomination being given, it is required to find how many Integers of a greater denomination are equal in value to that given number of the lesser: As when it is required to find how many *pence* are contained in 500 *farthings*: likewise how many *shillings* in 348 *pence*: or how many *days* in 864 *hours*: &c.

V. Reduction descending is performed by *Multiplication*, for if the given number of Integers of a greater denomination be multiplied by a number, which expresseth how many Integers of the lesser are equal to one of the Integers given, the Product is the number of Integers of the lesser denomination required.

Reduction descending is performed by Multiplication.

So 230*l.* of English Money will be reduced into 4600 *s.* for if 230 be multiplied by 20 (the number of *shillings* which are equal to 1 *pound*) the product is 4600; in like manner 4600 *s.* will be reduced into 55200 *d.* for if 4600 be multiplied by 12 (the number of *pence* contained in 1 *shilling*) the product is 55200. Also 55200 *pence* being multiplied by 4 because 4 *farthings* make a *Peny*) are reduced into 220800 *Farthings*, as by the operation in the *Margent* is evident.

```
   230 Pounds.
    20
  ----------
  4600 Shillings.
    12
  ----------
   92
  46
  ----------
 55200 Pence.
     4
  ----------
220800 Farthings.
```

The like method is to be observed in *Weights*, *Measures*, &c. So 345 *Ounces Troy* are reduced into 6900 *Peny weights*, and 6900 *Peny weights* to 165600 *Grains*, as by the operation in the *Margent* you may see.

```
   345 Ounces.
    20
  ----------
  6900 Peny W.
    24
  ----------
  276
 138
  ----------
165600 Grains.
```

Note, By this *Rule* the Learner is furnished with Skill to resolve that case in Division, when the *Dividend* is less than the *Divisor*:

Compare this with the Note upon the last Example of the 11th Rule of the 6th Chapter.

Example, Let it be required to divide 7 *pounds* of English Money equally amongst 8 persons; here it is evident that the *Dividend* 7 is less than the *Divisor* 8; that is, the number of *pounds* is less than the number of Persons, and consequently each share must be less than a Pound; so that in effect it is required to find how many *Shillings* and *Pence* belong to each Person for his share: First, therefore reduce the 7 *Pounds* into *Shillings*, which will be 140, these divided by 8 give 17 *Shillings* to each Person, and there will yet be a remainder of 4 *Shillings* to be also equally divided into 8 parts, but these 4 *Shillings* must be first reduced into *Pence*, which will be 48, then dividing 48 by 8, the *Quotient* will give 6 *Pence* more to every Person: so at last it appears that if 7 *Pounds* of English Money be equally divided into 8 parts, the entire *Quotient* (or one of the equal shares) will be 17 *Shillings* and 6 *Pence*.

In like manner, if 354 *Pounds* of English Money be given to be divided equally amongst 125 Persons, the share of each will be found to be 2 *Pounds*, 16 *Shillings*, 7 *Pence*, 2 *Farthings*, and somewhat more, but the parts of a *Farthing* being of no moment (and not properly to be handled in this place) are neglected.

Compare these two Examples with the last Example of the eleventh Rule of the sixth Chapter.

In *Reduction* descending, the Learner may receive help by the subsequent *Tables*.

1. *Of English Money.*

Pounds	*Multiplied by*	20	*Produce*	*Shillings.*
Shillings.		12		*Pence.*
Pence		4		*Farthings.*

2. *Of Troy Weight.*

Pounds	*Multiplied by*	12	*Produce*	*Ounces.*
Ounces		20		*Peny Weights.*
Peny Weight		24		*Grains.*

Also in *Apothecaries Weights.*

Ounces Troy	*Multiplied by*	8	*Produce*	*Drams.*
Drams		3		*Scruples.*
Scruples		20		*Grains.*

3. *Of Averdupois Weights.*

Hundred W.	*Multip. by*	4	*Produce*	*Quarters.*
Quarters		28		*Pounds.*
Pounds		16		*Ounces.*
Ounces		16		*Drams.*

4. *Of Liquid Measures.*

Hogsheads	*Multip. by*	63	*Produce*	*Gallons.*
Gallons		2		*Pottles.*
Pottles		2		*Quarts.*
Quarts		2		*Pints.*

5. *Of Dry Measures.*

	Multiplied by	Produce
Quarters	8	*Bushels.*
Bushels	4	*Pecks.*
Pecks	2	*Gallons.*
Gallons	2	*Pottles.*
Pottles	2	*Quarts.*
Quarts	2	*Pints.*

6. *Of Long Measures.*

	Multiplied by	Produce
English Miles	8	*Furlongs.*
Furlongs	220	*Yards.*
Yards	3	*Feet.*
Feet	12	*Inches.*
Inches	3	*Barley Corns.*

Also,

	Mult. by	Produce
Yards or *Ells.*	4	*Quarters.*
Quarters	4	*Nails.*

7. *Of Superficial Measures of Land.*

	Mult. by	Produce
Acres	4	*Roods.*
Roods	40	*Perches or Poles*

8. *Of Time.*

	Mult. by	Produce
Weeks	7	*Days.*
Days	24	*Hours.*
Hours	60	*Minutes.*

To reduce Integers of divers Denominations into the lowest of those Denominations.

VI. Integers of divers denominations may be reduced into the last of those denominations according to the fifth *Rule* aforegoing, by descending orderly to the next inferiour denomination,

nation, and adding to each Product such Integers (if there be any) which are of the same name.

So 12 *Pounds*, 13 *shillings*, and 10 *pence* may be reduced into 3046 *Pence* in this manner, *viz.* 12*l.* multiplied by 20 (because 20*s.* make one *l.*) produce 240 *Shillings*, unto which adding 13 *s.* the sum is 253 *Shillings.* Again, 253 *s.* multiplied by 12 (because 1 *shilling* is equal to 12 *Pence*) produce 3036 *Pence*, unto which if 10 *Pence* be added the sum is 3046 *Pence*, as by the operation in the *Margent* is manifest.

```
 l.   s.   d.
 12—13—10.
 20
 ---------
 240
add 13
 ---------
 253 Shillings.
 12
 ---------
 506
 253
 ---------
 3036
add 10
 ---------
 3046 Pence.
```

But after that general Method is well understood, the work of the last Example, and such like may be contracted thus; *viz.* To convert 12 *Pounds*, 13 *Shillings*, 10 *Pence*, all into *Pence*, First 12 multiplied by 0, (which stands in the units place of 20) produceth 0, but instead of 0, I write down 3 under the line (to wit, the three that stands in the units place of the 13 *shillings* in the sum propounded;) Then I proceed to multiply 12 by 2, saying twice 2 is 4, to which adding 1 (for the ten in the said 13 *Shillings*) it makes 5, which I set on the left hand of 3 before written; Lastly, twice 1 is 2, which I set on the left hand of 5; And so 12 *Pounds*, 13 *Shillings* and 10 *Pence* are converted into 253 *Shillings.*

```
 l.   s.   d.
 12—13—10
 20
 ---------
253 Shillings.
 12
 ---------
 516
253
 ---------
3046 Pence.
```

It

It remains to multiply the said 253 by 12 (because 12 *Pence* makes 1 *Shilling*) and to add 10 to the Product, which may be done thus; First, twice 3 is 6, to which adding 10 (to wit, 10 *Pence* in the Sum first propounded) it makes 16, wherefore (according to the *Rule* of Multiplication) I set 6 under the line, and keep 1 in mind; Again, twice 5 with 1 in mind making 11, I write down 1, and keep 1 in mind; likewise twice 2 and 1 in mind making 5, I write down 5; Then 253 multiplied by 1 makes 253, which I set orderly under 516; Lastly, those two Products added together make 3046, which is the number of *Pence* contained in 12*l.*—13*s.*—10*d.* as before was found out by the general method.

So 35 *Ounces*, 16 *Peny Weights*, and 12 *Grains Troy* will be reduced into 17196 *Grains*.

VII. Reduction ascending is performed by Division, for if the number of Integers given be divided by such a number of the same Integers, as are equal to one of the Integers required, the *Quotient* is the number of Integers sought.

Reduction ascending is performed by Division.

So 220800 *Farthings* being divided by 4 (the number of *Farthings* in a *Peny*) give 55200 *Pence* in the *Quotient*; In like manner if 55200 *Pence* be divided by 12 (the number of *Pence* in a *Shilling*) the *Quotient* is 4600 *Shillings*. Lastly, 4600 *Shillings* being divided by 20 (because 20*s.* make a *Pound sterling*) the *quotient* is 230 *Pounds sterling*) which are equal to 220800 *Farthings* first given. The operation is as followeth.


```
               12)      20)
4) 220800 (55200 (460|0 (230l.
            48
            --
            72
            72
            --
            00
```

In like manner, 34268 *Grains Troy* will be reduced to 5*l.* 11 *Ounces*, 7 *Peny Weight*, and 20 *Grains*. This kind of *Reduction* may be made the easier to the Learner by the following *Tables*.

1. *Of English Money.*

Farthings	*Divi. by*	4	*give*	*Pence.*
Pence		12		*Shillings.*
Shillings.		20		*Pounds.*

2. *Of Troy Weights.*

Grains	*Divi. by*	24	*give*	*Peny Weights.*
Peny Weight		20		*Ounces.*
Ounces		12		*Pounds Troy.*

Also in *Apothecaries Weights.*

Grains	*Divi. by*	20	*give*	*Scruples.*
Scruples		3		*Drams.*
Drams		8		*Ounces Troy.*

3. *Of Averdupois Weight.*

Drams	*Divided by*	16	*give*	*Ounces.*
Ounces		16		*Pounds.*
Pounds		28		*Quarters of C.*
Quarters		4		*Hund. Weight.*

4. *Of Liquid Measures.*

Pints	Divided by	2	give	*Quarts.*
Quarts		2		*Pottles.*
Pottles		2		*Gallons.*
Gallons		63		*Hogsheads.*

5. *Of Dry Measures.*

Pints	Divided by	2	give	*Quarts.*
Quarts		2		*Pottles.*
Pottles		2		*Gallons.*
Gallons		2		*Pecks.*
Pecks		4		*Bushels.*
Bushels		8		*Quarters.*

6. *Of Long Measures.*

Barly Corns	Divided by	3	give	*Inches.*
Inches		12		*Feet.*
Feet		3		*Yards.*
Yards		220		*Furlongs.*
Furlongs		8		*English Miles.*

Also

Nails	Divi. by	4	give	*Quarters of Yards, also of Ells.*
Quarters		4		*Yards, also Ells.*

Of Superficial Measures of Land.

Perches or Poles	Divi. by	40	give	*Roods or Quarters of Acres.*
Roods		4		*Acres.*

8. *Of Time.*

Minutes	Divi. by	60	give	*Hours.*
Hours		24		*Days.*
Days		7		*Weeks.*

Note,

Note, that if after Division is finisht in Reduction ascending there be any remainder, it is of the same denomination with the Dividend.

Note also that Reduction descending and ascending do mutually prove one another, by inverting the question; for as in 56 *Pounds* sterling, there will be found 53760 *Farthings*, by Reduction descending; So for Proof thereof, 53760 *Farthings* will be reduced to 56 *Pounds*, by Reduction ascending.

Questions to exercise Reduction.

1. In 257*l.* how many shillings? *Answer*, 5140.
2. In 3076*l.* how many shillings? *Answer*, 61520.
3. In 902 shillings how many pence? *An.* 10824.
4. In 2179 shillings how many farthings? *Answer*, 104592.
5. In 49*l.*—13*s.*—7*d.* how many pence? *Answer*, 11923.
6. In 2053*l.*—14*s.*—9*d.*—2*f.* how many farthings? *Answw.* 1971590.
7. In 354*lb.* of Troy weight how many grains, (of Gold-smiths weight?) *Answer*, 2039040.
8. In 300 English miles how many yards? *Answer*, 528000.
9. In 1 English mile, how many barley corns length? *Answ.* 190080.
10. In 560 Acres how many Perches? *Answer*, 89600.
11. In 225 Acres, 3 Roods, and 30 Perches, how many Perches? *Answ.* 36150.
12. In 11923 pence how many pounds? *Answer*, 49*l.*—13*s.*—7*d.*

13. In

13. In 5764684 farthings how many pounds? *Answ.* 6004*l.*—17*s.*—7*d.*

14. In 234678 Perches, how many Acres? *Answer*, 1466 Acres, 2 Roods, and 38 Perches.

15. In 525960 minutes of an hour, how many days? *Answ.* 365 days and 6 hours (or 1 year very near.)

16. In 10080 Pints, how many Hogsheads? *Answ.* 20.

17. In 34678 grains of Apothecaries weight how many ounces Troy? *Answ.* 72 Ounces, 1 Dram, 2 Scruples, and 18 Grains.

18. In 106735 Pints of wheat, how many Quarters? *Answ.* 208 Quarters, 3 Bushels, 2 Pecks, 1 Gallon, 1 Pottle, 1 Quart, 1 Pint.

19. In 3969301 Barley corns length, how many Miles? *Answ.* 20 Miles, 7 Furlongs, 12 Yards, 2 Feet, 4 Inches, and 1 Barley corns length.

20. In 1900800 Barley corns length, how many Miles? *Ans.* 10.

CHAP.

CHAP. VIII.

Of the Rule of Three Direct.

I. THE Rule of Three is so called, because by three numbers known or given, it teacheth to find a fourth unknown; it is also called the Golden Rule for the excellency thereof; Lastly, it is called the Rule of proportion for the reason hereafter declared.

II. The Rule of Three is either single or compound.

The Rule of Three.

III. The single Rule is, when three terms or numbers are propounded, and a fourth proportional unto them is demanded.

IV. Four numbers are said to be proportionals, when the first containeth the second, or is contained by the second in the same manner as the third containeth the fourth, or is contained by the fourth: so these 4 numbers are said to be Proportionals, 8, 4, 12, 6, for as 8 containeth 4 twice, so doth 12 contain 6 twice, and therefore 8 is said to have such proportion to 4 as 12 hath to 6; likewise these are Proportionals, 4, 8, 6, 12. For as 4 is the half of 8, so is 6 the half of 12, and therefore 4 is said to have such proportion to 8 as 6 hath to 12.

The divers denominations of the terms in the Rule of Three.

V. The terms or numbers of the Rule of Three (to wit, the three numbers given, and the fourth sought) consist of two different denominations, *viz.* two of the three given terms have one name, and the other given term with the term

required have another: ſo this queſtion being demanded, if four Students ſpend 19 pounds in certain months, how much money will ſerve 8 Students for the ſame time, and at the ſame rate of expence? Here Students and pounds are the two denominations of the terms in the queſtion, *viz.* 4 and 8 (being two of the terms propounded) have the denomination of Students, and 19 the other term given, together with the term required, have the denomination of pounds.

VI. In the Rule of Three, two of the three given terms imply a ſuppoſition, and the third moves a queſtion: ſo in the aforementioned queſtion a ſuppoſition is made, that 4 Students ſpend 19 pounds, and a queſtion is moved with the number 8, to wit, how many pounds will 8 Students ſpend.

The right ordering of the terms given.

VII. In the Rule of Three, the numbers given muſt be ſo ranked, that the known number, or term upon which the queſtion is moved, muſt poſſeſs the third place in the Rule; alſo of the other two that which hath the ſame denomination with the third, muſt be in the firſt place: laſtly, the other third, known term, which is of the ſame denomination with the fourth term ſought (or anſwer of the queſtion) muſt poſſeſs the ſecond place: ſo in the queſtion before mentioned, the terms 4, 19, and 8, are to be thus placed, *viz.* 8 is the term upon which the queſtion is moved, and therefore to poſſeſs the third place in the Rule; 4 is of the ſame denomination with 8 *viz.* of Students, and therefore to be in the firſt place; Laſtly, 19 being of the ſame denomination with the term ſought, *viz.* of money, is to be in the ſecond

ſecond place: and ſo they will be placed in the Rule thus,

Students. *Pounds.* *Students.*
If 4 —— 19 —— 8

That is to ſay, if 4 Students ſpend 19 pounds, what will 8 Students ſpend? And here for the better diſcerning of the term or number upon which the queſtion is moved, you may obſerve, that for the moſt part it is the known number in the queſtion which immediately followeth theſe or ſuch like words; *viz. How many? How much? What will? How long? How far? &c.*

VIII. The Rule of Three is either Direct or Inverſe.

The Rule of Three Direct.

IX. The Rule of Three Direct is, when the ſence or tenour of the queſtion requireth that the fourth number ſought muſt have ſuch proportion to the ſecond, as the third number hath to the firſt; ſo in the aforementioned queſtion, if 4 Students ſpend 19 pounds, how many pounds will 8 Students ſpend at the ſame rate of expence? It is evident that the thing required is to find a number which may have ſuch proportion to 19, as 8 hath to 4; that is, as 8 is the double of 4, ſo ought the fourth number to be the double of 19; for if 19 pounds be required to maintain 4 Students a certain time, as much more muſt needs be required for the maintainance of 8 Students the ſame time; and therefore in this caſe we may ſay in a direct proportion, as 4 *is* to 8, ſo is 19 to a number which ought to be as much more as 19.

X. In the direct Rule of Three, if you multiply the ſecond term by the third, or (which is all one) the third term by the ſecond, and then divide the Product by the firſt, the *quotient* will give the fourth term or fourth proportional required: ſo in the queſtion before propounded, if you multiply 19 by 8, the product is 152, which if you divide by 4 the quotient will give you 38 the fourth term demanded, and the work will ſtand thus.

How to work the Rule of Three Direct, the three given terms being ſingle numbers.

Stud l. Stud. l.
If 4—19—8—(38
8
4) 152(38 *pounds*
12
32
32
0

A ſecond Example may be this, if 8 yards coſt 9 pounds how much will 3 yards coſt? *Anſwer*, 3*l.*——7*s.*——6 *d.*

This queſtion being ſtated according to the ſeventh Rule of this Chapter, will ſtand as here you ſee; then multiplying (as before) the ſecond term 9 by the third term 3, the product is 27, which being divided by the firſt term 8, the quotient is 3 pounds, and there is a remainder of three pounds, which muſt be reduced into 60 ſhillings, and after thoſe ſhillings are divided by 8, and the reſt of the work proſecuted according to the Note

y. l. y. l. s. d.
8—9—3—(3:7:6
3
8) 27 (3 *pounds*
24
3 *the remainder*
20
8) 60 (7 *ſhillings.*
56
4 *the remainder*
12
8) 48 (6 *pence*

Note at the latter end of the 11th Rule of the 6th Chapter, at length the entire quotient or anſwer of the queſtion is 3*l.*——7*s.*——6*d.*

A third Example, if 51 ounces of Silver plate be ſold for 13 pounds ſterling, what is the price of 1 ounce of that plate? *Anſw.* 5*s.*—1 *d.* and ſomewhat more. The operation is thus: After the three known terms of this queſtion are rightly ordered they will ſtand as here you ſee in the Example; then multiplying the ſecond term 13 by the third term 1, the Product will be alſo 13 (for multiplication by 1 makes no alteration;) which 13 being divided by 51, after the manner of operation delivered in the note upon the 5th Rule of the 7th Chapter, the entire Quotient or anſwer of the queſtion will at length be found to be 5*s.*—1 *d.* and ſomewhat more, but the ſurpluſage being leſs than a farthing is omitted as uſeleſs.

oz. l. oz.
51—13—1
1
13
20
51) 260 (5 *ſhillings.*
255
5
12
51) 60 (1 *peny.*
51
9

Example 4. What muſt be paid to a labourer for his wages for 27 weeks at the rate of 4*s.* for 1 week? *Anſwer*, 5*l.*—8*s.*

After the three given terms are rightly placed in the Rule, they will ſtand as you ſee in the Example; then multiplying the third term 27 by the ſecond term 4, the product is 108, which I ſhould divide by the firſt term 1, but in regard diviſion

Week, Shil. Weeks
1——4——27
4
108

division by 1 makes no alteration, the *Quotient* is also 108, so that the fourth term sought is 108 shillings, which being reduced to pounds, according to the seventh Rule of the seventh Chapter, give 5*l.* 8*s.* for the answer of the question.

To prepare the terms of the Rule of Three, when they are compounded of divers denominations.

XI. In the Rule of Three, if after the question is stated according to the seventh Rule of this Chapter, any of the three given terms be a compound term consisting of divers denominations, as pounds, shillings, and pence; or weeks days, hours, &c. such compound term must first be reduced into the lowest of those denominations (by the 6th Rule of the seventh Chapter) to the end that the three given terms may be three single numbers; also of these three single numbers the first and third must always be of one and the same denomination: for if it happen that they express things of different names, such of the two which hath the greater name (or denomination) is to be reduced into the same name with the lesser (by the 5th Rule of the seventh Chapter.) These preparations being observed, the rest of the work is to be prosecuted according to the tenth Rule of this Chapter. *Example*, What will 48 ounces, 17 peny weight, and 20 grains of silver plate amount unto at the rate of 5*s.*——6*d.* the once? *Answer*, 13 *l.*—— 8*s.*——10*d*——3*f.* very near.

This

This question being stated according to the seventh Rule of this Chapter, will stand in the Rule as you see in the Example, to wit, if 1 ounce cost 5*s.*——6*d.* what will 48 *oz.*——17 *p. w.*——20 *gr.* cost? Here because the third term is compounded of divers denominations it must be reduced into the lowest of those denominations, to wit, grains; so by the sixth Rule of the seventh Chapter there will be found 23468 grains for the third term: likewise because the second term 5*s.*——6*d.* is a compound term, whose lowest name is pence, it must be reduced into pence (by the aforesaid Rule;) so there will be found 66 pence for the second term: Moreover because the first term hath the name ounce, and the third term the name grain, the first term 1 ounce must be converted into 480 grains (which are equal to 1 ounce;) then will the three terms or single numbers stand in the Rule, as here you see *viz.* if 480 grains cost 66 pence, how many pence will 23468 grains cost? Now proceeding according to the tenth Rule of this Chapter, there will arise in the quotient 3226 pence, besides a remainder of 408 pence, which being reduced to 1632 farthings, and

oz.	*s.*	*d.*	*oz.*	*p.w.*	*gr.*
1	5	6	48	17	20
20	12		20		
20	66		977		
24			24		
480			3928		
			1954		
			23468 grains		

gr.	*pence.*	*gr.*
480	66	23468

those

those divided by the first term 480 the quotient will be 3 farthings, so that the entire quotient is 3226 pence, 3 farthings, and somewhat more (but the parts of a farthing being of no moment, may be neglected.) Lastly, the said 3226 pence being reduced according to the seventh Rule of the seventh Chapter, give 13*l.* — 8*s.* — 10*d.* — 3*f.* so that 13*l.* — 8*s.* — 10*d.* — 3*f.* and somewhat more, will be the Answer of the Question.

The proof of the Rule of Three direct.

XII. For the proof of the Direct Rule of Three, multiply the fourth term by the first, which done, if that Product be equal to the Product of the second term multiplied by the third, the work is right otherwise it is erroneous: so in the first Example, 38 the fourth term, being multiplied by the first term 4, the Product is 152, which is also the Product of 19 multiplied by 8. But if it happen that after the fourth term, or answer of the question is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the first term, multiplied by such fourth term, and then the sum must be equal to the Product of the second and third terms (the second term consisting of the same denomination with the fourth:) so in the last Example the fourth term is 3226, and there happens to be a remainder of 408, which being added to the Product of the multiplication of the said 3226 by the first term 480, gives 1548888, which is the same with the Product of the third term 23468 multiplied by the second term 66, as will appear by the work.

XIII.

A compendious operation in the Rule of three direct, when the first term is 1 or unity.

XIII. When the first of the three given numbers in the Rule of three Direct, is 1 or unity, the question may oftentimes be answered more speedily than by the Rule of Three, even by those who have but little skill in Arithmetick, as will partly appear by the following Examples, *viz.*

1. At 17*s.* — 9*d.* the yard, what will 84 yards cost? *Answer*, 74*l.* — 11*s.* For reason sheweth that 84 yards must (at the said rate) cost 84 Angels, 84 Crowns, 84 half Crowns, and 84 Three pences, all which being computed and added together, will give the full value of 84 yards, *Viz.*

	l.	*s.*	*d.*
84 Angels make	42	00	00
84 Crowns	21	00	00
84 half Crowns	10	10	00
84 Three-Pences	1	01	00
Sum	74	11	00

2. At the rate of 9*s.* the Bushel of Wheat, what will 51 Quarters amount unto? *Answer*, 183 *l.* 12*s.* — 0*d.*

It

It is evident that the price of 1 Quarter (which consists of 8 Bushels) will be 8 Angels wanting 8 Shillings: therefore,

	l.	s.	d.
from 8 Angels, to wit,	4	00	00
subtract	0	08	00
remains the price of 1 Quarter	3	12	00

Then the value of 51 Quarters, at the rate of 3*l.*—12*s.*—0*d.* the Quarter, may be found in manner following, *Viz.*

	l.	s.	d.
51 times 3*l.* or 3 times 51*l.* is	51	00	00
	51	00	00
	51	00	05
51 Angels make	25	10	00
51 Shillings doubled make	5	02	00
the price of 51 Quarters	183	12	00

3. What is a Chest of Sugar worth, that weigheth neat weight (the Tare being subtracted) 7 *C.* 3 *q.* 7 *lb.* at the rate of 6*l.*—3*s.*—4*d.* for 1 *C*? *Answer*, 48*l.*—3*s.*—6*d.*—2*f.*

Tare is that wherein any thing is put, as a Bag for Pepper, a Chest for Sugar.

7. times

	l.	s.	d.	f.
7 times 6 pounds make	42	00	00	
7 times 3 Shillings	1	01	00	
7 Groats	0	02	04	
The half of 6*l.*—3*s.*—4*d.* for 2 *qu.* is	3	01	08	
The half of 3*l.*—1*s.*—8*d* for 1 *qu.* is	1	10	10	
The fourth part of 1*l.*—10*s.*—10*d.* (because 7*l.* is a fourth part of 28*l.* or of 1 *qu.*) is	0	07	08	2
	48	03	06	2

Practical Rules of this nature cannot be compleatly understood without some skill in fractions, as will hereafter appear in the second Chapter of the Appendix: And therefore I shall conclude this Chapter with the following Questions, whose Answers are annexed to them, and may be found out by the preceding Rules; but the operations are purposely omitted, and left as an exercise for the Learner.

Questions to exercise the Rule of Three direct.

1. If 17 yards of Cloath cost 19*l.* 2*s.* 6*d.* what will 35 yards cost at that rate? *Answer* 39*l.* 7*s.* 6*d.*

2. If 35 yards cost 39*l.* 7*s.* 6*d.* how many yards may be bought at that rate for 19*l.* 2*s.* 6*d*? *Answer*, 17 yards.

3. If 35 yards cost 39*l.* 7*s.* 6*d.* what are 17 yards worth at that rate? *Answer*, 19*l.* 2*s.* 6*d.*

4. If 17 yards be sold for 19*l.* 2*s.* 6*d.* how many yards will 39*l.* 7*s.* 6*d.* buy at that rate? *Answ.* 35 yards.

5. What

5. What must I pay for the carriage of 17 hundred weight, 3 quarters, and 11 pounds *Averdupois*, at the rate of 7 shillings the hundred weight? *Answ.* 6 *l.*—4*s.*—11*d.*—1 *farth.*

6. 6 *l.*—4 *s*—11 *d.*—1 *f.* be pay'd for the carriage of 17 hundred weight, 3 quarters, and 11 pounds, what was paid for the carriage of 1 pound weight? *Answ.* 3 Farthings.

7. What must I pay for 39 ounces, 7 peny weight, and 18 grains of white plate at the rate of 5*s.* and 5*d.* the ounce? *Ans.* 10*l.*—13*s.*—4*d.* and three quarters of a farthing.

8. What must 1 *l.* (or 20 *s.*) pay towards a Tax, when 326 *l.*—6*s.*—8 *d.* is assessed at 41 *l.*—16*s.*—2 *d.*—3 *f*? *Answ.* 2 *s.*—6*d.*—3*f.*

9. What will the Interest of 876*l.*—17*s.*—6*d.* amount unto for 1 year at the rate of 6*l.* for 100 *l.* for the same time? *Answ.* 52*l.*—12*s.*—3*d.*

10. If 3 yards in length of English measure be equal to 4 ells Flemish; how many Flemish ells are contained in 120 yards English? *Answer* 160 Flemish ells.

11. If 4 Flemish ells in length, be equal to 3 English yards; how many English yards in 300 Flemish ells? *Answ.* 225 English yards.

12. If 3 ells in length of English measure, be equal to 5 Flemish ells, how many Flemish ells in 120 English ells? *Answ.* 200 Flemish ells.

13. If 5 Flemish ells in length, be equal to 3 English ells; how many English ells in 145 Flemish ells? *Answ.* 87 English ells.

14. If 3 ounces of Silk weight, be equal to 4 ounces of Venice weight; how many ounces Venice are equal to 60 ounces of Silk weight? *Answer* 80 ounces Venice.

15.

15. A Merchant delivered at *London* 120*l.* sterling, to receive 207*l.* Flemish at *Amsterdam*; what was 1 *l.* sterling valued at in Flemish money? *Answ.* 1 *l.*—14 *s.*—6*d.*

16. If a Bill of Exchange be accepted at *London*, for payment of 400*l.* sterling, for the value deliver'd at *Amsterdam*, at 1 *l.*—13 *s.*—6*d.* Flemish for 1 *l.* sterling how much Flemish money was deliver'd at *Amsterdam*? *Answ.* 670 *l.* Flemish.

17. When the Exchange from *Antwerp* to *London* is at 1 *l.*—4 *s.*—7 *d.* Flemish for 1 *l.* sterling; how much sterling must I pay at *London* to receive 236 *l.* Flemish at *Antwerp*? *Answ.* 192*l.* sterling.

18. A Merchant deliver'd at *London* 370 *l.* sterling by Exchange for *Roan* at 74*d.* sterling for 50 *s.* Tournois; how much Tournois ought he to receive at *Roan*? *Answ.* 60000*s.* Tournois.

19. In 370 Ducats, at 4 *s.*—2*d* the Ducat; how many French Crowns at 6*s.*—2*d*? *Answ.* 250 Crowns; For if 74 *d.* give 1 Crown, 18500*d.* (or 370 Ducats) will give 250 Crowns.

20. In 516 Dollers, at 4*s*—5*d.* the Doller; how many Guinneas at 1 *l.*—1 *s.*—6 *d.* the peice? *Answ.* 106 Guinneas. For if 258 *d.* give 1 Guinnea, 27348 *d.* or 516 Dollers) will give 106 Guinneas.

CHAP.

CHAP. IX.

Of the Inverse Rule of Three.

I. THE Rule of Three Inverse is, when the fourth term required ought to proceed from the second term, according to the same rate or proportion that the first proceeds from the third: so this question being propounded, if 8 Horses will be maintained 12 days with a certain quantity of Provender, how many days will the same quantity maintain 16 Horses? Here as 8 is half 16, so ought the fourth term required to be half 12; for if certain bushels of Provender serve 8 Horses 12 days; 16 Horses will eat up as much Provender in half that time: and therefore you cannot say here in a direct proportion (as before in the Rule of Three direct) as 8 to 16, so is 12 to another number which ought to be in that case as great again as 12; but contrariwise by an *inverted Proportion*; beginning with the last term first, as 16 is to 8, so is 12 to another number, which ought to be in this case half 12.

horses *days* *horses*
8——12——16

And by the due observation of this definition, together with that of the Rule of Three Direct (propounded in the ninth Rule of the eighth Chapter) when any question belonging to the single Rule of Three is propounded, you may readily discern by which of those Rules it ought to be resolved; for if the three terms given look for a fourth

fourth in a direct proportion as they stand ranked in the Rule, you must resolve the question by the direct Rule; contrariwise when the proportion is inverted or turned backwards, it ought to be resolved by the Inverse Rule of Three, which here followeth.

How to work the Inverse Rule of Three.

II. In the inverse Rule of Three, after the three given terms are rightly placed in the Rule, and reduced (if there be need) according to the eleventh Rule of the eighth Chapter, multiply the first term by the second, or (which is the same) the second term by the first, and then divide the Product by the third term, so the *quotient* will give you the fourth term required, or answer of the question; thus in the question premised in the last Rule, if you multiply 12 by 8, the Product is 96, which if you divide by 16 the *Quotient* gives you 6, the fourth term required, as by the subsequent operation is manifest.

horses *days* *horses* *days*
8——12——16——(6
8
————
16) 96 (6
96
————
0

How to discern whether a question in the Rule of Three is to be resolved by the Rule Direct, or by the Rule Inverse.

III. For the more ready discovering, whether a question propounded belongs to the Rule of Three Direct, or to the Rule Inverse, observe the directions following, viz. 1. By the sence and tenour of the question consider whether more be

required

required or leſs; that is, whether the number ſought muſt be greater or leſs than the ſecond term. Secondly, eſteeming the firſt and third terms as extreams in reſpect of the ſecond, this will be a general Rule; namely, When more is required, the leſſer extream is the *Diviſor*; but when leſs is required, the greater extream is the *Diviſor*. Laſtly, the *Diviſor* being found out, it will be apparent whether the Rule be Direct or Inverſe, for when the *Diviſor* is the firſt term, it is a Rule Direct; but when the *Diviſor* is the third term, the Rule is Inverſe. Another Example of the Rule Inverſe may be this: If 12 Mowers do mow certain Acres in 4 days, in what time will 23 Mowers perform the ſame work? *Anſwer*, 2 days, 2 hours, and ſomewhat more. Here, the 3 known terms being rightly placed in the Rule, will ſtand as you ſee in the Example; and ſince it is evident that 23 men will require leſs time than 12 men to finiſh the ſame work, therefore (by the Rule aforegoing) the greater of the two extream Numbers 23 and 12 muſt be the *Diviſor*; and becauſe the *Diviſor* 23 ſtands in the third place, this queſtion is to be wrought by the Rule Inverſe; wherefore multiplying the firſt term 12 by the ſecond term 4, the product is 48, which being divided by the firſt term 23, the *Quotient* gives 2 days, and there is a remainder

M. *D.* *M.*
12—4—23
4
23) 48 (2 *days*
46
2
24
23) 48 (2 *hours*
46
2

mainder of 2 days, which being reduced to hours, and thoſe divided by 23, the *Quotient* will be 2 hours, and there is yet a remainder of 2 hours to be ſubdivided into 23 parts, if you pleaſe; ſo that the fourth term ſought, or anſwer of the queſtion is 2 days, 2 hours, and ſomewhat more.

Again, take this for a third Example, If I lend my Friend 356 pounds for one year and 35 days (the year being ſuppoſed to conſiſt of 365 days) how long time ought he to lend me 500 pounds to requite my courteſie? *Anſwer*, 284 days and ſomewhat more, there being a remainder, to wit, 400, after the Diviſion is finiſh'd, as by the ſubſequent operation is manifeſt.

l. *y.* *D.* *l.*
356—1. : 35—500
365
add 35
multiply {400
356
5|00) 1424|00 (284 *days.*

IV. The proof of the Inverſe Rule of Three is this, multiply the third term by the fourth, then if this Product be equal to the Product of the firſt term multiplied by the ſecond, the work is true, otherwiſe erroneous; ſo in the Example of the ſecond Rule, the Product of 16 and 6 equal to the Product of 8 and 12. But if it happen

The proof of the Rule of Three Inverſe.

happen that after the fourth term, or answer of the question, is found in the same denomination with the second term, there is yet a remainder, such remainder must be added to the Product of the third term multiplied by the fourth, and then the sum must be equal to the Product of the first and second terms (such second term being of the same particular denomination with the fourth:) so in the last Example, the fourth term is 284 days, and there remains 400 after the division is finisht, this 400 being added to the Product of the Multiplication of the third term 500 by the fourth term 284 gives 142400, which is equal to the Product of the first term 356, multiplied by the second term 400 days.

CHAP.

CHAP. X.

The double Golden Rule Direct, performed by two single Rules.

I. THE Compound Golden Rule is, when more than 3 terms are propounded.

II. Under the Compound Golden Rule, is comprehended the double Golden Rule, and divers Rules of plural proportion.

The doubee Golden Rule.

III. The double Golden Rule is, when five terms being propounded, a sixth proportional unto them is demanded: as in this question, if 4 Students spend 19 pounds in 3 months, how much will serve 8 Students 9 months? Or this, if 9 Bushels of provender serve 8 Horses 12 days, how many days will 24 Bushels last 16 Horses?

The parts into which the terms of the same Rule are distributed.

IV. The five terms given in this Rule consist of two parts, *Viz.* A supposition expressed in the three first terms; and a demand propounded in the two last: So in the first Example of the last Rule, this Clause (if four Students spend 19 pounds in 3 months) is the supposition, and this (how much will serve 8 Students nine

 months)

months) is the demand: likewiſe in the other Example of the ſame Rule, this clauſe (if nine Buſhels of Provender ſerve 8 Horſes 12 days) is the ſuppoſition, and this (How long, or how many days will 24 Buſhels laſt 16 Horſes) is the demand propounded.

V. Here for ranking the terms propounded in their due order, firſt obſerve amongſt the terms of ſuppoſition, which of them hath the ſame denomination with the term required; then reſerving that term for the ſecond place, write the other two terms of ſuppoſition one above another in the firſt place; And laſtly, the terms of demand likewiſe one above another in the third place of the Rule, in ſuch ſort that the uppermoſt may have the ſame denomination with the uppermoſt of thoſe in the firſt place. Example, If 4 Students ſpend 19 pounds in 3 months, how much will ſerve 8 Students 9 months? Here the three terms of ſuppoſition are 4, 19, and 3, and of theſe terms 19 hath the ſame denomination with the term required, *Viz.* of Pounds, for you are to enquire how much Money is requiſite for the maintenance of 8 Students 9 months; wherefore reſerving 19 for the ſecond place I write 4 and 3 one above another thus; then drawing a line upon the right hand of 4, I write 19 in the ſecond place; this done the work will ſtand as in the Margent; Laſt of all the terms of demand being 8 and 9, and 8 having the denomination of Students, I place it in the ſame line with 4 and 19, and write 9 under

The right ordering of the terms.

4——19
3

under it; all this performed, the terms in this queſtion rank themſelves as followeth:

Viz, *Thus,*

4——19——8
3 9

Or thus,

3——19——9
4 8

In like manner, if the ſecond queſtion of the third Rule of this Chapter were propounded; the terms thereof ought to be diſpoſed

Thus,

8——12——16
9 24

Or thus,

9——12——24
8 16

VI. Queſtions belonging to the double Golden Rule may be reſolved by two ſingle Rules of Three, or by the Golden Rule Compound of five Numbers.

VII. When Queſtions of this nature are reſolved by two ſingle Rules, the proportions are as followeth:

The Proportions of the double Golden Rule, when it is performed by two ſingle Rules.

I. As the uppermoſt term of the firſt place, is to the middle term; So is the uppermoſt term of the laſt place to a fourth Number:

 II. As

II. As the lower term of the first place is to that fourth number; so is the lower term of the last place to the term required.

So in this Example before recited,

```
4—19—8   using tacitly the lower term of the
3     9  first place as a common number in the
         first proportion, say, thus,
```

I. If 4 Students spend 19 pounds (in three months) what will serve 8 Students the same time?

Or thus, If 4 Students spend 19 pounds, what will 8 spend?

Which Rule of Three will be discovered to be direct (by the third Rule of the ninth Chapter?) therefore the fourth proportional proceeding from the said three given numbers 4, 19, and 8 is 38 (by the 10th Rule of the 8th Chap. aforegoing.) Again to find the term required, using tacitly the uppermost term of the third place as a common Number in this last proportion, *say as followeth.*

II. If in three months 38 pounds are spent (by 8 Students) how much will serve them for 9 months?

Or thus, If 3 give 38, what will 9 yield you?

Which Rule of Three will likewise be discovered to be direct (by the third Rule of the 9th Chapter; therefore the fourth proportional proceeding from the said 3 numbers, 3, 38, and 9, you shall likewise find (by the 10th Rule of the 8th Chapter before-recited) to be 114, for 38 being multiplied by 9, the Product is 342, which divided by 3, yields you in the *Quotient* 114; So that I conclude, if four Students spend nineteen pounds in three months, 114 pounds will serve 8 Students

dents

dents 9 months; as you may further observe by the Work following:

```
            4——19——8
            3                 9——(114
4—19—8—(38       | 3—38—9—(114
      8          |       9
4) 152 (38       | 3) 342 (114
   12            |    3
   32            |    04
   32            |    3
   0             |    12
                 |    12
                 |    0
```

In like manner if two single Rules of There be formed (according to the preceeding 7th Rule) out of the five numbers given in the last mentioned question, the same being ranked according to the latter manner of ordering the said numbers in the fifth Rule, each of the said two Rules of three will be a Rule direct, and the same answer of the question, to wit, 114 pounds will be discovered, as you may see by the subsequent operation.

```
            3——19——9
            4                 8—) 114
3—19—9—(57       | 4—57—8—(114
      9          |       8
3) 171 (57       | 4) 456 (114
   15            |    4
   21            |    05
   21            |    4
   0             |    16
                 |    16
                 |    0
```


VIII. The

VIII. The double Golden Rule is either Direct or Inverse.

IX. The Double Golden Rule Direct is, when both the single Rules do each of them look for a fourth term in a direct proportion: As in the Example of the seventh Rule, where each of the two single Rules of Three is a Rule Direct.

The double Golden Rule Direct.

For another Example take this, if the carriage of 8 C. weight 128 miles, cost 48 shillings, for how much may I have 4 C. weight carried 32 miles after the same rate? The terms of this question according to the fifth Rule of this Chapter, rank themselves in *this order:*

```
128 ——— 48 ——— 32
  8              4
```

Now taking tacitly the lower term of the first place as a common number, I form the first Rule of Three according to the seventh Rule, *saying,*

I. If the carriage of a certain weight (to wit, 8 C.) 128 miles, will cost 48 shillings, what will the carriage of the same weight 32 miles cost?

Here it is easie to discern, that the fewer miles any weight is carried, the less money will pay for the carriage of that weight; therefore the fourth number sought by the said Rule of three must be less than the second number 48: And forasmuch as by the third Rule of the ninth Chapter, when less is required, The greater extream (whether it be the first or third number) must be the Divisor; therefore the first number 128 is the Divisor, and consequently the Rule of Three above propounded is a Rule direct, wherefore finding out the fourth number

ber by the tenth Rule of the eighth Chapter, to be 12 shillings, I proceed to the second proportion, and say,

II. If the carriage of 8 C. (32 miles) cost 12 shillings, how much must I give to have 4 C. carried the same distance?

And here likewise finding a fourth number to be looked for in a direct proportion, I discover that fourth, by the said tenth Rule of the eighth Chapter, to be 6*s.* which is the term demanded, and the answer to the question propounded: so that at last I conclude, if the carriage of 8 C. 128 miles cost 48*s.* the carriage of 4 C. 32 miles will cost 6*s.* according to the same rate: see the whole Work.

```
128 ——— 48 ——— 32
  8             4 — (6
-----------------------------
128 ——— 48 ——— 32 — (12
        32
        --
        96
       144
       ----
128)  1536 (12
      128
      ---
       256
       256
       ---
         0
-----------------------------
8 — 12 — 4 — (6
      4
     --
8)   48 (6
     48
     --
      0
```

CHAP.

CHAP. XI.

The Double Golden Rule Inverse, performed by two single Rules.

THE Double Golden Rule Inverse is, when one of the single Rules looks for a fourth term in an inverted proportion: As in the last Example propounded in the fifth Rule of the last Chapter. For if you rank the terms of that question, according to the said fifth Rule, *thus*,

The double Golden Rule Inverse.

8 —— 12 —— 16
9   24

And then work by two single Rules of Three, formed according to the seventh Rule of the last Chapter, you shall find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be inverse, and the latter direct; for saying first, if 8 horses be maintained 12 days (by 9 bushels of Provender) how many days will 16 horses be kept by so much Provender? Here the answer 6 days will be found out by the Rule of Three inverse: Secondly, saying, if 9 bushels of Provender be eaten up (by 16 horses) in 6 days, in how many days will 24 bushels be spent? Here the answer 16 days will be found out by the Rule of Three direct.

But if you order the given terms of the same question, *thus*,

9——

9 —— 12 —— 24
8   16

And then work by two single Rules of Three, formed according to the seventh Rule of the last Chapter, you shall find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be Direct, and the latter Inverse; for saying first, If 9 bushels of provender will last 12 days (to maintain 8 horses) how many days will 24 bushels serve the same number of horses: The answer 32 days will be found out by the Rule of Three direct. Secondly, saying, If 8 horses will be maintained 32 days (by 24 bushels of Provender) how long will 16 horses be kept by the same quantity of Provender? Here the answer 16 days will be found out by the Rule of Three $\boxed{\text{direct}}$. inverse

Wherefore whensoever a question belonging to the double Rule of Three is severed into two single Rules of Three (according to the preceeding Rules) if one of them happens to be a Rule inverse.

Now the Resolution of the question propounded being ranked after the first manner, is as followeth.

8——

8——12——16
9 24—(16

8——12——16—(6
8

16) 96 (6
96

0

9——6——24—(16
6

9) 144 (16
9

54
54

0

Again, The Resolution of the same question, being ranked after the last manner, *is this.*

9——

9——12——24
8 16—(16

9——12——24—(32
12

48
24

9) 288 (32
27

18
18

0

8——32——16—(16
8

16) 256 (16
16

96
96

0

So that at last I say if 9 Bushels of Provender serve 8 Horses 12 days, 24 Bushels will last 16 Horses 16 days, which is the resolution of the Question propounded.

CHAP.

CHAP. XII.

The Golden Rule compounded of five Numbers.

I. THE Golden Rule compound of five numbers is, when the terms being ranked, as before, instead of the double terms we use their products, and then proceed to find the term required by one single Rule of Three.

The Golden Rule Compound of five numbers performed by one single Rule direct.

II. Here when the Question propounded ought to be performed by the double Rule direct, multiplying the terms of the first place, the one by the other, take their product for the first term, the middle number for the second, and the product of the two last terms for the third term; this done having found by the Rule of Three direct, a fourth proportional unto those three, that fourth term so found is the number you look for; so this question being again propounded, if 4 Students spend 19*l* in 3 months, how much will serve 8 Students 9 months? and the terms thereof being ranked as before, *viz. thus,*

4——19——8
3 9

The product of 4 multiplyed by 3 is 12, and the product of 8 multiplyed by 9 is 72; wherefore I *say*, As 12 to 19, so 72 to the term required, which I find by the single Rule of Three direct to be 114.

So

So that if 4 Students spend 19*l.* in three months 114*l.* will be requisite for the maintenance of 8 Students 9 months; see the whole operation, *as followeth.*

```
4——19——8
3       9—(114
——      ——
12      72
        19
        ——
        648
        72
    12)1368(114
        12
        ——
        16
        12
        ——
        48
        48
        ——
         0
```

In like manner this being the Question as before (in the last Rule of the tenth Chapter) if the carriage of 8 *C.* 128 miles, cost 48*s.* what will the carriage of 4 *C.* 32 miles stand me in? the Answer thereunto will be 6*s.* as appears by the Work.

128——

128——48——32
8   128   4

1024   384   128
96
48

1024) 6144 (6 *Shillings*
6144

0

III. When the Question propounded ought to be resolved by the double Rule Inverse, having multiplied the double terms across, that is, the uppermost term of the first place by the lower of the last, and the uppermost of the last place by the lower of the first, write each product under the lower term by which it is produced: and then if the Inverse proportion be found in the uppermost line, using those products as single terms, proceed to find the term required by the single Rule of Three direct: But in case you find the Inverse proportion in the lower line, perform the Work by the single Rule of Three Inverse.

The Golden Rule compound of five Numbers performed by one single Rule Direct or Inverse.

So in the Example above mentioned, if 9 bushels of Provender serve 8 horses 12 days, how long will 24 bushels last 16 horses? Here if you rank the terms *thus*, you shall find the Inverse proportion in the first line, as is observed in the last Chapter: And therefore having subscribed the products

8—12—16
9   24

products according to the direction given you in this Rule, I proceed to satisfie the demand of this question by the single Rule of Three direct, as appears by the Work following.

8——12——16
9   24—(16

144   192
12

384
192

144) 2304 (16
144

864
864

0

But the terms of this question being ranked *thus*, the Inverse proportion is found in the lower line, as you may observe likewise by the last Chapter: whereupon in this case

9——12——24
8   16

to resolve the Question, I proceed by the single Rule of Three Inverse, as appears by the Work hereunto annexed: howsoever therefore you work the Question, you shall find the term required to be 16; so that at last I conclude, as before in the last Chapter, If 9 bushels of Provender serve 8 horses 12 days, 24 bushels will last 16 horses 16 days.

```
9—12—24
8      16—(16
———    ———
192    144
 12
———
384
192
———
144)2304(16
    144
    ———
    864
    864
    ———
      0
```

CHAP. XIII.

The Rule of Fellowship.

Rules of plural Proportion.

I. THE Rules of plural Proportion are those, by which we resolve Questions, that are discoverable by more golden Rules than one, and yet cannot be performed by the double golden Rule mentioned before in the three last Chapters. Of these Rules there are divers kinds and varieties, according to the nature of the Question propounded; for here the terms given are sometimes four, sometimes five, sometimes more, and the terms required sometimes more than one, &c.

II. Two particular Rules of plural proportion are these, the Rule of Fellowship, and the Rule of Alligation.

The Rule of Fellowship.

III. The Rule of Fellowship is that, by which in accompts amongst divers men (their several stocks together with the whole gain or loss being propounded) the gain or loss of each particular man may be discovered: As in this Example, *A* and *B* were sharers in a parcel of Merchandize, in the purchace of which *A* laid out 7*l.* and *B* 11 *l.* and they having sold this Commodity, find that their clear gains amount to 54*s.* Now here the Question to be resolved by this Rule is, what part of that 54 *s.* accrews to *A*, and what to *B*, according to the rate of the several sums or stocks which they adventured? Again, *A*, *B*, and *C*, freight a Ship from the *Canaries* for *England*, with 108 Tuns of Wine, of which *A* had 48, *B* 36, and *C* 24, the Mariners meeting with a storm at Sea, were constrained for the safety of their lives, to cast 45 Tun thereof over-board; here the Question to be resolved is, How many of the 45 Tun each particular Merchant hath lost, according to the rate of his Adventure?

IV. The Rule of Fellowship is either single or double.

V. The single Rule is, when the stocks propounded do continue in the Adventure (or common Bank) equal times, to wit, one stock as long time as another.

How to work the single Rule.

VI. In the single Rule of Fellowship, take the total of all the stocks for the first term, the whole gain or loss,

for the second, and the particular stocks for the third terms; this done, repeating the Rule of Three so often, as there are particular stocks in the Question, the fourth terms produced upon those several operations, are the respective gains or losses of those particular stocks propounded: So in the first Example above-mentioned 7 *l.* and 11 *l.* are the stocks propounded, whose total is 18 *l.* which I take for the first term: Again, 54 *s.* the common gain, is the second term, and 7 *l.* the first particular stock, is the third term of the first proportion; whereupon I say, as 18 *l.* to 54 *s.* so 7 *l.* to another number, which by the direct Rule of Three I find to be 21 *s.* *viz.* the part of the gain due to *A*, that expended the 7 *l.* stock. Then for the second proportion, I say, as 18 *l.* to 54 *s.* so 11 *l.* to another number, which I likewise find by the Rule of Three direct to be 33 *s.* *viz.* the part of the gain due to *B*, for his 11 *l.* stock.

$$\left.\begin{matrix}7\\11\end{matrix}\right\}18 \text{———} 54\left\{\begin{matrix}7\text{———}21\\11\text{———}33\end{matrix}\right.$$

Again in the other premised Example, the particular loss that happens to *A*, is 20 Tun, to *B* 15, and to *C* 10 Tun.

$$\left.\begin{matrix}48\\36\\24\end{matrix}\right\}108 \text{———} 45 \text{———}\left\{\begin{matrix}48\text{———}20\\36\text{———}15\\24\text{———}10\end{matrix}\right.$$

2. *Double.*

VII. The double Rule of Fellowship is, when the stocks propounded are double numbers, *viz.* when each stock hath

hath relation to a particular time: Example, *A*, *B*, and *C*, hold a pasture in common, for which they pay 45 *l.* *per annum.* In this Pasture *A* had 24 Oxen went 32 days, *B* had 12 there 48 days, and *C* fed 16 Oxen there 24 days; now the Question to be resolved by this Rule is, what part each of these Tenants ought to pay of the 45 *l.* rent? and here you may *observe*, that the stocks propounded are *double* numbers, *viz.* each stock of Oxen hath reference to a *particular* time; for the respective stock of *A* is 24 Oxen, and its *particular* time is 32 days; again, the stock of *B* is 12 Oxen, and the *respective* time is 48 days; And lastly, the stock of *C* is 16 Oxen, and its *peculiar* time is 24 days, which as you see are *double* numbers.

How to work the double Rule.

VIII. In the double Rule of Fellowship, multiply each particular stock by its respective time, and take the total of their Products for the first term, the whole gain or loss for the second, and the said particular Products of the double numbers for the third term: This done, repeating, as before the Rule of Three, so often as there are Products of the double numbers; the fourth terms produced upon those several operations, are the numbers you look for: So in the *Example* of the last Rule, the Product of 24 and 32 is 768, the Product of 12 and 48 is 576, and the Product of 16 and 24 is 384, the sum of these Products is 1728, which is the first term in the Question, then 45 *l.* the rent, is the second term, and 768 the first Product, is the third term of the first proportion. Wherefore I say, as 1728 to 45 *l.* so 768 to another number, which I find by the di-

 rect

rect Rule of *Three* to be 20*l.* *viz.* the part of the Rent that *A* ought to pay: Then for the second proportion *I say*, as 1728 to 45*l.* so 576 to 15*l.* which is the part that *B* ought to *pay*: And lastly, as 1728 to 45*l.* so 384 to 10*l.* *viz.* the part that *C* must pay.

768				768	20
576	1728	45		576	15
384				384	10

A second Example of the eighth Rule. Three Merchants, *A*, *B*, and *C* enter Partnership, and agree to continue in a joint Adventure 16 months; *A* puts into the common stock at the beginning of the said Term 100 pounds, at 8 months end he takes out 40 pounds, and 4 months after such taking out he puts in 140 pounds. *B* puts in at first 200 pounds, at 6 months end he puts in 50 pounds more, and 4 months after the putting in of the 50 pounds, he takes out 100 pounds. *C* puts in at first 150 pounds, at 4 months end he takes out 50 pounds, and 8 months after such taking out puts in 100 pounds. Now at the end of the said 16 months, they had gained 357 pounds, the Question is how much of the said gain belongs to each Merchant for his share.

In Questions of this Nature, two things are principally to be observed. 1. The whole time of Partnership. 2. The respective time belonging to each man's particular stock; so here, it is evident that the whole time is 16 months, and the particular stocks and times belonging to each Merchant will be as followeth, *viz.*

A Had

A had 100*l.* in the common stock for 8 months, therefore 100 multiplied by 8 produceth	800
Also 60*l.* for 4 months, therefore 60 multiplied by 4 produceth	240
Also 200*l.* for 4 months, therefore 200 multiplied by 4 produceth	800
The total of the products of money and time for *A*, is	1840

B. had 200*l.* in the common stock for 6 months, therefore 200 multiplied by 6 produceth	1200
Also 250*l.* for 4 months, therefore 250 multiplied by 4 produceth	1000
Also 150*l.* for 6 months, therefore 150 multiplied by 6 produceth	900
The total of the products of money and time for *B*, is	3100

C. had 150*l.* in the common stock for 4 months, therefore 150 multiplied by 4 produceth	600
Also 100*l.* for 8 months, therefore 100 multiplied by 8 produceth	800
Also 200*l.* for 4 months, therefore 200 multiplied by 4 produceth	800
The total of the products of money and time for *C*, is	2200

Then adding the said three totals together, to wit, 1840, 3100 and 2200, the sum is 7140, wherefore proceeding as in the last Example, I say by the Rule of three direct, as 7140 is to the total gain 357

 pounds;

pounds; ſo is 1840 to 92 pounds the gain of *A*: again, As 7140 is to 357; ſo is 3100 to 155 the gain of *B*: Laſtly, as 7140 is to 357; ſo is 2200 to 110 the gain of *C*.

The Proof.

IX. The Rule of fellowſhip is proved by addition of the terms required, whoſe ſum ought to be equal to the ſecond term in the Queſtion, otherwiſe the whole Work is erroneous: ſo in the firſt *Example* of the ſixth Rule afore-going, 21*s.* and 33*s.* being added together are equal to 54*s.* the *ſecond term* in that Queſtion. Likewiſe in the laſt *Example* of the ſame Rule, as alſo in the firſt *Example* of the laſt Rule, the *ſum* of 20, 15, and 10, the terms required, are equal to 45, the *ſecond term* propounded.

CHAP. XIV.

The Rule of Alligation.

I. THE Rule of Alligation is that, by which we reſolve Queſtions, that concern the mixing of divers ſimples together.

II. Alligation is either Medial or Alternate.

Alligation Medial.

III. Alligation Medial is, when having the ſeveral quantities and rates of divers ſimples propounded, we diſcover the mean rate of a mixture compounded of thoſe ſimples. So 10 buſhels of Wheat at 4*s.* or (which is all one) 48*d.* the buſhel; 40 buſhels of Rye at 3*s.* or 36*d.* the buſhel; and 50 buſhels of Barley at 2*s.* or 24*d.* the buſhel; being mixed with

with 20 buſhels of Oats at 12 *d.* the buſhel, the Rule of *Alligation medial* ſheweth you the mean price of that miſtling.

The Operations and Proportions of the ſame Rule.

IV. In Alligation medial, firſt ſum the given quantities, then find the total value of all the ſimples: this done, the proportion will be as followeth.

As the ſum of the quantities it to the total value of the ſimples:

So is any part of the mixture propounded to the required mean rate or price of that part.

Repeating again the premiſed Example of the third Rule, I demand how much one buſhel of that miſtling is worth? Now the ſum of 10, 40, 50, 20, (the given quantities) is 120 buſhels, and the value of the 10 Buſhels of Wheat at 48 *d.* the buſhel amounts to 480 *d.* for 48 being multiplyed by 10, the product is 480: Again the value of the 40 buſhels of Rye at 36 *d.* the buſhel, is 1440 *d.* The value of the 50 buſhels of Barley at 24*d.* the buſhel is 1200 *d.* And the value of 20 buſhels of Oats at 12 *d.* the buſhel is 240 *d.* All theſe values being added together, their total is 3360 *d.* I ſay then by the Rule of *Three Direct*, if 120 buſhels give 3360 *d.* what will 1 buſhel yield? The Rule preſently anſwers me 28 *d.* whereupon I conclude, that a buſhel of that Miſtling may be afforded for 28*d.* that is, 2*s.* 4*d.* which is the reſolution of the *Queſtion propounded.*

120 —— 3360 —— 1 —— 28

In

In like manner if it be demanded what 8 Bushels or a Quarter of that Mistling is worth, the *Answer* will be 224*d.* which being divided by 12, and by that means reduced into *shillings*, is 18*s.* 8*d.*

120——3360——8——224

V. In *Alligation Medial*, the trial of the Work is by comparing the total value of the several Simples with the value of the whole mixture: For when those sums accord, the operation is perfect; So in the first Example of the last Rule.

The Proof.

		l.	*s.*	*d.*
The Value of	10 Bushels of Wheat at 4*s.* the Bushel is	2	0	0
	40 Bushels of Rye at 3*s.* the Bushel is	6	0	0
	50 Bushels of Barley at 2*s.* the Bushel is	5	0	0
	And 20 Bushels of Oats at 12*d.* the Bushel is	1	0	0
	All which amount to	14	0	0

which is likewise the value of 120 Bushels at 28*d.* or 2*s.* 4*d.* the Bushel, for that also amounts to 14*l.*

VI. *Alligation Alternate* is, when having the several rates of divers Simples given, we discover such quantities of them, as are necessary to make a mixture, which may bear a certain rate propounded.

Alligation Alternate.

Example: A man being determined to mix 10 Bushels of Wheat at 4*s.* or 48*d.* the Bushel, with Rye

Rye of 3*s.* or 36*d.* the Bushel, with Barley of 2*s.* or 24*d.* the Bushel, and with Oats of 1*s.* or 12*d.* the Bushel, the Rule of *Alligation Alternate* will discover unto you how much Rye, how much Barley, and how much Oats he ought to add unto the 10 Bushels of Wheat; in such sort that the mixture of them altogether may bear a certain rate or price propounded.

VII. In Questions of *Alligation Alternate*, you must rank the terms in such sort, that the given rate of the mixture may represent the root, and the several rates of the Simples may stand as branches issuing from the root: So the Example of the last Rule being propounded, I demand how much Rye, Barley, and Oats, ought to be added to the 10 Bushels of Wheat, that the mixture of all together may bear the rate or price of 28*d.* or 2*s.* 4*d.* the Bushel: And therefore drawing a line of connexion, I place 28*d.* the given rate of the mixture, upon the left hand thereof, by it self, representing the *Root*, and likewise write the other rates propounded, *viz.* 48*d.* 36*d.* 24*d.* and 12*d.* one above another upon the right hand of that line of Connexion, which rates are conceived to issue from 28*d.* as branches from the *Root*, the fabrick hereof appears plainly in the Margent.

The right ordering of the Terms.

$$28\begin{cases}48\\36\\24\\12\end{cases}$$

VIII. Having ranked the terms in their due order, link the branches together by certain Arches, in such sort, that one that is greater than the *Root* or rate of the mixture, may always be coupled with another

How to couple the Terms.

ther

ther that is less than the same: So in the premised Example, 48 may be linked with 12, and 36 with 24, or otherwise 48 may be coupled with 24, and 36 with 12, and then the Work will stand

$$\textit{Thus,}\ 28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \qquad \textit{Or thus,}\ 28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right.$$

IX. Having alligated the branches, and found the differences betwixt them and the Root, write the differences of each branch just against his respective yoke-fellow. So the branches of the example aforegoing being linked after the first manner, and the difference between 28 and 48 (by the third or fourth Rule of the fourth Chapter of this Book) being 20, I place 20 just against 12, the respective yoke-fellow of 48. Again, 16, being the difference betwixt 28 and 12, I write it just against 48. In like manner 8 being the difference between 28 and 36, I place it right against 24. And lastly, 4 the difference betwixt 28 and 24, I write just against 36: In the end the whole *Fabrick* of the Work (as the branches are thus linked) will stand as in the Example.

How to order the differences.

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \quad \left|\begin{array}{r}16\\4\\8\\20\end{array}\right.$$

But

But the branches being linked after the other manner, the work will be thus disposed:

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \quad \begin{array}{r}4\\16\\20\\8\end{array}\Bigg|$$

For in this case 48 hath 24 for his yoke-fellow, and the respective *Comerado* of 36 is 12; and here the interchangeable placing of the *difference* (as in the premised Examples) is that which is more particularly termed *Alternation*.

X. When one branch is linked to divers other branches, and not to one alone, the *differences* ought to be as often transcribed, as it is so diversly linked. So in the premised Example, you may (if you please) conceive 12 to be coupled both with 48 and 36; likewise 24 may be conceived to be linked with the same 48 and 36; wherefore the *difference* betwixt 28 and 12 being 16, I write it both just against 48 and 36; In like manner the *difference* between 28 and 24 being 4, I write it likewise over against the same numbers 48 and 36 Again, 20 being the *difference* betwixt 28 and 48, I place it just against 24 and 12; and 8 being the *difference* between 28 and 36 I write it likewise over against the same numbers 24 and 12; All this performed, the whole frame of the work will stand as in the *Margent*.

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \quad \left|\begin{array}{l}16.\ 4.\\16.\ 4.\\20.\ 8.\\20.\ 8.\end{array}\right.$$

2. Take this for another Example: It is requi-red

red to mix 10 Buſhels of Wheat at 48*d.* the buſhel with Rye of 36 *d.* the buſhel, with Barley of 24 *d.* the buſhel, and with Oats of 12 *d.* the buſhel, and the queſtion now is, How much Rye, Barley, and Oats ought to be added to the 10 buſhels of Wheat, that the entire mixture may be afforded at 16 *d.* the buſhel? Here the branches of this Queſtion (according to the eighth Rule of this Chapter) ought to be linked *thus,*

16	48
	36
	34
	12

And as for the *Alternation* of the differences, it is evident (by the preſent Rule) that the difference between 16 and 12 being 4 ought to be thrice tranſcribed, *viz.* firſt juſt againſt 48, then againſt 36, and laſt of all againſt 24. Again, 32 the difference betwixt 16 and 48, as alſo 20 the difference between 16 and 36; and laſtly, 8 the difference betwixt 16 and 24, ought all to be placed juſt againſt 12.

16	48	4
	36	4
	24	4
	12	32.20.8.

3. I determining to mix 10 buſhels of Wheat at 48 *d.* the buſhel, with Rye of 36 *d.* the buſhel, with Barley of 24 *d.* the buſhel, and with Oats of

of

of 12*d.* the Buſhel, deſire to know how much of each I ought to take, that I might afford the whole mixture at 40*d.* the buſhel: Here the whole Work being ordered according to the Rules aforegoing, it will ſtand as *followeth.*

40	48	16. 28.
		4.
	36	8.
	24	8.
	12	8.

4. A man intending to mix 10 Buſhels of Wheat at 48*d.* the Buſhel, with Rye of 36*d.* the Buſhel with Barley of 24*d.* the Buſhel, with Peaſe of 16*d.* the Buſhel, and with Oats of 12*d.* the Buſhel, deſires to know how much Rye, Barley, Peaſe, and Oats he ought to add to the 10 Buſhels of Wheat, that the whole maſs of Corn ſo mixed might be afforded at 20*d.* the Buſhel. This *Queſtion* being thus propounded, the terms thereof (by the Rules aforgoing) may be *Alligated*, and the differences of the terms *Alternated*, as *followeth.*

20	48	4.
	36	4.
	24	4. 8.
	16	28. 16. 4.
	12	4.

5. Laſtly, A Goldſmith hath ſome Gold of 24 *Carects*, other of 21 *Carects*, and other ſome of 19 *Carects fine*, which he would ſo mix with *Alloy*, that 192 Ounces of the entire *mixture* might bear

17

17 *Carects fine*; now the *Question* is, how much of each sort, as also how much *Alloy* he must take to accomplish his desire? Before you can well understand this *Question*, it will be necessary to explain what a *Carect fine*, and what *Alloy* is: the Mint-Masters and Goldsmiths to distinguish the different *fineness* of Gold, esteem an entire ounce to contain 24 *Carects*, and one ounce of Gold that being tryed in the fire loseth nothing of the weight, is said to be 24 *Carects fine*: again the ounce that being tryed loseth one four and twentieth part of the weight, Is said to be 23 *Carects fine*: In like manner that which loseth two four and twentieth parts of the ounce, is esteemed to be 22 *Carects fine*, and so consequently of the rest: And as for *Alloy*, it is silver, copper, or some other baser metal, with which the Goldsmiths use to mix their Gold, to the intent they may moderate, or abate the *fineness* thereof. Here you may also observe, that as the *fineness* of Gold is measured by *Carects*, so is the *fineness* of *Silver* estimated by *ounces*: In such sort, that a *pound* of *Silver* which being tryed a certain time in the fire, loseth nothing of the *weight*, is said to be 12 *ounces fine*: But a *pound*, that being tryed loseth somewhat of the *weight*, is said to be the *remainder* of the *weight fine*. *Example*; a *pound* of *Silver*, that loseth in the fire one *ounce* 8 *p.* is estimated to be 10 *ounces* 12 *p. fine*; and that which loseth 2 *ounces* 8 *p.* 10 *grains*, is said to be 9 *ounces* 11 *p.* 14 *grains fine*, &c. Now to rank the terms of the last mentioned *Question*, as also the differences of the terms in their due order, because the three given branches (*viz.* 24

What a Carect fine and what Alloy is.

Carects,

Carects, 21 *Carects*, and 19 *Carects*) are all greater than 17 *Carects* the *root* or rate of the *mixture*. I add 0 as another branch, which I conceive to be less than the *root*, and then proceed as in the former operations; the whole *frame* of the Work is expressed here, as followeth.

17	24	17
	21	17
	19	17
	0	7. 4. 2.

XI. When in one and the same line there are found more differences than one, add them together, and write the sum just against the same differences before a straight line drawn towards the right hand of the Work.

How to add the differences.

So the first Example of the last Rule being propounded, the sum of 16 and 4 (the differences placed just against the first branch) being 20, I write it over against the same differences, before the new line drawn upon the right hand of the Work, and so consequently the rest in their due order, as appears by the Example hereunto annexed.

28	48	16. 4.	20
	36	16. 4	20
	24	20. 8.	28
	12	20. 8	28

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In like manner the last Example of the last Rule being offered, the whole Fabrick of the Work will stand, as followeth:

$$17\left\{\begin{array}{l}24\\21\\19\\0\end{array}\right. \quad \begin{array}{r}17\\17\\17\\7.\,4.\,2.\end{array}\left|\begin{array}{l}17\\17\\17\\13\end{array}\right.$$

XII. Alligation Alternate is, either Partial or Total.

XIII. Alternation Partial is, when having the several rates of divers Simples, and the quantity of one of them given, we discover the several quantities of the rest, in such sort that a mixture of those Simples being made according to the quantity given, and the quantities so found, that mixture may bear a certain rate propounded: Of this kind is the Example of the sixth Rule, as also all the Examples of the tenth Rule except the last.

Alternation Partial.

XIV. In Questions of Alternation Partial, the proportion is as followeth.

The Proportions used in this Rule.

As the difference annexed to the first branch is to the several differences of the rest:

So is the quantity propounded to the several quantities required.

So the Example of the sixth and seventh Rules of this Chapter being again repeated, and the terms thereof, as also the differences of the terms being ordered after the first manner (shewed you in the ninth Rule aforegoing) it is evident that for

every 16 Bushels of Wheat that I take in the mixture, I ought to take 4 Bushels of Rye, 8 Bushels of Barley, and 20 Bushels of Oats; and therefore *I say,*

The first Case.

$$28\left\{\begin{array}{l}48\\36\\24\\12\end{array}\right. \quad \left|\begin{array}{r}16\\4\\8\\20\end{array}\right.$$

I. As 16 the difference annexed to the first branch (being the rate of the Wheat) is to 4, the difference annexed to the next, being the rate of the Rye; so is 10 the given quantity of the Wheat to another number, which being found by the Rule of *Three direct*, to be two bushels and an half (or two pecks) is the quantity of Rye necessary in the *mixture.*

II. As 16 to 8, so is 10 to another number; which being likewise found by the *Rule of Three* to be five bushels, is the quantity of Barley necessary in the *mixture.*

III. As 16 to 20, so is 10 to another number, which being in like sort found by the Rule of *Three* to be 12 bushels, and half of a bushel, is the quantity of Oats requisite in the *mixture.*

So that at last I conclude, a heap of Corn being composed of 10 bushels of Wheat, 2 bushels and a half of Rye, 5 bushels of Barley, and 12 bushels and an half of Oats (when those several Grains bear the *prices* aforesaid) may be aforded at 2 s. 4 *d.* the bushel.

The same Example being ordered after the second manner (expressed likewise in the 9th Rule of this present Chapter) *I say,*

2 Case.

 1. As

I. As 4 the *difference* annexed to the *rate* of the Wheat, is to 16 the *difference* annexed to the *rate* of the Rye; ſo is 10 the *given* quantity of the Wheat, to 40 buſhels the *required* quantity of the Rye.

II. As 4 to 20, ſo is 10 to 50 buſhels the requiſite quantity of the Barley.

III. As 4 to 8, ſo is 10 to 20 buſhels the quantity of the Oats *neceſſary* in the mixture.

28	48	4
	36	16
	24	20
	12	8

So that I conclude *again*, a maſs of Corn being compounded of 10 buſhels of Wheat, 40 buſhels of Rye, 50 buſhels of Barley, and 20 buſhels of Oats, (when thoſe Grains bear the prices propounded in this Example) may be afforded at 2 *s.* 4 *d.* the buſhel as before.

3Caſe. 3. That Example being diſpoſed after the third manner (expreſſed in the tenth and eleventh Rules of this Chapter) *I ſay*,

I. As 20 the *ſum* of the differences annexed to the *rate* of the Wheat, is to 20 the *ſum* of the differences annexed to the *rate* of the Rye; ſo is 10 the *given* quantity of the Wheat, to 10 buſhels the *required* quantity of the Rye.

II. As 20 to 28, ſo is 10 to 14 buſhels the *requiſite* quantity of the Barley.

III. As 20 to 28, ſo is 10 to 14 buſhels, the quantity of Oats *demanded* in the mixture.

28	48	16.4	20
	36	16.4	20
	24	20.8	28
	12	20.8	28

Whereupon this *third time* likewiſe I conclude, that (thoſe Grains ſtill retaining the given rates) 10 buſhels of Wheat, 10 buſhels of Rye, 14 buſhels of Barley, and 14 buſhels of Oats being all mixed together will conſtitute a *maſs* of Corn, that may be afforded at 28*d.* or 2*s.* 4*d.* the buſhel.

By this Example thus *diverſified* it plainly appears, that the quantities required may be altered as often as the Queſtion given will admit divers *Alligations*, and yet the mixture produced will ſtill hold the *rate* propounded; but when the *Queſtion* propounded will admit but one only way of *Alligation*, the quantities required to make the *mixture*, cannot be varied; ſo the ſecond *Example* of the tenth Rule of this Chapter, being again produced, and ordered according to the direction of the eleventh Rule aforegoing, *I ſay*,

I. As 4 to 4, ſo 10 to 10 buſhels of Rye.

II. As 4 to 4, ſo 10 to 10 buſhels of Barley.

III. As 4 to 60, ſo 10 to 150 buſhels of Oats.

16	48	4	4
	36	4	4
	24	4	4
	12	32, 20, 8.	60

So that for this question *I conclude*, to 10 bushels of Wheat you ought to add 10 bushels of Rye, 10 bushels of Barley, and 150 of Oats, to the end that a *mixture* of Corn might be made, which may be sold at 16 *d.* the bushel: And here the quantities found (*viz.* 10, 10, and 150) cannot be *altered*, because the terms of this Question will not admit any other variety of *Alligation*.

XV. In Alternation Partial, the proof is likewise by comparing the total value of the several simples, with the value of the whole mixture: So in the second example of the last Rule, the total *value* of the 10 bushels of Wheat, 40 bushels of Rye, 50 bushels of Barley, and 20 bushels of Oats amounts to 14 *l.* which is also the value of the whole mixture at 2 *s.* 4 *d.* the bushel, as appears by the example of the fifth Rule of this present Chapter.

The Proof.

XVI. Alternation total is, when having the total quantity of all the simples, together with their several rates, we produce their several quantities, in such sort, that a mixture of them being made according to the quantities so found, that mixture may bear a certain rate propounded: Of this sort is the last example of the tenth Rule aforegoing; as also *this*: A Goldsmith having divers sorts of Gold, *viz.* some of 24 Carects, other of 22 Carects, some of 18 Carects, and other some of 16 Carects *fine*, is desirous to melt of all these sorts so much together, as may make a *mass* containing 60 ounces of 21 Carects *fine*: Now this Rule of *Alternation total* sheweth you how much you are to take of each sort, to the end the whole mass

Alternation total.

may contain just 60 ounces of 21 Carects, the *fineness* propounded.

XVII. In Questions of Alternation total the proportion is, as followeth.

The Proportions.

As the sum of all the differences is to the total quantity of all the simples: So is the correspondent difference of each rate to the respective quantity of the same rate.

So the last example of the last Rule being propounded, *I say*,

I. As 12 the sum of the differences is to 60 ounces the *total* quantity of all the simples: so is 5 the correspondent *difference* of 24 Carects the first rate, to 25 ounces, *viz.* the required quantity of the Gold of the same rate, which may be taken to make the mixture propounded.

II. As 12 to 60, so is 3 the correspondent *difference* of 22 Carects the second rate, to 15 ounces, *viz.* the quantity of the Gold of 22 Carects, that ought to be used in the mixture.

III. As 12 to 60; so is 1 to 5 ounces of the Gold of 18 Carects fine.

IV. As 12 to 60, so is 3 to 15 ounces of the Gold of 16 Carects *fine*, which are requisite to be taken for the mixture propounded.

$$21\begin{cases}24\\22\\18\\16\end{cases}\begin{array}{|l}5\\3\\1\\3\\\hline 12\end{array}$$

Whereupon *I conclude*, that 25 ounces of 24 Carects *fine*, 15 ounces of 22 Carects, 5 ounces of 18 Carects, and 15 ounces of 16 Carects *fine*, being all melted together will produce a *mass* of Gold containing 60 ounces of 21 Carects *fine*, which is the resolution of the *Question* propounded.

Again, The last Example of the tenth Rule being here repeated, and ordered according to the direction of the eleventh Rule, I *say*,

I. As 64 to 192, so 17 to 51 ounces of 24 Carects *fine*.

II. As 64 to 192, so is 17 to 51 ounces of 21 Carects *fine*.

III. As 64 to 192, so is 17 to 51 ounces of 19 Carects *fine*.

IV. As 64 to 198, so is 13 to 39 ounces of *Alloy*.

17	24	17	17
	21	17	17
	19	17	17
	0	7.4.2.	13
			64

And therefore for *conclusion* I say, that 51 ounces of Gold, 24 Carects *fine*, 51 ounces of 21 Carects *fine*, 51 ounces of 19 Carects *fine*, and 39 ounces of *Alloy* being all mixed together, will produce a *mass* containing 192 ounces of Gold, 17 Carects *fine*, which is the *satisfaction* of the *question* premised.

And here *observe* (as before in the Exposition of the fourteenth Rule of this Chapter) that the operations of the first of these Examples may be *varied* according to the diversity of the *Alligations* which

which it will admit, whereas the last Example is not subject to any *variety*, the *Alligations* thereof remaining always the same.

XVIII. Here the operation is perfect, when the sum of the quantities found agrees with the total quantity propounded; So in the first Example of the last Rule, 25, 15, 5, and 15 (the quantities found) being all added together amount to 60, which is the total quantity propounded. *The Proof.*

CHAP. XV.

The Rule of False.

I. THE Rule of False is always performed by false and supposititial numbers taken at pleasure after the Proposition is made, and the question propounded; for things are said to be found out by the *Rule of False*, when by false terms *supposed*, we *discover* the true terms *required.*

II. The *Rule of False*, is either of single or double Position.

III. The Rule of single Position is when at once, *viz.* by one false Position, we have means to discover the true resolution of the Question propounded. *The Rule of single Position.*

For Example: *A*, *B*, and *C*, determining to buy together a certain quantity of Timber, that should cost them 36*l.* agree amongst themselves that *B* shall pay of that sum a *third* part more than *A*, and that C shall pay a *fourth* more than *B*. Now the Question is, What particular sum each of these parties

parties ought to pay of the 36*l.* To resolve this Question; first, put the case that *A* ought to pay 6*l.* of the 36*l.* and then *B* must pay 8*l.* because he pays one third part more than *A.* And lastly *C* ought to pay 10*l.* because he is to lay out one fourth part more than *B.* This done although by addition of these three sums, *viz.* 6, 8, and 10, I find that I have made a wrong *Position* (their total amounting only to 24*l.* which ought to have been 36*l.*) nevertheless by those *suppositional* Numbers, I have means to discover the true sums which the several parties ought to pay: For I say by the Rule of *Three Direct*,

I. As 24 to 36, so is 6 to 9*l.* the part that *A* must pay.

II. As 24 to 36, so is 8 to 12*l.* the part that *B* ought to pay.

III. As 24 to 36, so is 10 to 15*l.* the part of the 36*l.* that *C* must pay.

The Proof.

IV. Here for trial of this Rule the total of the sums found ought to accord with the sum given: So in the Example of the last Rule, 9, 12, and 15 being all added together amount to 36, the sum propounded.

The Rule of double Position.

V. The Rule of double *Position* is, when two false *Positions* are supposed for the resolution of the question propounded. As in *this*, A Workman having thresht out 40 quarters of Grain (part thereof being Wheat, and the rest Barley) received for his labour 28*s.* being paid after the rate of 12*d.* for every quarter of Wheat, and 6*d.* for each quarter of Barley: Now here the question is how many of those 40 quarters were Wheat, and how many Barley? Here therefore I first suppose at random, that there were 26 quarters of Wheat, and 14 of Barley, and then to discover whether I have guessed right or wrong, I find how much money is due to the Workman at the rate of 12 *d.* the Quarter of Wheat, and 6 *d.* the Quarter of Barley, which I find to be 33 *s.* (*viz.* 26*s.* for the 26 Quarters of Wheat, and 7*s.* for the 14 quarters of Barley) which he ought to have received, if my *supposition* had been right; but because it differs from 28 *s.* the true sum that he received, I perceive I have mist the mark, and therefore discovering how much I have err'd by finding the *difference* betwixt 28*s.* and 33 *s.* I keep in mind 5 their *difference*, which is called the *first errour*, or the *errour of the first Position:* Again, I propound for the *second Position*, that there was 30 quarters of Wheat, and 10 quarters of Barley; and then the *second errour* I find to be 7; for there is then due to the Workman for the 30 quarters of Wheat 30 *s.* and for the 10 quarters of Barley 5 *s.* in all 35 *s.* which differs from 28 *s.* the true sum that he received, by 7*s.* and here by these two *false Posiions*, together with their *errours*, you may discover how many quarters of Wheat, and how many of Barley the Workman thresht, as shall be further explained by the *Rules following*.

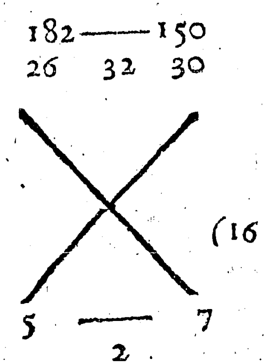
The Operation.

VI. In the Rule of double *Position* having drawn two lines across, and placed the terms of the false *Position* (*viz.* those that have the same Denomination) at the uppermost end of that Cross, as also each *errour* under his respective *Position* at the lower end of the same Cross, multiply each *errour* by the contrary *Position*

Position; that is, the second *errour* by the first *Position*, and the first *errour* by the second *Position*; this done, when both the *errours* are of one and the same kind (*viz* both excesses or both defects) subtract the less product out of the greater, and then the remainder is your Dividend; but if the *errours* be of differing kinds, (*viz.* one of them an excess, and the other a defect) add those Products together, and then the sum will be your dividend, which if you divide by the difference of the *errours*,) when they are of one and the same kind) or by their sum (when they are of different kinds) the *Quotient* will give you a number you look for, having the same Denomination with the false *Positions* placed at the upper end of the Cross.

1. *Example.* The Question of the last Rule being again propounded, I place these terms, *viz.* 26 (having the denomination of the Quarters of Wheat in the *first Position*) and 30 (having the same Denomination in the *second Position* at the upper end of the Cross:) As also 5 and 7 the two *errours* respectively under them at the lower end of the same Cross, as you may see it exemplified by the Pattern *following*.

Note, That this Character.—— signifies that the lesser of the two Numbers, betwixt which it is found, ought to be substracted from the greater.

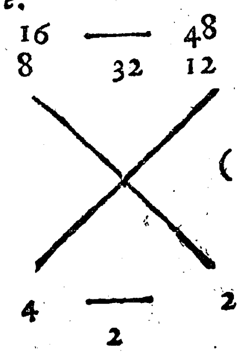


This

This done, having multiplied 26 by 7, the *Product* is 182, and likewise 30 by 5, the *Product* is 150, which being deducted out of 182 (because the *errours* here are both of the same kind, that is, are each of them an excess above 28*s.* the sum that the workman received) the *remainder* is 32, which being divided by 2 (the difference betwixt 5 and 7 the two errours) leaves in the *Quotient* 16, for the quarters of Wheat that the Workman thresht, whose complement to 40 *viz.* 24, are the quarters of Barley, that he likewise thresht; so at last I conclude, the workman receiving 28 *s.* for his wages in threshing out 40 quarters of Grain being part Wheat, part Barley) at 12 *d.* the quarter of Wheat: and 6*d.* the quarter of Barley, threshed in all 16 quarters of Wheat, and 24 quarters of Barley.

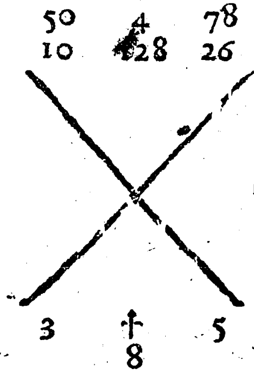
2. *Example.* The same question being again propounded, I suppose for my *first Position* that there are 8 quarters of Wheat, and 32 quarters of Barley, and then the first errour will be 4 *s.* for 8 *s.* being accounted for the 8 quarters of Wheat, and 16 *s.* for the 32 quarters of Barley, make in all 24*s.* which wants 4*s.* of 28*s.* the sum received: Again, *supposing* that there are 12 quarters of Wheat, and 28 quarters of Barley, the second errour will be 2 *s.* for 12 *s.* being allowed for the 12 quarters of Wheat, and 14*s.* for the 28 quarters of Barley, the sum is 26 *s.* which comes 2*s.* short of 28 *s.* the right sum: now then 8 being multiplied by 2, the Product is 16; likewise 12 by 4 produceth 48, out of which if you deduct 16 (because the errours in this case happen to be both defects under 28 *s.* the sum received) the remainder is 32, which being

being divided by 2 (the difference of the errours) gives you in the quotient 16, *viz.* the quarters of Wheat, *as before.*



3 *Example.* The same demand being the third time produced, I take for my *first Position* 10 quarters of Wheat, and 30 quarters of Barley, and then proceeding as before, the first errour will prove 3 *s.* which upon that *Position* I want of 28 *s.* the right sum: Again here for the *second Position* I take 26 quarters of Wheat, and 14 quarters of Barley, and then the second errour will be 5 *s.* which upon that *Position* I have exceeded 28 *s.* the true sum: now then multiplying 10 by 5, the Product is 50, and 26 by 3, the Product is 78: And here (because the errours are of different kinds, one of them being a *defect*, and the other an *excess* of 28 *s.* the true sum) you are to add 50 and 78 the two Products together, whose sum is 128, which being divided by 8, the sum of 3 and 5 the two errours, gives you in the quotient 16 for the quarters of Wheat, *as before* in the former resolutions. So that what *Positions* soever you take in this *Question*, you shall always find, that the Workman threshed 16 quarters

ters of Wheat, and 24 quarters of Barley, which is the Resolution of the Question propounded.

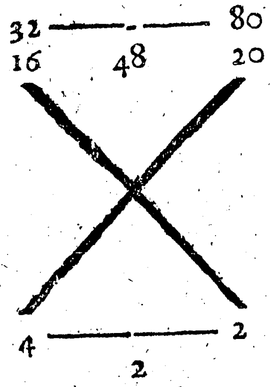


Note that this Character † intimates that the Numbers, betwixt which it is found, ought to be added together.

VII. Here the trial is the same with that which is used in finding out the errours: So in the Example premised 16 and 24 being the numbers found, and 16*s.* being allowed for the 16 quarters of Wheat, likewise 12*s.* for the 24 quarters of Barley, their sum is 28*s.* which was the sum received by the *Workman.*

4. *Example.* A certain man being demanded what was the age of each of his 4 Sons? Answered, that his eldest Son was 4 years elder than the second; his second was 4 years elder than the third; his third Son was 4 years elder than the fourth or youngest; and his fourth or youngest was half the age of the eldest; the Question is, what was the age of each Son? Here I guess the age of the eldest Son to be 16, then it may be inferr'd from the Question, that the age of the second Son was 12, the age of the third 8, and the age of the fourth or youngest 4, this 4 should be half 16 (for the Question saith, that the age of the youngest was half the age of the eldest,) but it wants 4 of what it ought

ought to be; wherefore I make a second Position, and take 20 for the age of the eldest, then the age of the second must necessarily be 16, the age of the third 12, and the age of the fourth 8, which should be half 20, but it wants 2: now (according to the Rule) multiplying 16 (the first Position) by 2 (the second errour) the Product is 32; also multiplying 20 (the second Position) by 4 (the first errour) the Product is 80, and because the errours are both of one kind, to wit, both defective; I subtract the lesser Product from the greater, so the remainder is



2) 48 (24

48 for a Dividend, also subtracting the lesser errour from the greater, the remainder is 2 for a Divisor: Lastly, dividing 48 by 2, the quotient is 24, and such was the age of the eldest Son, therefore the age of the second was 20; the age of the third 16; and the age of the fourth 12, which is half the age of the eldest, as was declared by the question.

The Doctrine of Vulgar Fractions.

CHAP. XVI.

Notation of Vulgar Fractions.

I. THus far of *Arithmetick* in *whole numbers*, only the Doctrine of *Fractions* ensueth, which depends upon this supposition, that Unity, or at least one whole thing, whatsoever it be, may in mind be conceived divisible into any number of equal parts: some will not allow 1 or unity to be a number, when it is consider'd in the abstract, and separated from matter, but forasmuch as that Prince of Arithmeticians, *Diophantus* of *Alexandria*, in divers of his subtil Problems doth mention unity as a number, and propounds it to be divided into numbers, I shall take the like liberty to esteem 1 or unity as a number, and likewise suppose it divisible into any number of equal parts.

II. A broken number, otherwise called a *Fraction*, is only part of an integer or whole thing, as if you would express in figures the length of a piece of cloth, that contains *three fourths*, or (which is all one) *three quarters* of a yard, you are to write it thus, $\frac{3}{4}$, that is, an entire yard being supposed to be divided into four equal parts, the length of the piece pro- *A Fraction.*

pounded is three of those four parts: In like manner (a Foot being divided into 12 inches) you must write six inches thus $\frac{6}{12}$, that is, *six twelve parts* of a foot; or if the foot be divided into one hundred equal parts, to express five and twenty of those parts, set them down thus, $\frac{25}{100}$ that is five and twenty hundredth parts of a foot.

III. A Fraction consists of two parts, the *Numerator* and the *Denominator*, which are placed one above the other, and separated by a little line.

IV. The *Numerator* is the number placed above the line, and the *Denominator* is the number placed underneath:

$\frac{3}{4}$ *Numerator.* *Denominator.*

so in the aforementioned *Fraction* $\frac{3}{4}$, the number 3 placed above the line is the *Numerator*, and the number 4 placed underneath is the *Denominator*. Also in this Fraction $\frac{6}{12}$, the *Numerator* is 6, and the *Denominator* is 12. The *Denominator* is so called, because it denominates or declares into how many equal parts the Integer or whole thing is supposed to be divided, and the *Numerator* is so called, because it numbreth or expresseth how many of those equal parts of the Integer are signified by the Fraction.

V. A Fraction is either proper or improper.

A proper Fraction. *VI.* A proper Fraction is that whose Numerator is less than the Denominator, such are the *Fractions* before-mentioned $\frac{3}{4}$ $\frac{6}{12}$ $\frac{25}{100}$, and the like.

VII. A proper Fraction is either single or compound.

A single Fraction. *VIII.* A single Fraction is that which consists of one Numerator, and one Denominator; such are $\frac{3}{4}$ $\frac{6}{12}$ $\frac{25}{100}$, and the like.

IX. A single Fraction doth often arise in Division of whole Numbers, for when Division is finisht, if any number remain, it is to be esteemed as the Numerator of a Fraction, which hath the Divisor for a Denominator, and is to be annexed to the Integer or Integers in the quotient as part of the quotient; which Fraction doth always express certain parts (or at least a part) of an integer or entire unity, which hath the same Denomination with one of the Integers in the quotient; so if 17 pounds be given to be divided equally amongst 5 persons, there will arise 3 entire pounds in the quotient, and there will be a remainder or surplusage of 2 pounds, which 2 is to be placed, as the Numerator of a *Fraction*, over the Divisor 5 as a *Denominator*; so will the *Fraction* be $\frac{2}{5}$, and the compleat quotient will be $3\frac{2}{5}$, that is, 3 pounds, and 2 fifth parts of a pound for each persons share.

5) 17 ($3\frac{2}{5}$

A single Fraction doth likewise arise, when a lesser whole number is given to be divided by a greater, for in such case the *Dividend* is to be made the *Numerator* of a Fraction, and the *Divisor* the *Denominator*; which Fraction is the true quotient, and doth always express certain part (or at least a parts) of an Integer, which hath the same name with the *Dividend*: so if 3 pounds sterling be given to be divided equally amongst 4 Persons, the share of each, that is, the quotient will be $\frac{3}{4}$, to wit, 3 fourth parts of a pound.

In like manner, if 5 be given to be divided by 8, the quotient is $\frac{5}{8}$, so that the *Numerator* of a Fraction is always a *Dividend*, the *Denominator* is a *Divisor*, and the *Fraction* it self is the *quotient*.

A Compound Fraction. *X.* A Compound Fraction (otherwise called a Fraction of a Fraction) is that which hath more *Numerators* and *Denominators* than one, and may be discovered by the word [*of*] which is interpos'd between the parts of such compound Fraction: so $\frac{2}{3}$ of $\frac{3}{4}$ is a Fraction of a Fraction, or *compound Fraction*, and expresseth two thirds of three fourths of an *Integer*, viz. a pound sterling being supposed the *Integer*, and first divided into four parts, three of those four parts are equal to 15 *s.* Again, if the said 15 *s.* be divided into three parts, two of those three parts are equal to 10 *s.* therefore the *compound Fraction* $\frac{2}{3}$ of $\frac{3}{4}$ of a pound sterling doth express 10 *s.* In like manner the compound Fraction $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a pound sterling, that is, one fourth of three fourths of four fifths of a pound sterling doth express 3 *s.* as will be farther manifest by the sixteenth and ninth Rules of the seventeenth Chapter.

An improper Fraction. *XI.* An improper Fraction is that, whose Numerator is either greater, or at least equal unto the Denominator: so this Fraction $\frac{16}{4}$, that is 16 fourths, is called an *Improper Fraction*, and so is this $\frac{4}{4}$; for indeed a Fraction of this kind may well be surnamed *Improper*, because it will not admit the definition of a true Fraction, since it is always greater than an entire unity, or at least equal unto it; so sixteen Farthings, or $\frac{16}{4}$ of a peny are equal to 4 entire pence; and 4 Farthings, or $\frac{4}{4}$ of a peny are equal to 1 peny; therefore when the Numerator is greater than the Denominator, such *improper* Fraction signifieth more than 1 or an Integer, but when the Numerator is equal to the Denominator (be it what number soever) such *improper* Fraction is always equal to unity, or 1 Integer.

A mixt number. *XII.* A mixt number consists of entire unities (or Integers) or at least of unity (or 1 Integer) and a Fraction annexed: So $5\frac{11}{12}$, $1\frac{3}{4}$, and such like; are called mixt numbers; So that if a piece of Timber be five feet and eleven inches in length, you are to write that length thus, $5\frac{11}{12}$; In like manner, one mile and three quarters or fourths of a mile are to be writen thus, $1\frac{3}{4}$.

CHAP. XVII.

Reduction of Vulgar Fractions.

I. The same parts of *Numeration*, as have been wrought in *whole Numbers* in the preceeding Chapters, are likewise to be performed in *fractions*, but first of all *Reduction* of *Fractions* in divers kinds must be known, which being the principal skill in the doctrine of Fractions, must be diligently observed by the Learner.

II. A number is said to be a common Measure or Divisor unto two or more numbers given, when it will measure or divide every one of the numbers given, and leave no remainder; so 4 is a common measure unto the numbers 12 and 20; for if 12 be divided by 4, the *Quotient* will be exactly 3, without any remainder or surplusage; also if 20 be divided by the same Divisor 4, the quotient will be precisely

precisely 5 without any remainder; in like manner 5 is a *common Divisor* unto these three numbers 10, 25 and 40.

To find the greatest common measure unto any two numbers.

III. Two numbers being given their greatest common Divisor; that is, the greatest number which will measure or divide each of the numbers given without leaving any remainder, may be found out in this manner; *viz.* Divide the greater number by the less, then divide the Divisor by the remainder (if there be any) and so continue dividing the last Divisors by the remainders, until there be no remainder (neglecting the quotients;) so is the last Divisor the greatest common Divisor unto the numbers given.

Thus, If the greatest *common Divisor* unto the numbers 91 and 117 be sought, divide the greater number 117 by 91, the remainder is 26, by which dividing 91, the remainder is 13; by which dividing 26 the remainder is 0; so is 13 the greatest *common Divisor* unto the numbers 117 and 91, as is manifest in dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 117 precisely 9 times. In like manner, 29 will be found a common Divisor unto 116 and 145; And 51 a common Divisor unto 561 and 612.

```
91) 117 (1
    91
    ——
    26) 91 (3
        78
        ——
        13) 26 (2
            26
            ——
             0
```

To reduce a Fraction into the least terms viz. 1. By a general Rule.

VI. A single Fraction may be reduced into the least terms by dividing the Numerator & Denominator

minator by their greatest common measure (or divisor;) for the quotients will be the Numerator and Denominator of a fraction equal to the former, and in the least terms.

So if the fraction $\frac{91}{117}$ be given to be reduced into the *least terms*, search out the greatest common Divisor unto 91 and 117 by the last *Rule*, which will be found 13, and then dividing 91 by 13, the quotient will be 7 for a new Numerator; also dividing 117 by 13, the quotient will be 9 for a new Denominator: so the fraction $\frac{91}{117}$ is reduced into the *least terms, viz.* into the fraction $\frac{7}{9}$. In like manner $\frac{116}{145}$ will be reduced unto $\frac{4}{5}$; And $\frac{561}{612}$ unto $\frac{11}{12}$: But here you are to observe, that if the greatest *common Divisor* unto the Numerator and Denominator be 1, such Fraction is in its *least terms* already: so the fraction $\frac{13}{151}$ cannot be reduced into lower terms, because the greatest *common Divisor* will be found 1, (by the third *Rule* of this Chapter;) the like may happen of infinite others: And although the last be a general *Rule* for the Reduction of Fractions into their *least terms*, yet there are other practical *Rules*; which in some cases will be more ready (especially unto beginners) *viz.*

2. By particular Rules.

V. When the Numerator and Denominator are even numbers, they may be measured or divided by 2. Therefore in such case you may (as is taught in the Rules of the 6th Chapter) take the half of the Numerator for a new Numerator, also the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at Length the line which separates the Numerator from the Denominator, and

16	8	4	2	1
64	32	16	8	4

cross

cross'd the same with a downright stroke near the Fraction, as you may see in the *Margent*, then take the half of 16, which is 8, for a new Numerator, also the half of 64, which is 32, for a new Denominator; Again, the half of 8 is 4, for a new Numerator, also the half of 32 is 16, for a new Denominator and proceeding in like manner, there will be found $\frac{1}{4}$ equivalent unto $\frac{16}{64}$.

225	45	9
475	95	19
50	10	2
425	85	17

VI. When the Numerator and Denominator do each of them end with 5, or one of them ending with 5, and the other with a Cypher, they may be both measured or divided by 5. So $\frac{225}{475}$ will be reduced into $\frac{9}{19}$ and $\frac{50}{425}$ into $\frac{2}{17}$, as by the operation in the *Margent* is manifest.

28	7	1
84	21	3

VII. Whensoever you can espy any other number, which will exactly divide the Numerator and Denominator (although it be not the greatest common Divisor) you may divide the Numerator and Denominator by such number as before: So $\frac{28}{84}$ may be first reduced into $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced into $\frac{1}{3}$ by 7, as by the operation is manifest.

4	00
5	00
7	00
90	00

VIII. When the Numerator and Denominator do each of them end with a Cypher or Cyphers, cut off equal Cyphers in both, and the fraction will be reduced into lesser terms: So $\frac{400}{500}$ is reduced into $\frac{4}{5}$, and $\frac{700}{9000}$ into $\frac{7}{90}$.

IX.

To find the value of a single Fraction in the known parts of the Integer.

IX. The value of a single fraction in the known parts of the Integer, may be found out in this manner, *viz.* multiply the Numerator of the fraction propounded by the number of known parts of the next inferiour denomination which are equal to the Integer, and divide that product by the Denominator, so is the quotient the value of the fraction in that inferiour denomination, and if there happen to be any fraction in the quotient, you may find the value thereof in the next inferiour denomination, by the same Rule, and so proceed till you come to the least known parts.

So the value of $\frac{9}{16}$ of a pound *sterling* will be found 11*s.* 3*d.* *viz.* multiply the Numerator 9 by 20 (the number of shillings which are equal to 1 pound *sterling*) the Product is 180, which being divided by the Denominator 16, the quotient in $11\frac{4}{16}$ shillings. In like manner, the value of $\frac{4}{16}$ of a shilling will be found 3 pence, for multiplying the Numerator 4 by 12 (the number of pence in a shilling) the product is 48, which being divided by the Denominator 16, the quotient is 3 pence. Also the value of $\frac{7}{13}$ of a pound *sterling*, will be found 10*s.* $9\frac{3}{13}$*d.* And $\frac{31}{96}$ of a pound *Troy* will be found equivalent unto 3 ounces 17 peny weight and 12 grains.

```
       9
      20
     ----
16) 180 (11 4/16
    16
    ----
     20
     16
    ----
      4
     12
    ----
16)  48 (3
     48
    ----
      0
```

X. A

To reduce a mixt number into an improper fraction.

X. A mixt number may be reduced into an improper fraction equivalent unto the mixt number, In this manner, *viz.* Multiply the Integer or integers in the mixt number by the Denominator of the fraction annexed to the Integer or Integers, and unto the Product add the Numerator of the said fraction; so is the sum the Numerator of an improper fraction, whose Denominator is the same with that of the said fraction annexed.

So $4\frac{11}{12}$ will be reduced into the improper fraction $\frac{59}{12}$, for 4 being multiplied by 12, the Product is 48, unto which adding the Numerator 11, the sum is 59 for a new Numerator, which being placed over the Denominator 12, gives the improper fraction $\frac{59}{12}$; which is equivalent unto $4\frac{11}{12}$ (as will appear by the 13th *Rule* of this Chapter.) In like manner $7\frac{1}{2}$ will be reduced into $\frac{15}{2}$.

To reduce a whole number into an improper fraction.

XI. A whole number is reduced into an improper fraction, by placing the whole number given as a Numerator, and 1 as a Denominator.

So 14 Integers will be reduced into the *improper fraction* $\frac{14}{1}$, and one Integer into the *improper fraction* $\frac{1}{1}$.

XII. A whole number is reduced into an improper fraction which shall have any Denominator assigned, in multiplying the whole number given by the Denominator assigned, and placing the Product as a Numerator over the said Denominator.

As if 13 be given to be reduced into an *improper fraction*, whose Denominator shall be 4, multiply 13 by 4, the Product is 52, which being placed over 4, gives the *improper fraction* $\frac{52}{4}$ equivalent unto 13 (as will appear by the next *Rule.*) In like manner 13 may be reduced into $\frac{91}{7}$.

To reduce an improper Fraction into its equivalent whole or mixt number.

XIII. An improper fraction may be reduced into its equivalent whole number or mixt number in this manner, *viz.* divide the Numerator by the Denominator, and the quotient will give the whole number or mixt number sought; So the improper fraction $\frac{59}{12}$ will be reduced into this *mixt number* $4\frac{11}{12}$, for if 59 be divided by 12, the quotient is $4\frac{11}{12}$. Also this improper fraction $\frac{52}{4}$ will be reduced into the whole number 13.

To reduce fractions to a common denominator, viz. 1. *When two fractions are propounded.*

XIV. Fractions having unequal Denominations may be reduced into fractions of the same value, which shall have equal denominators, by this Rule and the next following, *viz.* when two fractions having unequal Denominators are propounded to be reduced into two other fractions of the same value, which shall have a common Denominator, multiply the Numerator of the first fraction (that is either of them) by the Denominator of the second, and the Product shall be a new Numerator (correspondent unto the Numerator of that first fraction;) also multiplying the Numerator of the second fraction by the Denominator of the first, the Product is a new Numerator (correspondent unto the Numerator of the second fraction;) lastly, multiply the Denominators one by the other, and the Product

Product is a common Denominator to both the new Numerators.

Thus, If the fractions $\frac{2}{3}$ and $\frac{4}{5}$ be propounded, multiply 2 by 5, the product 10 is a new Numerator correspondent unto 2: also multiply 4 by 3, the product 12 is a new Numerator correspondent unto 4: lastly, multiply 3 by 5, and the product 15 shall be a common *Denominator* unto the new Numerators, so the fractions $\frac{10}{15}$ and $\frac{12}{15}$ are found out, which have *equal Denominators*, and each of these new fractions is equal unto its correspondent fraction first given, *viz.* $\frac{10}{15}$ is equal unto $\frac{2}{3}$ and $\frac{12}{15}$ is equal unto $\frac{4}{5}$, as will be manifest by the 4th *Rule* of this Chapter.

$$\begin{array}{cc} 2 & 4 \\ \text{--} & \text{--} \\ 3 & 5 \\ 10 & 12 \\ \hline 15 & 15 \end{array}$$

2 When three or more Fractions are to be reduced into others that shall have a Common Denominator.

XV. When three or more Fractions having unequal Denominators, are given to be reduced into other Fractions of the same value with those given, but such as shall have one common Denominator; multiply *continually* (according to the thirteenth Rule of the fifth Chapter) the Numerator of the first Fraction into all the Denominators, except the Denominator of that first Fraction; and reserve the last Product for a new Numerator instead of that first Numerator: In like manner, multiply *continually* the Numerator of the second Fraction into all the Denominators, except the Denominator of the second Fraction, and reserve the last Product for a new Numerator, instead of the second Numerator; Proceed in like manner to find out new Numerators for the rest of the given Fractions: Lastly, multiply *continually* all

all the Denominators one into another, and the last Product shall be a common Denominator to all the new Numerators.

As for Example, if these three Fractions, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{7}$, having unequal (or different) Denominators, be given to be reduced into three other Fractions of the same value, which shall have equal Denominators (or one common Denominator.) First, I multiply *continually* the first Numerator 3 into the second and third Denominators 5 and 7, saying 3 times 5 makes 15, which multiplyed by 7 produceth 105, for a new Numerator instead of the first Numerator 3; Secondly, I multiply *continually* the second Numerator 2 into the first and third Denominators 8 and 7, saying, twice 8 is 16, which multiplyed by 7 produceth 112, for a new Numerator instead of the second Numerator 2; Thirdly, I multiply *continually* the third Numerator 5 into the first and second Denominators 8 and 5, saying 8 times 5 makes 40, which multiplyed by 5 produceth 200, for a new Numerator instead of the third Numerator 5; Fourthly and lastly, I multiply *continually* all the Denominators 8, 5 and 7 one into another, saying, 8 times 5 makes 40, which multiplyed by 7 produceth 280 for a Denominator to each of the three new Numerators 105, 112 and 200 before found out; And so these three Fractions $\frac{105}{280}$, $\frac{112}{280}$, and $\frac{200}{280}$, are discovered, which have one common Denominator 280, and each of them is equal in value unto its correspondent Fraction first given, *viz.* $\frac{105}{280}$ is equal unto $\frac{3}{8}$; Also $\frac{112}{280}$ is equal unto $\frac{2}{5}$; and $\frac{200}{280}$ is equal unto $\frac{5}{7}$; as may easily be proved

$$\begin{array}{ccc} \frac{3}{8}, & \frac{2}{5}, & \frac{5}{7} \\ \frac{105}{280}, & \frac{112}{280}, & \frac{200}{280} \end{array}$$

ved by the Fourth Rule of this Chapter.

After the ſame manner theſe four Fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ are reducible into theſe, $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$ and $\frac{300}{360}$, which have 360 for a common Denominator, and are equal in value reſpectively to the four Fractions given to be reduced.

Note, Although by the foregoing fourteenth and fifteenth Rules, any multitude of Fractions may be reduced to a common Denominator; yet becauſe Fractions in their leaſt Terms are fitteſt for uſe, I ſhall ſhew how leſſer Denominators, than thoſe that will be diſcovered by the ſaid Rules, may often times be found out, *viz.*

I. When the unequal Denominators of two Fractions have a common Diviſor greater than 1, divide the Denominators ſeverally by their greateſt common Diviſor (found out by the fore-going third Rule of this Chapter;) and then multiply croſs-wiſe in this manner, *viz.* the Numerator of the firſt Fraction by the latter Quotient, and the Numerator of the latter Fraction by the firſt Quotient, and reſerve the Products for new Numerators; Laſtly, multiply the Denominator of the Firſt Fraction by the latter Quotient (or the Denominator of the latter Fraction by the firſt Quotient,) ſo ſhall the Product be a common denominator to the ſaid new Numerators: As for Example, if $\frac{5}{12}$ and $\frac{7}{18}$ be propoſed to be reduced to a common Denominator, I divide each of the Denominators 12 and 18 by their greateſt common Diviſor 6, and the

$$\begin{array}{rcc} 6) & \frac{5}{12} \times & \frac{7}{18} \\ \hline & 2 & 3 \\ \hline & \frac{15}{36} & \frac{14}{36} \end{array}$$

the Quotients are 2 and 3; then I multiply 5 the Numerator of the firſt Fraction by 3 the latter Quotient, alſo 7 the Numerator of the latter Fraction by 2 the firſt Quotient, and the Products 15 and 14 I reſerve for new Numerators inſtead of 5 and 7; Laſtly, I multiply 12 the Denominator of the firſt Fraction by 3 the latter Quotient (or 18 the Denominator of the latter Fraction by 2 the firſt Quotient,) and the Product 36 is a Denominator to each of the new Numerators 15 and 14: So $\frac{15}{36}$ and $\frac{14}{36}$ are found out, which have the leaſt common Denominator unto which the given Fractions $\frac{5}{12}$ and $\frac{7}{18}$ can be reduced; Alſo $\frac{15}{36}$ is equal to $\frac{5}{12}$, and $\frac{14}{36}$ to $\frac{7}{18}$.

II. Whenſoever the Denominator of a Fraction can be divided by the Denominator of a ſecond Fraction, without any Remainder; then if by the Quotient you multiply ſeverally the Numerator and Denominator of ſuch ſecond Fraction, a third will ariſe, having the ſame value with the ſecond, and the ſame Denominator with the firſt Fraction: By this Rule Three or more Fractions may often times be reduced to a leſſer common Denominator, than that which will be diſcovered by the foregoing Rule *XV.* As for Example, let theſe ſix following Fractions be given to be reduced to a common Denominator, *viz.*

$$\frac{11}{36},\ \frac{11}{18},\ \frac{7}{12},\ \frac{4}{9},\ \frac{5}{6},\ \frac{2}{3}.$$

Becauſe 36 the Denominator of the firſt Fraction, being divided by the five other Denominators ſeverally,

rally will give these Quotients 2, 3, 4, 6, and 12 without any Remainder, I muitiply the Numerator and Denominator of each of the five latter Fractions, by its correspondent Quotient; *viz.* 11 and 18 by 2 the first Quotient; Also 7 and 12 by 3 the second Quotient, and in like manner the rest; So instead of those five latter Fractions, five others (hereunder placed after the first of those six) are produced, *viz.*

$\frac{13}{36}$, $\frac{22}{36}$, $\frac{21}{36}$, $\frac{15}{36}$, $\frac{30}{36}$, $\frac{24}{36}$.

All which Fractions last express'd have a common Denominator 36, and are equal in value respectively to those given to be reduced.

To reduce a compound fraction to a single fraction. See continual multiplication in the last Rule of the 5th Chapter.

XVI. A compound fraction (otherwise called a *fraction of a fraction*) may be reduced into a single fraction in this manner, *viz.* Multiply all the Numerators *continually*, and take the Product for a new Numerator, also multiply all the Denominators *continually*, and the Product shall be a new Denominator.

$\frac{2}{3}$ of $\frac{3}{4}$

$\frac{6}{12}$ or $\frac{1}{2}$

Thus, if the *compound fraction* $\frac{2}{3}$ of $\frac{3}{4}$ be given to be reduced into a single fraction, multiply the Numerators 2 and 3, one by the other, so is the Product 6 a new Numerator. Also multiplying the Denominators 3 and 4 one by the other, the product 12 is a new Denominator, so $\frac{6}{12}$ (or $\frac{1}{2}$ is the single fraction sought, being equivalent unto $\frac{2}{3}$ of $\frac{3}{4}$, the compound fraction given to be reduced: in

In like manner, this compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will be reduced unto $\frac{24}{60}$, or $\frac{2}{5}$; for the Numerator 2, 3, 4 being multiplied *continually* produce the new Numerator 24, and the Denominators 3, 4, 5 multiplied *continually* produce the new Denominator 60: Lastly, the new Fraction $\frac{24}{60}$ (by the fourth Rule of this Chapter) will be reduced unto $\frac{2}{5}$, which is equal to $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$: But to make the meaning hereof more evident, suppose the Integer to be one pound of English money; then

$\frac{4}{5}$ of 1 *l.* (*viz.* of 20 *s.*) is——16 *s.*

$\frac{3}{4}$ of those $\frac{4}{5}$ (*viz.* of 16 *s.*) is——12 *s.*

$\frac{2}{3}$ of those $\frac{3}{4}$ (*viz.* of 12 *s.*) is ——8 *s.* or $\frac{2}{5}$ *l.*

whereby 'tis manifest that $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ *l.* is equal to $\frac{2}{5}$ *l.*

By this Rule a fraction or mixt number of a lesser name may be reduced to a fraction of a greater name. As if 3 $\frac{1}{2}$ pence be propounded to be reduced into an improper fraction of a pound sterling, the operation will be in this manner, *viz.* 3 $\frac{1}{2}$ or $\frac{7}{2}$ of a penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound sterling, which compound fraction will (by the aforesaid Rule) be reduced, to $\frac{7}{480}$ *l.* In like manner 42 $\frac{3}{16}$ minutes of an hour are equal to $\frac{45}{64}$ of an hour, for $\frac{675}{16}$ (that is 42 $\frac{3}{16}$) of $\frac{1}{60}$ are equal to $\frac{675}{960}$ (or in its least terms) $\frac{45}{64}$.

Here you may also observe, that when a compound fraction is one of the given terms in any question, it is first of all to be reduced to a single fraction by the aforesaid sixteenth Rule.

XVII. Two or more fractions bing given, there may be whole numbers found, which shall have the same reason or proportion as the

To find whole numbers, which shall have the same reason as any fractions or mixt numbers given.

L fractions

fractions given, *viz.* When the fractions given have unequal denominators, reduce them into equivalent fractions which shall have a common denominator (by the 14th or 15th Rule of this Chapter;) then rejecting the common denominator, the Numerators shall have the same reason or proportion as the fractions first given.

So $\frac{3}{5}$ and $\frac{5}{8}$ being given, will first of all be reduced into their equivalent fractions $\frac{24}{40}$ and $\frac{25}{40}$; then rejecting the common denominator 40, the Numerators 24 and 25 have the same reason with $\frac{3}{5}$ and $\frac{5}{8}$ *viz.* As $\frac{3}{5}$ is to $\frac{5}{8}$, so is 24 to 25: Also if the fractions $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ were given, there will be found 8, 16, and 32, which are in the same proportion one to the other as the fractions given. In like manner if mixt numbers be given, there may be whole numbers found which shall have the same *reason* or *proportion*, as the mixt numbers; so $5\frac{2}{3}$ and $3\frac{5}{8}$ being given, will be first reduced into the improper fractions $\frac{17}{3}$ and $\frac{29}{8}$ (by the tenth Rule of this Chapter:) also the said $\frac{17}{3}$ and $\frac{29}{8}$ will be reduced into $\frac{136}{24}$ and $\frac{87}{24}$; then rejecting the common Denominator 24, the Numerators 136 and 87 will have the same *reason* as $5\frac{2}{3}$ and $3\frac{5}{8}$, *viz.* As 136 is to 87, so is $5\frac{2}{3}$ to $3\frac{5}{8}$: Also $16\frac{1}{2}$ and 18 being given, there will be found 33 and 36, which being divided by their common Divisor 3 (found by the third Rule of this Chapter) will give out 11 and 12 which have the same *reason* as $16\frac{1}{2}$ and 18.

CHAP.

CHAP. XVIII.

Addition of Vulgar Fractions and mixt Numbers.

I. WHen the numbers given to be added are single fractions, and have equal denominators, add all the Numerators together, so is the sum the Numerator of a fraction, whose denominator is the same with the common denominator, which new fraction is the sum of the fractions given to be added.

To all single fractions, viz. *1. When they have equal denominators.*

So $\frac{3}{9}$ and $\frac{2}{9}$ being given to be added, their sum will be found $\frac{5}{9}$ *viz.* the sum of the Numerators, 3 and 2, is 5, which being placed over the common Denominator 9, gives $\frac{5}{9}$: In like manner the sum of these fractions $\frac{7}{8}$ $\frac{5}{8}$ $\frac{3}{8}$ and $\frac{2}{8}$ will be found $\frac{17}{8}$, which (by the 13 *Rule* of the seventeenth Chapter) will be found equivalent unto $2\frac{1}{8}$; so that $2\frac{1}{8}$ is the sum of the fractions given to be added.

II. When the fractions given to be added have unequal denominators, they are first to be reduced into fractions of the same value, which shall have a common Denominator (by the fourteenth or fifteenth Rule of the seventeenth Chapter;) and then they may be added by the first Rule of this Chapter.

2. When they have unequal denominators.

So if $\frac{2}{3}$ and $\frac{3}{5}$ were given to be added, their sum will be found $1\frac{4}{15}$ for (by the fourteenth Rule of

the ſeventeenth Chapter) $\frac{2}{3}$ and $\frac{3}{5}$ will be reduced into their equivalent fractions $\frac{10}{15}$ and $\frac{9}{15}$ which having equal Denominators may be added according to the firſt rule of this Chapter, and ſo the ſum will be found $1\frac{4}{15}$: In like manner the ſum of theſe fractions $\frac{1}{2}$ $\frac{3}{8}$ and $\frac{3}{4}$ will be found $1\frac{5}{8}$. Alſo the ſum of theſe ſix Fractions $\frac{13}{36}$, $\frac{11}{18}$, $\frac{7}{12}$, $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, after they are reduced to be a common Denominator (according to the latter Example in the Note at the end of the fifteenth Rule of the ſeventeenth Chapter) will be found $\frac{126}{36}$, that is, $3\frac{1}{2}$.

$$\frac{2}{3} \times \frac{3}{5}$$

$$\frac{3 \quad 5}{10}$$

$\frac{19}{15}$ *that is* $1\frac{4}{15}$

III. When any of the fractions given to be added is a compound Fraction, ſuch compound fraction is firſt of all to reduced into a ſingle fraction (by the ſixteenth Rule of the ſeventeenth Chapter) and then you may proceed as before.

The Addition of compound Fractions.

So $\frac{3}{5}$ and $\frac{2}{3}$ of $\frac{1}{4}$ being given to be added, their ſum will be found $\frac{23}{30}$ for the compound fraction $\frac{2}{3}$ of $\frac{1}{4}$ will (by the ſixteenth Rule of the 17th. Chapter) be reduced to $\frac{2}{12}$ (or in its leaſt terms) $\frac{1}{6}$, which added to the ſingle fraction $\frac{3}{5}$ (according to the ſecond Rule of this Chapter) gives $\frac{23}{30}$. Here you may obſerve, that the fractions given to be added in all the former caſes, are ſuppoſed to be fractions of Integers, which have one and the ſame *particular denomination*, *viz.* If one of the fractions, given to be added, be a fraction of a pound *ſterling*, all the reſt ought to be fractions of a pound

By Denomination is meant the name of any Integer or thing.

pound *ſterling*, and the like is to be underſtood of other denominations.

IV. When fractions of Integers, of different denominations are given to be added, they are firſt of all to be reduced into fractions of Integers which ſhall have one and the ſame particular denomination (by the ſixteenth Rule of the ſeventeenth Chapter;) and then they may be added by the firſt or ſecond Rule of this Chapter.

To add fractions of Integers which have different denominations.

So if $\frac{7}{9}$ of a pound *ſterling*, $\frac{3}{5}$ of a ſhilling, and $\frac{5}{8}$ of a peny were given to be added, reduce the two latter into fractions of a pound *ſterling* (by the ſixteenth Rule of the ſeventeenth Chapter) *viz.* $\frac{3}{5}$ of a ſhilling is $\frac{3}{5}$ of $\frac{1}{20}$ of a pound *ſterling*, which compound fraction being reduced into a ſingle fraction gives $\frac{3}{100}$ *li.* Likewiſe $\frac{5}{8}$ of a peny, is $\frac{5}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound *ſterling*, which compound fraction being reduced, gives $\frac{1}{384}$ *li.* Laſtly, $\frac{7}{9}$ *li.* $\frac{3}{100}$ *li.* and $\frac{1}{384}$ *li.* being added according to the ſecond Rule of this Chapter, their ſum will be found $\frac{280068}{345600}$, or in its leaſt terms $\frac{23339}{28800}$ *li.*

V. When mixt numbers are given to be added, find firſt of all the ſum of the fractions (by the firſt and the ſecond Rule of this Chapter;) then add the Integer or Integers (if there be any found) in the ſum of the fractions, unto the whole numbers, and collect the ſum of them as you were taught by the Rules of the third Chapter.

To add mixt numbers.

So if $3\frac{1}{2}$ $4\frac{1}{3}$ and $16\frac{5}{8}$ were given to be added, their ſum will be found $24\frac{11}{24}$ *viz.* the ſum of the fractions $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{5}{8}$ will be found (by the ſecond Rule of this Chapter) to be $1\frac{11}{24}$, and the ſum of the whole

whole numbers, 3, 4, and 16, is 23, unto which adding 1 (the Integer found in the sum of the fractions) the sum is 24; so that 24 $\frac{11}{32}$ is the sum of the mixt numbers given to be added.

CHAP. XIX.

Subtraction of Vulgar Fractions and mixt Numbers.

The subtraction of single fractions, viz.
1. When they have a common Denominator.

I. WHen the numbers given are both single fractions and have equal Denominators, subtract the lesser Numerator from the greater, and place the remainder over the common Denominator, so is such new fraction the difference between the fractions given.

Thus the difference between the fractions $\frac{9}{11}$ and $\frac{7}{11}$ is $\frac{2}{11}$, which is found by subtracting the lesser Numerator 7 from the greater Numerator 9, and placing the remainder 2 over the common denominator 11; also the difference between the fractions $\frac{11}{21}$ and $\frac{17}{21}$ is $\frac{6}{21}$, that is, the fraction $\frac{17}{21}$ exceeds $\frac{11}{21}$ by $\frac{6}{21}$.

2. When they have unequal Denominators.

II. When the numbers given are both single Fractions and have not a common Denominator, reduce them into fractions of the same value which shall have a common Denominator (by the fourteenth or fifteenth Rule of the seventeenth Chapter,) and then find their difference by the last Rule.

So the difference between the fractions $\frac{6}{7}$ and $\frac{7}{8}$ will be found $\frac{1}{56}$, *viz.* reducing the fractions given into their equivalent fractions $\frac{48}{56}$ and $\frac{49}{56}$, which have a common Denominator, the difference sought will be found $\frac{1}{56}$ by the first Rule of this Chapter. Likewise $\frac{7}{12}$ being subtracted from $\frac{11}{13}$, there will remain $\frac{41}{156}$.

The subtraction of mixt numbers, viz.
1. By a general Rule.

III. When one of the numbers given is a whole number or a mixt number, also when both of them are mixt numbers, reduce such whole, or mixt numbers into an improper Fraction or Fractions, by the tenth or eleventh Rule of the seventeenth Chapter, and then the operation will be according to the first or second Rule of this Chapter.

So 7 $\frac{3}{5}$ being given to be subtracted from 12, the remainder will be found 4 $\frac{2}{5}$; *viz.* First 7 $\frac{3}{5}$ will be reduced into the improper Fraction $\frac{38}{5}$, also 12 will be reduced to $\frac{12}{1}$, then these two improper fractions $\frac{38}{5}$ and $\frac{12}{1}$ will be reduced into their equivalent fractions $\frac{38}{5}$ and $\frac{60}{5}$ (which have a common Denominator.) Lastly, the difference between $\frac{38}{5}$ and $\frac{60}{5}$ is $\frac{22}{5}$, or 4 $\frac{2}{5}$. In like manner 9 $\frac{1}{2}$ being given to be subtracted from 12 $\frac{1}{5}$, the remainder will be found 2 $\frac{7}{10}$; as by the subsequent operation is manifest.

12	7 $\frac{3}{5}$	12 $\frac{1}{5}$	9 $\frac{1}{2}$
$\frac{12}{1}$	$\frac{38}{5}$	$\frac{61}{5}$	$\frac{19}{2}$
60		122	
38		95	
$\frac{22}{5}$ that is 4 $\frac{2}{5}$		$\frac{27}{10}$ that is 2 $\frac{7}{10}$.	

Although

Although the three last Rules be sufficient for all cases in *Subtraction* of *Fractions*, mixt numbers, or whole and mixt; nevertheless the following *Rules* will be more expeditious in the subtraction of mixt numbers, or whole and mixt, especially when the Integers consist of many places, as will be manifest by the operation, *viz.*

2. By particular Rules, viz. 1. A whole number from a mixt number.

IV. When a whole number is given to be subtracted from a mixt number, subtract the said whole number from the integer or Integers of the mixt number (as is taught by the Rules of the fourth Chapter) and unto the remainder annex the fractional part of the mixt number given, so is the mixt number thus found, the remainder or difference sought.

$$\begin{array}{r} 24\frac{5}{8} \\ 7 \\ \hline 17\frac{5}{8} \end{array}$$

As if 7 be given to be subtracted from $24\frac{5}{8}$, the remainder will be $17\frac{5}{8}$ as by the operation is manifest.

2. A Fraction from an Integer.

V. When a fraction is given to be subtracted from an Integer, subtract the Numerator from the Denominator, and place that which remains over the Denominator, which new fraction thus found, is the remainder or difference sought.

So $\frac{3}{5}$ being subtracted from an Integer, or 1, the remainder is $\frac{2}{5}$: Also $\frac{11}{13}$ being subtracted from 1, the remainder is $\frac{2}{13}$.

3. A Fraction from a whole number greater than 1.

VI. When a fraction is given to be subtracted from a whole number greater than 1, subtract the said fraction from one of the Integers given by the last Rule;) so the remaining

remaining fraction being annexed to the number of Integers lessened by unity or 1, gives the remainder or difference sought.

Thus $\frac{5}{7}$ being subtracted from 17, the remainder is $16\frac{2}{7}$; also $\frac{7}{12}$ being subtracted from 39, the remainder is $38\frac{5}{12}$.

4. A mixt number from a whole number.

VII. When a mixt number is given to be subtracted from a whole number, subtract first of all (by the fifth Rule of this Chapter) the fractional part of the mixt number from an Integer borrowed from the whole number given, and set down the remaining fraction, then adding the Integer borrowed unto the Integer or Integers of the mixt number, subtract the said sum from the whole number given (as is taught in subtraction of whole numbers;) so that which remains, together with the remaining fraction before found, is the remainder or difference sought.

So if $9\frac{7}{12}$ be subtracted from 50, the remainder is $40\frac{5}{12}$, as by the operation is manifest.

$$\begin{array}{r} 50 \\ 9\frac{7}{12} \\ \hline 40\frac{5}{12} \end{array}$$

5. A Fraction from a mixt number by this and the next Rule.

VIII. When a fraction is given to be subtracted from a mixt number, and the said fraction is less than the fractional part of the mixt number, subtract the lesser fraction from the greater by the first or second Rule of this Chapter, then the remaining fraction being annexed to the Integer or Integers of the mixt number, gives the remainder or difference sought.

So $\frac{5}{9}$ being subtracted from $12\frac{7}{8}$ the remainder is $12\frac{23}{72}$, as by the operation is manifest.

$$\begin{array}{r} 12\ \frac{7}{8} \\ 0\ \frac{5}{9} \\ \hline 12\ \frac{23}{72} \end{array}$$

IX. When a fraction is given to be subtracted from a mixt number, and the said Fraction is greater than the fractional part of the mixt number, subtract the said greater fraction from an Integer borrowed from the mixt number (by the fifth Rule of this Chapter) and add the remaining fraction unto the fractional part of the mixt number (by the first or second Rule of the eighteenth Chapter) so the fraction found by that addition, being annexed to the Integers of the mixt number lessened by an Integer, or 1, gives the remainder or difference sought.

Thus $\frac{5}{9}$ being subtracted from $13\frac{3}{8}$, the remainder is $12\frac{59}{72}$, *viz.* subtracting $\frac{5}{9}$ from 1, the remainder is $\frac{4}{9}$, which added to $\frac{3}{8}$ gives $\frac{59}{72}$, which being annexed to 12 (the number of Integers in the mixt number lessened by 1 or unity) gives $12\frac{59}{72}$ the remainder sought.

$$\begin{array}{r} 13\ \frac{3}{8} \\ 0\ \frac{5}{9} \\ \hline 12\ \frac{59}{72} \end{array}$$

6. A mixt number from a mixt number by this and the next Rule.

X. When a mixt number is given to be subtracted from a mixt number, and the fractional part of the mixt number to be subtracted, is less than the fractional part of the mixt number from which you are to subtract, subtract the lesser fraction from the greater (by the first or second Rule of this Chapter) and set down the remaining Fraction: also subtract the Integers of the lesser mixt number from the Integers of the greater (as in Subtraction of whole numbers,) so is the mixt number thus found, the remainder or difference sought.

So

So if $17\frac{3}{8}$ be given to be subtracted from $20\frac{5}{7}$, the remainder will be found $3\frac{19}{56}$, *viz.* subtracting $\frac{3}{8}$ from $\frac{5}{7}$, the remainder is $\frac{19}{56}$ also subtracting 17 from 20, the remainder is 3.

$$\begin{array}{r} 20\ \frac{5}{7} \\ 17\ \frac{3}{8} \\ 3\ \frac{19}{56} \end{array}$$

XI. When a mixt number is given to be subtracted from a mixt number, and the fractional part of the mixt number to be subtracted is greater than the fractional part of the mixt number from which you are to subtract, subtract the said greater Fraction from an Integer borrowed from the greater mixt number (by the fifth Rule of this Chapter) and add the remaining fraction unto the fractional part of the greater mixt number (by the first or second Rule of the 18th Chapter;) so is the sum to be reserved as the fractional part of the remainder sought; then add the Integer borrowed unto the Integer or Integers of the lesser mixt number, and subtract the sum from the Integers of the greater mixt number (as in subtraction of whole numbers;) so that which remains, together with the fraction before reserved, is the remainder or difference sought.

Thus if $20\frac{7}{8}$ be given to be subtracted from $35\frac{3}{5}$ the remainder will be found $14\frac{29}{40}$, *viz.* subtracting $\frac{7}{8}$ from an Integer or 1, the remainder is $\frac{1}{8}$, which added to $\frac{3}{5}$ gives $\frac{29}{40}$, then adding the Integer borrowed unto 20, it will be 21, which subtracted from 35, the remainder is 14, so the remainder or difference sought is $14\frac{29}{40}$.

$$\begin{array}{r} 35\ \frac{3}{5} \\ 20\ \frac{7}{8} \\ 14\ \frac{29}{40} \end{array}$$

When

When you cannot clearly discern which is the greater of two fractions, having unequal Denominators, reduce them into fractions of the same value which shall have a common Denominator (by the fourteenth Rule of the seventeenth Chapter) and then it will be apparent which of the two fractions is the greater. As if it be desired to know which of these two fractions $\frac{5}{7}$ and $\frac{11}{13}$ is the greater, after they are reduced to $\frac{78}{91}$ and $\frac{77}{91}$, it is evident that the former exceeds the latter by $\frac{1}{91}$.

To discern the greater of two fractions.

CHAP. XX.

Multiplication of Vulgar Fractions, and mixt Numbers.

I. WHen the numbers given to be multiplied are both single fractions, multiply the Numerators one by the other, and take the Product for a new Numerator; also multiply the denominators one by the other, and the Product is a new denominator, which new Fraction is the Product sought.

To multiply single Fractions.

So $\frac{7}{12}$ and $\frac{5}{8}$ being given to be multiplied, the Product will be found $\frac{35}{96}$, for 7 multiplied by 5 produceth 35 for a new Numerator, and 12 multiplied by 8 produceth 96 for a new Denominator: also $\frac{5}{7}$ and $\frac{3}{7}$ being multiplied one by the other, the Product will be found $\frac{15}{49}$. Here you may observe that in the multiplication of proper *Fractions*, the Product is always less than either of the terms given; for in multiplication such proportion as

as unity or 1 hath to either of the terms given, the same proportion hath the other term to the Product.

II. When one of the numbers given is a whole number or a mixt number; also when both of them are mixt numbers, reduce such whole number or mixt number or numbers into an improper fraction or fractions by the tenth or eleventh Rules of the seventeenth Chapter, and then the operation will be the same as in the last Rule.

To multiply mixt numbers.

So $8\frac{2}{3}$ being given to be multiplied by 5, the Product will be found $43\frac{1}{3}$; *viz.* $8\frac{2}{3}$ being reduced into the improper fraction $\frac{26}{3}$: Also 5 unto $\frac{5}{1}$, multiply 26 by 5, the Product is 130 for a new Numerator: Also multiplying 3 by 1, the Product is 3 for a new Denominator, which new Fraction $\frac{130}{3}$ being reduced (according to the thirteenth Rule of the seventeenth Chapter) will be $43\frac{1}{3}$ the Product sought. In like manner $7\frac{1}{2}$ being multiplied by $5\frac{3}{5}$, the Product will be found 42. Here observe, that when either of the terms given is a compound fraction, it is first of all to be reduced into a single fraction, and then the operation is as before.

Note 1. Sometimes the work of Multiplication in Fractions may be very usefully contracted by this following Rule, *viz.*

When two fractions propos'd to be multiplied (whether they be proper or improper) are such, that the Numerator of the one, and the Denominator of the other, may be severally divided by some common Divisor without a remainder; you may take

take the Quotients instead of the said Numerator and Denominator, and then multiply as before in the first Rule of this Chapter: As for example, if $\frac{6}{7}$ be to be multiplyed by $\frac{5}{12}$; because 6 the Numerator of the first, and 12 the Denominator of the latter Fraction, being severally divided by their common Divisor 6, give the Quotients 1 and 2, I set these (or imagine them to be set) in the places of 6 and 12; by which exchange there arise $\frac{1}{7}$ and $\frac{5}{2}$, these multiplied one by the other (according to the first Rule of this Chapter) produce $\frac{5}{14}$ the desired Product of $\frac{6}{7}$ into $\frac{5}{12}$, in the smallest terms.

Again to multiply $\frac{16}{48}$ by $\frac{3}{16}$; because the Numerator of the first Fraction and the Denominator of the latter, being each divided by 16 give the Quotients 1 and 1, I set 1 and 1 in the places of 16 and 16; likewise because 48 the Denominator of the first, and 3 the Numerator of the latter Fraction, being each divided by their common Divisor 3, give 16 and 1, I take 16 and 1 instead of 48 and 3, so by those exchanges there arise $\frac{1}{16}$, and $\frac{1}{1}$, which multiplyed one by the other produce $\frac{1}{16}$, which is the Product in the smallest terms made by the multiplication of $\frac{16}{48}$ into (or by) $\frac{3}{16}$.

2. To take any part or parts of a number propounded, is nothing else but to multiply the said number by the Fraction which declareth what part is to be taken: So if you desire to know what is $\frac{5}{8}$ of 320, multiply $\frac{320}{1}$ by $\frac{5}{8}$, or $\frac{40}{1}$ by $\frac{5}{1}$, and the Product will be 200. In like manner $\frac{2}{3}$, of $45\frac{3}{8}$ is $30\frac{1}{4}$. Also $\frac{1}{4}$ of 120 is 30.

3. Sometimes the work of multiplication in mixt numbers

numbers

numbers may be compendiously performed after the manner of these following examples, *viz.* if it be required to multiply $120\frac{1}{4}$ by $48\frac{1}{2}$, first multiply the whole numbers mutually, to wit, 120 by 48, and place the particular products orderly one under the other as in Multiplication of whole numbers; then multiply the said whole numbers first given by the fractions alternately, *viz.* take $\frac{1}{4}$ of 48 which is 12, also take $\frac{1}{2}$ of 120 which is 60, and place the said 12 and 60 orderly to be added to the former particular Products: Lastly, add all together, and to the sum annex the product of the two fractions, to wit in this example, the product of the Multiplication of $\frac{1}{4}$ by $\frac{1}{2}$, which is $\frac{1}{8}$, so the total Product required will be $5832\frac{1}{8}$, as you see by the example in the Margent. In like manner, if $18\frac{1}{2}$ be multiplied by $40\frac{1}{3}$, the Product will be $746\frac{1}{6}$; and if $29\frac{1}{2}$ be multiplied by 50, the Product will be 1475, as you see by the examples following.

$120\frac{1}{4}$
$48\frac{1}{2}$
900
480
12
60
$5832\frac{1}{8}$

$18\frac{1}{2}$	$29\frac{1}{2}$
$40\frac{1}{3}$	50
720	1450
20	25
6	1475
$746\frac{1}{6}$	

4. When a fraction is to be multiplied by a number which happens to be the same with the Denominator, take the Numerator for the Product; so if this fraction $\frac{3}{4}$ be propounded to be multiplied by the Denominator 4, the Product will

be

be $\frac{12}{4}$ that is 3, which is the same with the Numerator 3. In like manner if $\frac{5}{8}$ be multiplied by the denominator 8, the Product is equal to 5 the Numerator of the said $\frac{5}{8}$.

CHAP. XXI.

Division of Vulgar Fractions and mixt Numbers.

I. WHen the numbers given are both single fractions, multiply the Denominator of the Divisor by the Numerator of the Dividend, and take the Product for a new Numerator: also multiply the Numerator of the Divisor by the Denominator of the Dividend, and the Product is a new Denominator; which new fraction is the quotient sought.

The Division of singl Fractions.

So if $\frac{4}{9}$ be given to be divided by $\frac{3}{5}$, the quotient will be found $\frac{20}{27}$; *viz.* multiplying 5 by 4 the Product is 20 for a new Numerator, also multiplying 3 by 9, the Product is 27 for a new Denominator, so is $\frac{20}{27}$ the quotient sought; in like manner if $\frac{5}{8}$ be given to be divided by $\frac{2}{7}$, the quotient will be found $\frac{35}{16}$, that is $2\frac{35}{16}$, as you see in the Example: here you may observe, that in Division by proper fractions, the quotient is always greater than either of the fractions given; for in Division, as the divisor is in proportion to 1 or unity, so is the dividend to the quotient.

$\frac{3}{5}) \frac{4}{9} (\frac{20}{27}$

$\frac{2}{7}) \frac{5}{8} (\frac{35}{16}$

II. When

II. When one of the numbers given is a whole number or a mixt number; also when both are mixt numbers, reduce such whole number or mixt number or numbers into an improper fraction or fractions, by the tenth or eleventh Rule of the seventeenth Chapter, and then the operation will be the same as in the last Rule.

So if 42 be divided by $7\frac{1}{2}$ the quotient will be found $5\frac{3}{5}$, for $7\frac{1}{2}$ and 42 will be reduced into these improper fractions $\frac{15}{2}$ and $\frac{42}{1}$, then multiplying 42 by 2, the Product is 84 for a new Numerator, also multiplying 15 by 1, the product is 15 for a new Denominator, so is $\frac{84}{15}$ the quotient sought, which is equal to $5\frac{3}{5}$ (as is evident by the thirteenth Rule of the seventeenth Chapter.) In like manner, if $6\frac{1}{2}$ be divided by $3\frac{2}{5}$, the quotient will be $1\frac{31}{34}$. Also if $5\frac{2}{3}$ be divided by $12\frac{1}{5}$ the quotient will be $\frac{32}{75}$.

$7\frac{1}{2}) 42 ($

$\frac{15}{2}) \frac{42}{1} (\frac{84}{15}$

$15) 84 (5\frac{3}{5}$

Note, Sometimes the Work of Division in Fractions may be very usefully contracted by this following Rule, *viz.* When either the two Numerators, or the two Denominators of the Fractions proposed can be divided severally by some common Divisor without a remainder, you may take the Quotients instead of the said Numerators or Denominators, and then divide by the first Rule of this Chapter: As for Example if $\frac{12}{17}$ be to be divided by $\frac{8}{5}$, because the Numerators 12 and 8 being each divided by their common Divisor 4 will give the Quotients 3 and 2; I take these instead of 12 and 8, by which exchange there arise $\frac{3}{17}$ and $\frac{2}{5}$, the former of which being divided by the latter, (accord-

ing

ing to the first Rule of this Chapter) given $\frac{15}{34}$, which is the Quotient in the least terms that ariseth by dividing $\frac{12}{17}$ by $\frac{8}{5}$.

Again, to divide $\frac{25}{8}$ by $\frac{15}{8}$; because the Numerators 25 and 15 being severally divided by their common Divisor 5 give the Quotients 5 and 3, likewise because the Denominators 8 and 8 being each divided by 8 give the Quotients 1 and 1, I set 5 and 3 in the places of the Numerators 25 and 15, also 1 and 1 in the places of the Denominators 8 and 8, whence arise $\frac{5}{1}$ and $\frac{3}{1}$. Lastly, dividing $\frac{5}{1}$ by $\frac{3}{1}$, that is 5 by 3, there ariseth $\frac{5}{3}$, that is $1\frac{2}{3}$, which is the desired Quotient of $\frac{25}{8}$ divided by $\frac{15}{8}$.

Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. The difference of two numbers is $1\frac{13}{24}$, the lesser number is $2\frac{1}{8}$, what is the greater? *Answ.* $3\frac{2}{3}$, (found by *Addition.*)

Q. 2. What number is that, which if added to $3\frac{5}{8}$ gives the sum $8\frac{23}{88}$? *Answ.* $4\frac{7}{11}$ (found by *Subtraction.*)

Quest. 3. There is in three bags the sum of $121\frac{9}{48}$*l.* *viz.* in the first bag $50\frac{5}{8}$*l.* in the second $40\frac{4}{13}$*l.* what is in the third bag? *Answer* $30\frac{1}{3}$*l.* (found by *Addition* and *Subtraction.*)

Quest. 4. Two Merchants *A* and *B*, having certain shares in a Ship, the share of *A* is $\frac{7}{10}$ of the Ship, that of *B* $\frac{9}{13}$, what is the difference between their parts? *Answ.* The share of *A* exceeds the share of *B* by $\frac{1}{130}$. (found by *Subtraction.*)

Quest.

Quest. 5. What is $\frac{5}{8}$ of $130\frac{2}{3}$? *Answ.* $81\frac{2}{3}$ (found by *Multiplication.*)

Quest. 6. What number is that, which being multiplied by $\frac{3}{5}$ produceth $25\frac{2}{5}$? *Answ.* $42\frac{1}{3}$ (found by *Division.*)

Now followeth the doctrine of *Decimal Fractions.*

The Doctrine of Decimal Fractions.

CHAP. XXII.

Notation of Decimal Fractions.

I. IT is hard to determine, who was the first that brought *Decimal Arithmetick* to light, though it be a late Invention; but without doubt it hath received much improvement within the compass of a few years, by the industry of *Artists*, and now seems to be arrived at perfection. The excellency thereof is best known to such as can apply it to the practical part of the *Mathematicks*, and to the Construction of *Tables*, which depend upon standing or constant proportions, such are *Trigonometrical Canons*, *Tables* for computing of *compound Interest*, &c. in which cases decimal operations do afford so great help, (that in my opinion) many ages have not produced a more useful invention.

The proper use of Decimal Arithmetick.

But it may be objected, that *Decimal Arithmetick* for the most part gives an imperfect solution to

a question. This I grant, yet the answer so given may be as useful as that which is exactly true; for in common affairs, the loss of $\frac{1}{1000}$ part of a *grain*, or of an *inch*, &c. to wit, any quantity which cannot be seen, is inconsiderable: but I could not be mistaken, for in extolling *Decimals* I do not cry down *Vulgar Fractions*, since experience sheweth that *Decimal Fractions* are commonly abused, by being applyed to all manner of questions about *money*, *weight*, &c. when indeed many questions may be resolved with much more facility by *Vulgar Arithmetick*, as may partly appear by this Example, *viz.* at 9*l.*—6*s.*—8*d.* the hundred weight of Tabaco, what will 987 hundred weight cost? *Answ.* 9212*l.* which by the common *Rule* of *Practice* by *Aliquot parts* is found out in a quarter of the time, that will necessarily be required to work it by *Decimals*, which at last will give an imperfect Answer; I might instance the like inconvenience divers ways, were it not for loss of time; so that the right use of *Decimals* depends upon the discretion of the *Artist*.

Decimal Fractions sometimes abused.

II. When a single Fraction hath for its denominator a number consisting of 1 or unity in the extream place towards the left hand, and nothing but a Cypher or Cyphers towards the right, it is more particularly called a *Decimal*: of this kind are these that follow, $\frac{5}{10}$, that is five tenths, $\frac{5}{100}$, five hundredth parts; likewise these are decimal fractions, $\frac{34}{1000}$, $\frac{205}{10000}$, $\frac{1023}{10000}$, &c.

The definition of a Decimal Fraction.

III. A Decimal fraction may be express'd without the Denominator, by prefixing a point or comma before (to wit, on the left hand of) the Numerator, so $\frac{5}{10}$ may be written thus, .5 or thus, ,5, and $\frac{25}{100}$ thus, .25, or thus, ,25.

IV. In Decimals when the Numerator consists not of so many places as the Denominator hath Cyphers, fill up the void places in the Numerator with Cyphers prefixed on the left hand: so $\frac{5}{100}$ is written thus .05; likewise $\frac{50}{1000}$ thus, .050; and $\frac{205}{10000}$, thus, .0205, likewise $\frac{6}{1000}$, thus, .006.

V. In Decimals thus express'd, the Denominator is discoverable by the places of the Numerator: for if the Numerator consists of one place, the Denominator consists of 1 or unity with one Cypher; if of two places, the Denominator consists of 1 with two Cyphers annexed; if of three, the Denominator consists of 1 or unity with three Cyphers annexed: so the Denominator of .25 is 100; the Denominator of .050 is 1000, and the Denominator of .096 is 1000.

VI. Cyphers at the end of a Decimal do neither augment nor diminish the value thereof: so .2, .20, .200, .2000 are *decimals*, which have one and the same value, for $\frac{20}{100}$ being abbreviated by the eighth *Rule* of the seventeenth *Chapter*, will be made $\frac{2}{10}$ and so will $\frac{200}{1000}$ or $\frac{2000}{10000}$.

VII. Wherefore Decimal fractions are easily reduced to a common Denominator (which is a troublesome work in *Vulgar Fractions*;) for if all the Numerators of as many decimal fractions as are given, be made to consist of the same number of places, by annexing a Cypher or Cyphers at the

end (that is on the right hand) of ſuch Numerators as are defective, they will all be reduced to a common Denominator, ſo theſe *Decimal* .2, .03, .027 (which ſignifie $\frac{2}{10}$, $\frac{3}{100}$, $\frac{27}{1000}$) may be reduced into theſe, .200, .030, .027, which have a 1000 for a common Denominator.

VIII. The order of places in any Decimal proceedeth from the left hand to the right, contrary to the order of places in Integers, which is from the right hand to the left: ſo in this *Decimal*, .247 the figure 2 ſtandeth in the firſt place (being the outermoſt towards the left hand, and next to the point,) the figure 4 ſtandeth in the ſecond *place*, and 7 in the *third*. Alſo in this *Decimal* .0245, a Cypher ſtands in the *firſt place*, 2 in the *ſecond*, 4 in the *third*, and 5 in the *fourth*.

IX. Every place in the Numerator of a Decimal Fraction hath a peculiar Denominator, or proper value, *viz.* the Denominator of the firſt place is 10, of the ſecond, 100; of the third, 1000, &c. ſo that the firſt place of a Decimal ſignifies tenth parts of an unite or Integer; the Second place, hundreth parts of an Integer; the third place, thouſandth parts of an Integer; &c. Hence it is manifeſt, that this *Decimal* .3254 (every place thereof being conſidered apart by it ſelf) conſiſts of .3, .02, .005, .0004 (*viz.* $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$) which being reduced to a common Denominator (by the ſeventh *Rule* of this *Chapter*) will give theſe, .3000, .0200, .0050, .0004 (to wit, $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$) all which collectively make .3254 (or $\frac{3254}{10000}$)

X. In whole numbers, the firſt place above (that is on the left hand of) the place of unities, ſignifies

Tens of Unites; but the firſt place beneath (that is on the right hand of) the place of Unities, ſignifies tenth parts of 1 or Unity, and is called the firſt place of Decimal parts, or place of Primes; likewiſe the ſecond place above the place of Unities, ſignifies hundreds of Unities, but the ſecond place beneath the place of Unities, ſignifieth hundredth parts of 1 or Unity, and is called the *ſecond place of Decimals*, or place of ſeconds; ſo that as the values of the places in Integers, do aſcend in a decuple proportion from the place of Units towards the left hand, ſo the values of the places of decimals do deſcend in a ſubdecuple proportion beneath the place of Units towards the right hand: *viz.* Among the places of Integers, every following place towards the left hand, is ten times the value of the next preceeding place: But among the places of decimal parts, every following place towards the right hand is one tenth part of the value of the next preceeding place: All which will be evident by the following *Table*.

A TABLE for the Notation of Integers and Decimals.

Integers.		
&c.		*&c.*
Ten Thousands	7	*Fifth place*
Thousands	3	*Fourth place*
Hundreds	2	*Third place*
Tens	8	*Second place*
Unites (under 10)	5	*First place*
of Unities.		

Decimal parts		
Ten parts	8	*First place*
Hundreth parts	2	*Second place*
Thousandth parts	3	*Third place*
Ten Thousandth parts	7	*Fourth place*
&c.		*&c.*
of 1 or Unity.		

In the foregoing Table you may obſerve, that the places of Integers,or whole numbers are ſeparated from the places of *Decimal parts* of 1 (or unity) by a point; ſo the number on the left hand of the

the point expreſſeth 73285 Integers or Unities, but the number on the right hand of the point expreſſeth only 8237 parts of 1 (or an Integer) ſuppoſed to be divided into 10000 equal parts. In like manner this nomber 5 .8 ſignifies 5 Integers and eight tenth parts of an Integer, and this number 285 .82 ſignifies 285 Integers (or Unities) and $\frac{82}{100}$ parts of an Integer.

CHAP. XXIII.

Concerning the Reduction of Vulgar Fractions to Decimal Fractions.

I. IF the greateſt Integer of *money*, as alſo of *weight*, *meaſure*, &c. were ſubdivided decimally, to wit, a *pound* of Engliſh money into ten equal pieces of coin, and every one of theſe into ten other equal pieces, &c. and *weights*, *meaſures*, &c. after the ſame manner: the doctrine of Arithmetick would be taught with much more eaſe and expedition than now it is; but it being improbable that ſuch a reformation will ever be brought to paſs, I ſhall proceed in directing a courſe to the ſtudious for obtaining the frugal uſe of ſuch Decimal fractions as are in his power.

II. Foraſmuch as in *Arithmetical* queſtions, ſome of the given numbers do for the moſt part happen to be fractions, a way muſt be ſhew'd how to reduce a *Vulgar Fraction* to a *Decimal Fractions*, yet in ſome

some cases there is no need of this *Reduction*; for
example, a *foot* in length is vulgarly subdivided in-
to 12 inches, an inch into 4 quarters, and each
quarter into 2 half quarters; but a *foot* may as
easily, and a great deal more commodiously be di-
vided, first into ten equal parts, and then each of
those into ten other equal parts, and each of these
into ten other equal parts; (or at least such divi-
sion must be supposed or imagined when it cannot
actually be made.) This *foot* in length so divided be-
ing applied to the sides of *superficial figures*, or of
solids will at first sight give the quantities of lines in
feet and *dicimal parts* of a *foot* (as readily as a *foot*
vulgarly divided will shew you how many *feet*, *in-
ches*, *quarters*, and *half quarters* are contained in
any line) from whence the *superficial* or *solid con-
tent* may be found in *feet* by *multiplication* only; and
how much this excels the *vulgar way*, I shall part-
ly manifest in the fifth Rule of the 26th Chapter.
The like subdivision I would have to be made
of a *Yard*, *Perch*, &c.

How to reduce a vulgar fraction to a decimal fraction.

III. A single fraction, which is no decimal
fraction, may be reduced into a deci-
mal of the same value, or infinitely
near (for all vulgar fractions cannot
be exactly reduced to decimals) by
the Rule of Three direct; for as the
Denominator of any single fraction whatsoever, is
to the Numerator thereof, so is any other Denomi-
nator to his correspondent Numerator: *Example*,
let it be required to reduce $\frac{5}{8}$ into a Decimal, whose
Denominator is assigned to be 1000, say by the *Rule
of three*, if the Denominator 8 hath 5 for a Nume-
rator, what will the Denominator 1000 require for
a

a Numerator? Multiply and divide as the Rule of
Three direct doth require, so will the fourth *propor-
tional* be found to be 625, which is the Numerator
sought; therefore $\frac{625}{1000}$ or 625, is a *decimal fraction*
equal in value to $\frac{5}{8}$. Another *Example*. Let it be re-
quired to reduce $\frac{7}{240}$ into a *decimal fraction*, whose
Denominator shall be 100000, say by the Rule of
three, if 240 the Denominator give 7 for a Nume-
rator, what will the *Denominator* 100000 require
for a *Numerator*? *Answ.* 2916 and somewhat more;
but that which the said 2916 wants of being a true
Numerator is less than $\frac{1}{1000}$ part of an Integer,
therefore the *decimal fraction* $\frac{2916}{100000}$ or .02916 is al-
most equal to $\frac{7}{240}$, which $\frac{7}{240}$ cannot be exactly re-
duced into a *decimal fraction*. The like will happen
in the reduction of most *vulgar fractions* to *decimals*,
in which case, the *Denominator* of the *decimal* must
be assigned to be so great, that what is wanting in
the *Numerator* may be an inconsiderable value.

IV. Upon the aforesaid ground, the known or
accustomary parts of *Money*, *Weight*, *Measure*, *Time*,
&c. may be reduced to *decimals*: for if you desire to
know what *decimal fraction* of a pound *sterling* is e-
qual in value to one *shilling*, consider first that a
pound is the *Integer*, and that 20 *shillings* are equal
to that *Integer*, therefore 1 shilling is $\frac{1}{20}$ of a pound,
how if we conceive one pound to be divided into
100000 parts, *viz*. if we assign 100000 for the *Deno-
minator* of a *decimal fraction*, the *Numerator* will be
found by the last Rule to be 5000, so that $\frac{5000}{100000}$ or
.05000 or 05 (for Cyphers at the end of a *decimal*
are of no use, as hath been shewn in the 6th *Rule* of
the 22 *Chapter*) is a *decimal fraction* of a *pound*, and is
exactly

actly equal to 1*s.* or $\frac{1}{20}$ part of a pound *sterling*.

In like manner forasmuch as 240 *pence* are equal to a pound of English money, 7 *pence* are $\frac{7}{240}$ parts of a pound, which *fraction* will be reduced into this *decimal* .02916*l.* which is very near equal to $\frac{7}{240}l.$ for it wants not $\frac{1}{100000}$ part of a pound. Moreover since 960 *farthings* are equal to a *pound* English, one *farthing* is $\frac{1}{960}$ part of a pound, which will be reduced into this *decimal* .00104*l.* very near; but if you please to proceed near to the truth, you will find this *decimal* .00104166, &c. to answer a *farthing*, and so by augmenting the Denominator with Cyphers, you may proceed infinitely near, when you cannot attain unto the truth it self. After the same method may the vulgar *Sexagenary fractions* used in *Astronomy* be reduced to *decimals*; for since a *degree* is usually subdivided into sixty parts called *minutes* or *primes*; a *prime* or *minute* into sixty parts called *seconds*; a *second* into sixty *thirds*; a third into sixty *fourths*, &c. and consequently a *degree* is equal unto 60 *minutes* (or *Primes*) or unto 3600 *seconds*, or 216000 *thirds*, or 12960000 *fourths*, &c. It is evident that 7 *minutes* (or *Primes*) are $\frac{7}{60}$ parts of a *degree*, which by the third *Rule* of this *CHAPTER* may be reduced into the Decimal .1166, &c. Also 29 *thirds* are $\frac{29}{216000}$ parts of a *degree* which may be reduced into the Decimal .000134, &c.

Moreover, 58 : 33 : 14 : 12, that is, 58 *primes*, 33 *seconds*, 14 *thirds*, and 12 *fourths* may be reduced to a *decimal* in this manner, *viz.* reduce them all into *fourths* (according to the sixth *Rule* of the seventh *Chapter*) so will you find 12647652 *fourths*, which are

are $\frac{12647652}{12960000}$ parts of a *degree*, which *vulgar fraction* may be reduced into this *decimal* of a *degree*, to wit .975899, &c. (by the third *Rule* of this *Chapter*.)

This to the ingenious will be a sufficient light for the finding of the *Decimals* congruent to the *shillings*, *pence*, and *farthings*, which are under a *pound sterling*; also the *Decimals* of the known parts of *Weight*, *Measure*, *Time*, &c. as they are express'd in the following *Table*, wherein you may observe, that most of the *Decimals* consist of 7 or 8 figures, yet in ordinary practice, you shall have occasion to use only the first five, and sometimes fewer.

THE

THE TABLE OF REDUCTION.

TABLET I.
Of English Money, the Integer being a Pound.

Shillings.	Decimals of a Pound.
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

Pence with Farthings.	Decimals of a Pound.
	.0489583
	.0479166
	.046875
11	.0458333
	.0447916
	.04375
	.0427082
10	.0416666
	.040625
	.0395833
	.0385416
9	.0375
	.0364583
	.0354166
	.034375
8	.0333333
	.0322916
	.03125
	.0302083
7	.0291666
	.028125
	.0270833
	.0260416
6	.025
	.0239583
	.0229166
	.021875
5	.0208333
	.0197916
	.01875
	.0177708
4	.0166666
	.015625
	.0145833
	.0135416
3	.0125
	.0114583
	.0104166
	.009375
	.0083333
2	.0072916
	.00625
1 Pen. & 1 Far.	.0052083
Peny 1	.0041666
3. Farth.	.003125
2. Farth.	.0020833
1. Farth.	.0010416

TABLET II.
Of Troy weight, the Integer being an Ounce.

Peny weights.	Decimals of an Ounce.
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

Grains.	Decimals of an Ounce.
23	.0479166
22	.0458333
21	.04375
20	.0416666
19	.0395833
18	.0375
17	.0354166
16	.0333333
15	.03125
14	.0291666
13	.0270833
12	.025
11	.0229166
10	.0208333
9	.01875
8	.0166666
7	.0145833
6	.0125

5	.0104166
4	.0083333
3	.00625
2	.0041666
1	.0020833

TABLET III.
Of Averdupois great weight, the Integer being an hundred weight, to wit, 112 pounds.

quarters of 1 hundred	*decimals of 1 hundred.*
3	.75
2	.5
1	.25

Pounds.	*decimals of 1 hundred.*
27	.2410714
26	.2321428
25	.2232142
24	.2142857
23	.2053571
22	.1964285
21	.1875
20	.1785714
19	.1696428
18	.1607142
17	.1517857
16	.1428571
15	.1339285
14	.125
13	.1160714
12	.1071428
11	.0982142
10	.0892857
9	.0803571
8	.0714285
7	.0625
6	.0535714
5	.0446428
4	.0357142
3	.0267857
2	.0178571
1	.0089285

Ounces.	*decimals of 1 hundred.*
15	.0083705
14	.0078125
13	.0072544
12	.0066964
11	.0061383
10	.0055803
9	.0050223
8	.0044642
7	.0039062
6	.0033482
5	.0027901
4	.0022321
3	.0016741
2	.0011160
1	.0005580

quarters of 1 Ounce.	*decimals of 1 hundred.*
3	.0004185
2	.0002790
1	.0001395

TABLET

TABLET IV.
Of Averdupois little weight, the Integer being a Pound.

Ounces.	*Decimals of a pound*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

Drams.	*Decimals of a pound.*
15	.05859375
14	.0546875
13	.05078115
12	.046875
11	.04296875
10	.0390625
9	.03515625
8	.03125
7	.02734375
6	.0234375
5	.01953125
4	.015625
3	.01171875
2	.0078125
1	.00390625

quarters of a dram	*decimals of 1 pound.*
3	.0029296
2	.0019531
1	.0009765

TABLET V.
Of liquid Measures, the Integer being a gallon.

Pints.	*decimals of 1 gallon.*
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

quarters of a pint.	*decimals of a gallon.*
3	.09375
2	.0625
1	.03125

N TABLET

TABLET VI.

Of dry measures, the Integer being a Quarter.

Bushels.	decimals of a quarter.
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Pecks.	decimals of a quarter.
3	.09375
2	.0625
1	.03125

Quarters of a peck.	decimals of a quarter.
2	.0234375
	.015625
1	.0078125

Pints.	decimals of a quarter
3	.005859
2	.003906
1	.001953

TABLET VII.

Of long measures, one Yard or one Ell being the Integer.

quarters of 1 yard or 1 ell.	decimals of 1 yard or 1 ell.
3	.75
2	.5
1	.25

Nails.	decimals of 1 ya. or 1 ell.
3	.1875
2	.125
1	.0625

quarters of 1 nails.	decimals of 1 ya. or 1 ell.
3	.046875
2	.03125
1	.015625

TABLET VIII.

Of the Reduction of Inches, &c. to decimals, the Integer being a foot in length.

Inches	decimals of a foot.
11	.9166666
10	.8333333
9	.75

Inches	decimals of a foot.
8	.6666666
7	.5833333
6	.5
5	.4166666
4	.3333333
3	.25
2	.1666666
1	.0833333

quarters of an Inch.	decimals of a foot.
3	.0625
2	.0416666
1	.0208333
half a quart. of an Inch.	.0104166

TABLET IX.

Of dozens, the Integer being a gross.

dozens.	decimals of a gross.
11	.9166666
10	.8333333
9	.75
8	.6666666
7	.5833333
6	.5
5	.4166666
4	.3333333
3	.25
2	.1666666
1	.0833333

parts of a dozen.	decimals of a gross.
11	.076388
10	.069944
9	.0625
8	.055555
7	.04861
6	.041666
5	.034722
4	.027777
3	.020833
2	.013888
1	.006944

TABLET X.

Of Time, a day being the Integer.

Hours.	decimals of a day.
23	.9583333
22	.9166666
21	.875
20	.8333333
19	.7916666
18	.75
17	.7083333
16	.6666666
15	.625
14	.5833333
13	.5416666
12	.5
11	.4583333
10	.4166666

9	.375
8	.3333333
7	.2916666
6	.25
5	.2083333
4	.1666666
3	.125
2	.0833333
1	.0416666

Minutes.	*decimals of a day.*
59	.0409722
58	.0402777
57	.0395833
56	.0388888
55	.0381944
54	.0375
53	.0368055
52	.0361111
51	.0354166
50	.0347222
49	.0340277
48	.0333333
47	.0326388
46	.0319444
45	.0312500
44	.0305555
43	.0298611
40	.0291666
41	.0284722
40	.0277777
39	.0270833
38	.0263888
37	.0256944
36	.0249999
35	.0243055
34	.0236111
33	.0229166
32	.0222222
31	.0215277
30	.0208333
29	.0201388
28	.0194444
27	.01875
26	.0180555
25	.0173611
24	.0166666
23	.0159722
22	.0152777
21	.0145833
20	.0138888
19	.0131944
18	.0125
17	.0118055
16	.0111111
15	.0104166
14	.0097222
13	.0090277
12	.0083333
11	.0076388
10	.0069444
9	.00625
8	.0034722
7	.0048611
6	.0041666
5	.0034722

4

4	.0027777
3	.0020833
2	.0013888
1	.0006944

V. This Table aforegoing consists of ten several Tablets, of which the first (intituled *English money*) contains in the first column thereof the particular Fractions (*viz.* the *shillings*, *pence*, and *farthings*) of a pound *sterling*; and in the other column the *decimals*, unto which they may be respectively reduced: So in the same Tablet .65 is the *decimal*, answerable to 13*s.* .0208333 to 5*d.* and .003125 to 3*f.* Likewise, .0489583 is the decimal of 11*d.* together with 3 farthings: Also .03125 is the decimal of 7 pence half-peny.

Tablet 1. Of English money.

VI. The next Tablet (intituled *Troy weight*) contains in the first column thereof the particular fractions (*viz.* the *Peny-weights*, and *Grains*) of an ounce *Troy*, and in the other their respective decimals: so .6 is the correspondent *decimal* of 12 peny weight, and .0020833 of 1 grain. Likewise .025 is the decimal of 12 grains.

2. Of Troy weight.

VII. The third Tablet (intituled *Averdupois great weight*) contains in the first column thereof the Fractions (*viz.* the *Quarters*, *Pounds*, *Ounces*, and the *Quarters of an Ounce* of an Hundred according to *Averdupois weight*, and in the other their proper decimals: so .5 is the *decimal* of two quarters or half a hundred, .1517857 of 17 pounds

3. Of Averdupois great weights.

 .0033482

.0033482 of 6 ounces, and .0004185 the *decimal* of 3 quarters of an ounce.

VIII. The fourth (intituled *Averdupois little weight*) sheweth you the fractions (*viz.* the *Ounces*, *Drams*, and *quarters* of a *dram*) of a pound *Averdupois*, together with their respective decimals: So the *decimal* of 3 Ounces is .1875, the *decimal* of 9 Drams is .03515625, and the *decimal* of one quarter of a Dram is .00097 5.

4. Of Averdupois little weight.

IX. The fifth (intituled *Liquid measures*) hath the fractions (*viz.* the *Pints* and *quarters of a pint*) of a Gallon, and likewise their several decimals: So the *decimal* of 5 Pints is .625, and the *decimal* of two quarts or half a pint is .0625.

5. Of Liquid measures.

X. The sixth (intituled *Dry Measures*) gives you the fractions (*viz.* the *Bushels*, *Pecks*, *quarters* of *Pecks* and *Pints*) of a quarter, together with their peculiar decimals: so .375 is the *decimal* of three Bushels, .03125, of one Peck, .0234375 of $\frac{3}{4}$ of a Peck, and .003906 of two Pints.

6. Of Dry measures.

XI. The seventh (intituled *Yards* and *Ells*) offers you the fractions (*viz.* the *Quarters*, *Nails*, and *quarters* of *Nails*) of Yards or Ells, and their respective decimals: so .25 is the *decimal* of one quarter of a Yard or Ell, .125 of two Nails, and .046875 of three quarters of a Nail.

7. Of Long measures.

XII. The eighth (intituled *Reduction of Inches, &c. to decimals of a foot*) presents unto you the fractions (to wit, the Inches, quarters and half quarter of an Inch) of a *foot*, together with their correspondent decimals: So .4166666 is the *decimal* of 5 Inches, .0625 of $\frac{3}{4}$ of an Inch, and 0.0104166 of $\frac{1}{8}$ or half a quarter of an Inch.

XIII. The ninth Tablet (intituled *Dozens*) yields you the Fractions (*viz.* the Dozens and particulars) of a Gross, as also their respective *decimals*: so .25 is the *decimal* of 3 Dozen and .048611 of 7 particulars.

8. Of things accompted by the Dozen.

XIV. The tenth and last Tablet (intituled *Time*) gives you the Fractions (*viz.* the Hours and Minutes) of a Day: So .625 is the *Decimal* of 15 hours .0375 of 54 minutes, and .0006944 of one minute.

9. Of Time.

XV. When a single Fraction of any of the premised Tablets is propounded to be reduced to a decimal, find it in the first Column of the Tablet, unto which it belongs; this done, just against that Fraction so found, you shall have the decimal required: So 13 *s.* being propounded, taking the first premised *Tablet*, I find 13*s*. in the first Column of the Tablet of *money*, and just against the same thirteen shillings, I *observe* .65, before which having prefix d a point, and by that means signed it for a *decimal* (according to the third Rule of the 22 Chapter of this Book) I conclude the same .65 so ordered, to be the correspondent *decimal* of thirteen shillings the fraction propounded. In like manner .0229166 is the *decimal* of 11 grains in the Tablet of *Troy* weight; and .0357142 the *decimal* of 4 *lb.* in the Tablet of *Averdupois great weight*, &c.

The use of the same Table for the Reduction.
1. Of single fractions to decimals.

XVI. When

XVI. When two or more Fractions are propounded, and it is required to find a decimal equivalent unto the ſum of them, find the decimal of each of the Fractions given according to the laſt Rule; then adding together the decimals ſo found, that intire ſum is the decimal ſought: So 13*s* 5*d.* being reduced to a decimal, is .670833; for the decimal of 13*s.* is .65, and the decimal of 5*d* .020833, which being added together (by the ſecond Rule of the 24th Chapter of this Book) amount to .670833, *viz.* the decimal which repreſents 13*s.* 5*d.* the Fraction propounded: In like manner the decimal of 9 peny weight, and 13 Grains is .4770833, and the decimal of $\frac{1}{2}$ *C.* 19 *lb.* 7 Ounces, is .67354, &c.

13*s.*	.65
5*d.*	.020833
	.670833

9 *p. w.*	.45
13 *gr.*	.027083
	.477083

$\frac{1}{2}$ *C.*	.5
19 *lb.*	.16964
7 *ounc.*	.00390
	.67354

And here as you ſee meer Fractions reduced, ſo likewiſe may the Fractions of mixt numbers be reduced to Decimals; for example, theſe numbers 97 *lb.*

lb. 7 ounces, 13 $\frac{1}{4}$ drams. *Item* of 67 Gallons 5 $\frac{3}{4}$ pints. *Item* 28 Quarters, 0 Buſhels and 2 $\frac{1}{2}$ Pecks, after reduction are 97 .4891, 67 .7187, and 28 .0781.

97.4375	67.625	28.0625
.0507	.0937	.0156
.0009		
97.4891	67.7187	28.0781

Again 22 $\frac{1}{2}$ yards, 3 $\frac{3}{4}$ Nails; *Item* 36 Groſs, 3 Dozen and 5 particulars, being reduced, are 22 .7031, 36 .2847.

22.5	36.25
.1875	.0347
.0156	
22.7031	36.2847

3. *Of Decimals to ſingle Fractions.*

XVII. When a Decimal is propounded to know what Fraction it repreſents, ſearch the ſame Decimal in the ſecond Column of the Tablet, unto which it belongs, where if you find it expreſly, the number juſt againſt it in the firſt Column is the fraction you look for: So .65 (repreſenting the fraction of a Pound ſterling) being given, I find it in the ſecond Column of the Tablet of Money, and over againſt it in the firſt Column I find 13*s.* which is the fraction repreſented by .65, the decimal propounded. In like manner 3 .025 (repreſenting 3 ounces and .025 of an ounce *Troy*) being propounded, the number repreſented by it, is 3 Ounces, 0 *p. w.* 12 grains.

XVIII. When in the ſecond Column of the Tablet,

Tablet, unto which you are directed, you cannot precisely find the decimal propounded, search that which being less, comes nearest unto it, and take the number that answers unto it in the first Column for the greatest fraction of the number required: Then deducting the decimal so found out of the decimal given, find likewise the remainder as another decimal, and take his correspondent number for the next fraction of the number required; And so proceed in that order, till you have discovered the intire number represented by the decimal propounded.

Example: .6739 being propounded, I demand the fraction of a pound sterling represented by it; the decimal in the Tablet of money, which being less comes nearest to .6739 is .65, whose correspondent number in that Tablet is 13, which are the shillings of the number required; then subtracting (by the 1 Rule of the 25 Chapter of this Book) .65 out of .6739, the remainder is .0239, and the nearest decimal in the same Tablet to .0239, is .0208, whose correspondent number is 5, which are the pence of the number required. Last of all deducting .0208 out of .0239, the remainder is .0031, which gives you in the first Column 3, being the farthings of the number required: So that I conclude the intire fraction represented by the decimal .6739, is 13 *s.* 5 *d.* 3 *f.*

	.6739*l. sterling.*
Subtract 13 *s.*	.65
	.0239
Subtract 5 *d.*	.0208
3 *f.*	.0031

In

In like manner 7.359*C*, being reduced by the Tablet of *Averdupois great weight* is $7\frac{1}{4}$ *C.* 12*lb.*4 ounc. And 94.58*lb.* reduced by the Tablet of *Averdupois little weight* is 94 *lb.* 9 ounces and 6 drams.

	7.359 *C.*
Subtract 1 *quarter*	.25
	.109
Subtract 12 *lb*	.107
4 *oz.*	.002
	94.58 *lb.*
Subtract 9 *oz.*	.56
6 *Drams.*	.02

CHAP. XXIV.

Addition of Decimal Fractions.

I. TO such as well understand the *Notation* of *Decimal Fractions*, all the varieties of their *Numeration*, to wit, *Addition*, *Subtraction*, &c. will be as easie as the operations by whole numbers; therefore he that would be a good Proficient in *Decimal Arithmetick*, must throughly understand the 22 and 23 *Chapters* aforegoing.

II. When divers decimal fractions are given to be added together, they must first of all be orderly placed one under another according to the Doctrine of their Notation. So if these *Decimal Fractions*, to wit, .125, .39 and .7 were given to be added, they must be written down thus;

.125
.39
.7

or

or if you will have the ſame number of places to be in all the *decimals* given, without altering their values, they may be written thus,

.125
.390
.700

Not thus,

.125
. 39
. 7

For the Figures or Cyphers, which are of like degrees or places muſt be ſubſcribed directly one under another, *viz. tenth parts of* or *primes* muſt be written down directly underneath *tenths*; alſo *hundredth parts* or *ſeconds* muſt be placed under *hundredth parts*, as you ſee in the firſt Example, where .3 or three tenth parts in the ſecond decimal ſtands directly under .1 or one tenth part in the firſt decimal; likewiſe .7 or ſeven tenths in the third decimal ſtands directly under the tenths in the former, and ſo of the reſt.

In like manner, when mixt numbers, which conſiſt of Integers and decimal parts are given to be added, due reſpect muſt be had of their ſubſcription one under another: ſo if theſe mixt numbers, to wit, 32 .056, 7 .07, and 1 .9 were given to be added, they muſt be written down thus,

32 .056
7 .07
1 .9

III. Having placed the decimals and drawn a line underneath in manner aforeſaid, add them together,

gether, beginning with the outermoſt rank towards the right hand (as hath been taught in Addition of whole numbers of one denomination in the third Chapter:) ſo if the decimals in the firſt Example of the ſecond *Rule* of this *Chapter* were given to be added, I firſt ſubſcribe 5, which is all that ſtands in the firſt rank towards the right hand, then proceeding to the ſecond rank, I ſay 9 and 2 make 11, wherefore I write down 1, which is the exceſs of 11 above 10, and for the 10 I carry 1 in mind to the next rank, ſaying 1 in mind added to 7 makes 8 which added to 3 and 1 make 12, wherefor I write 2, which is the exceſs of 12 above 10, under the line, reſerving 1 in mind for the 10, then I prefix a point before 2, which ſtands in the firſt place of decimals; and on the left hand of the point, to wit in the place of Units or firſt place of Integers, I write down 1 (being the 1 in mind) which done, I find that the ſum of the Decimals given is 1.215, that is, one Integer (whether it be a Perch, Yard, Foot, &c.) and $\frac{1215}{1000}$ parts of an Integer, as you ſee in the Example.

.125
.39
.7
1.215

In like manner theſe mixt numbers 32.056; 7.07 and 1.9 being given to be added, their ſum will be found to be 41 .026, that is, 41 Integers and $\frac{26}{1000}$ parts of an Integer, as you ſee in the Margent; more *Examples* for the learners exerciſe are theſe.

32 .056
7 .07
1 .9
41 .026

.65	24 .7	503 .75
.025	0 .35	0 .32
.03	5 .27	0 .12
.705	30 .31	504 .19

CHAP.

CHAP. XXV.

Subtraction of Decimal Fractions.

I. HAving first written down the greater of the two numbers given (whether it be a whole number, mixt number, or decimal) and the lesser underneath the greater, according to the directions in the second Rule of the 24 Chapter, proceed as you are taught in Subtraction of whole numbers (by the Rules of the 4th Chapter:) So if this decimal fraction .784 were given to be subtracted from this decimal .837, the remainder will be .053, that is $\frac{53}{1000}$ parts of an Integer; in like manner if this mixt number 78 .919 were given to be subtracted from 295 .094, the remainder will be 216$\frac{175}{1000}$. In each of which examples you may observe that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole numbers of one denomination: Note also, when the decimals in both the numbers given consist not of the same number of places, that decimal which is defective in places towards the right hand, must have the void places filled up with cyphers, or at least cyphers must be supposed to be annexed: So if this decimal .04338 be given to be subtracted from this .65, the remainder will be found to be .60662, and the Work will stand as in the Margent, where you see the three void places are supplyed with cyphers, and then the operation is as in whole numbers, by borrowing 10 as often as the lower figure cannot be subtracted from the upper. More Examples of subtraction of Decimals are these following.

```
.837
.784
----
.053

295 .094
 78 .919
--------
216 .175

.65000
.04338
------
.60662
```

```
24.04338      .37     .394
  .65        0.104    .35
--------     -----    ----
23.39338     36.896   .044
```


CHAP XXVI.

Multiplication of Decimal Fractions.

I. VVHen two numbers are given to be multiplied, and are both mixt numbers, or both Decimal fractions, or one of them a whole number, and the other a decimal or mixt number (which are all the cases that can happen) there is no necessity of writing them down precisely one under the other as in Addition and Subtraction, for the product or number sought in Multiplication depends not upon any regular placing of the two numbers given: So if this mixt number 56.3 were given to be added to this mixt number 1. 30526, thy ought to be written one under the other, as you see (according to the second Rule of the 24th Chapter;) but if they are to be multiplied one by the other, they may be written thus,

```
1. 30526
  56 .3
--------
```

```
1.30526
   56.3
-------
```

II. In any of the Cases which may happen in Multiplication of Decimals, multiply the numbers given as if they were whole numbers, then cut off always from the product by a point, comma, or line

line, so many places towards the right hand, as there are places of decimal parts in both the numbers given to be multiplied; that done, the figure or figures (if any happen to be) on the left hand of the said point or line of separation doth declare the Integer or Integers in the product, and those on the right hand of the point are decimal parts of an Integer: So if this mixt number 56.3 (that is, 56 Integers and $\frac{3}{10}$ of an Integer) be given to be multiplied by this mixt number 1.30527, the product will be found 73.486138, that is, 73 Integers and $\frac{486138}{1000000}$ parts of an Integer; for having chosen that to be the Multiplicator, which will cause least work, and subscribed it under the Multiplicand (to wit, 56.3 underneath 1.30526) I proceed according to the Rules of Multiplication of whole numbers, *viz.* having drawn a line underneath the numbers given, I multiply all the Multiplicand, to wit, 1.30526, as if it where a whole number, by 3 the first multiplying figure, and subscribe the product thereof, which is 391578 underneath the line, and proceeding in like manner with the other multiplying figures 6 and 5, at last I find the total of the particular products to be 73486138; and because there are 6 places of decimal parts in both the numbers given (to wit, 5 places of parts in the multiplicand, and I place in the multiplicator) I cut off 6 places to the right hand from the total before produced, so will it stand thus 73|486138: Wherefore I conclude that the true product is 73 $\frac{486138}{1000000}$ or 73.486138, that is, 73 Integers and almost $\frac{1}{2}$ of an Integer.

```
    1.30526
       56.3
   --------
     391578
    783156
   652630
   --------
   73|486138
```

In

In like manner, if this mixt number 246.25 (that is 246 $\frac{25}{100}$) were given to be multiplied by 35 Integers, the true product will be found 8618.75, that is 8618 Integers and $\frac{75}{100}$ parts of an Integer, as you see by the operation in the Margent, where you may observe that two places are cut off from the total number produced of the multiplication, towards the right hand, because there are two places of decimals in the multiplicand (the multiplicator consisting of Integers only;) but if there had been decimal parts also in the multiplicator, so many more places should have been cut off, as was shewed in the first *Example*.

```
   246.25
       35
  -------
   123.125
   73875
  -------
   8618|75
```

Again, If these two decimals .87 and .9 (to wit $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true product will be found to be .783, that is $\frac{783}{1000}$ parts of an Integer as you see in the *Example*, where you may observe that the product is a fraction only; for after 3 places (being the number of places of decimals in both the numbers given to be multiplied) are cut off to the right hand, there remains no Integer on the left hand.

```
   .87
   .9
  ----
  .783
```

III. When the multiplication is finisht, if there arise not so many places in all as ought to be cut off by the second Rule of this Chapter (which may often happen when the product is a fraction;) in such case, as many places as are wanting, so many cyphers must be prefixed to the product on the left hand thereof, and then a point must be prefixt

to

to ſign the product ſo increaſed for a decimal: So theſe decimals .0375 and .05 being given to be multiplied one by the other, I multiply 375 by 5, and there ariſeth 1875: Now according to the ſecond Rule of this Chapter, I ſhould cut off 6 places to the right hand, and here are but 4 in all; wherefore I prefix two Cyphers, to wit, as many as there are places wanting, and then prefixing a point, the true product will be .001875 or $\frac{1875}{1000000}$. In like manner if this mixt number 5.525 be multiplied by this decimal .0026, the true product will be found to be .0143650 (or $\frac{143650}{10000000}$) as you may ſee by the operation in the Margent, where one cypher is prefixed to the numbers ariſing from the total Multiplication to diſcover the true product.

```
 .0375
   .05
-------
.001875
-------
 5.525
 .0026
-------
 .33150
 .11050
-------
.0143650
```

IV. Decimal parts of an Integer may be reduced to the known or accuſtomed parts of ſuch Integer by Multiplication only, for if the decimal fraction given be multiplied by that number which declareth how many known parts are equal to the Integer, the Product gives the number of known parts required: So this decimal fraction of a pound ſterling, to wit, .8687 *l.* being propounded, I multiply it firſt by 20 (the number of ſhillings contained in a pound) and the product gives 17 ſhillings and .3740 parts of a ſhilling;

To reduce decimals to the known parts of the Integer.

ſhilling; which decimal .3740 being multiplied by 12 (the number of pence in a ſhilling) produceth 4 pence, and .488 parts of a peny; Laſtly, multiplying .488 by 4 (the number of farthings, which make a peny) the product gives 1 farthing, and .9520 parts of a farthing, which are very near in value to another farthing, ſo it appears that .8687 parts of a pound ſterling are 17 *s.* 4 *d.* 2 *f.* very near. After the ſame manner, a *decimal fraction* of any Integer whatſoever may be reduced into the known or accuſtomed parts of ſuch Integer.

```
           .8687 l.
              20
        ----------
Shil.     17|3740
              12
        ----------
             7480
             3740
        ----------
Pence      4|4880
               4
        ----------
Farth.     1|9520
```

A briefer way to value any decimal part of a pound of *Engliſh money*, without loſs of a farthing, may be this, *viz.* the figure (if any happen) in the firſt place of the decimal being doubled, gives ſhillings; alſo if there be 5, or a figure greater than 5 in the ſecond place, one ſhilling more is to be added to the former; laſtly, when 5 is taken from the figure in the ſecond place, if every unit in the remainder be accounted as ten, and the figure in the third place as unities, theſe tens and units taken as one number and leſſened by 1, give the number of farthings, which with the ſhillings before found declare the value of the decimal propounded; likewiſe if the figure in the ſecond place

A brief way to find the value of any decimal fraction of a pound of Engliſh moneys.

(when any happens) be leſs than 5, every unit in ſuch figure is to be accounted ten as before: ſo in the decimal before mentioned, to wit, .8687*l.* the figure 8 in the firſt place being doubled gives 16 ſhillings, alſo becauſe 5 is contained in 6 which ſtands in the ſecond place, one ſhilling more is to be added to the aforeſaid 16 ſhillings, which will now be made 17*s.* that done, the remainder of the ſaid 6 after 5 is ſubtracted, to wit, 1 being eſteem'd as 10, and added to 8 (which ſtands in the third place, and to be eſteemed as units) gives 18, from which abating 1, the remainer is 17 farthings or 4 pence and a farthing; ſo that the value of the ſaid decimal .8687*l.* is found as before to be 17 ſhillings 4 pence 1 farthing. After the ſame manner this decimal of a pound of *Engliſh money*, to wit 319*l.* will be reduced to 6 ſhillings and 18 farthings, or 6 ſhillings 4 pence 2 farthings, which wants leſs than a farthing of the exact value of the decimal .319*l.*

V. Having explained all the caſes in *Multiplication of Decimals*, I ſhall here give the learner a taſte of their excellent uſe by ſome familiar queſtions whereby it will be evident, that what is oftentimes performed by many tedious *Multiplications* and *Diviſions* in the vulgar way, is effected for the moſt part by one or two *Multiplications* in *Decimals*.

See the queſtions from 49 to 73 in the 10th Chapter of the Appendix.

The firſt *Example* may be this: Suppoſe there is a certain piece of *Wainſcot*, in form of a *rectangled Parallegram*, commonly called a *long ſquare*, whoſe breadth is 3 *yards*, $\frac{3}{4}$ of a *yard*, 1 *nail* and $\frac{1}{4}$ of a *nail*; and the length of 6 *yards*, and $\frac{1}{2}$ of a *yard*, the queſtion is to know

know how many *ſquare yards* are contained in that piece of *Wainſcot*; here becauſe it is deſired that the *ſuperficial content* may be given in *yards*, the parts of a *yard* aswell in the breadth as in the length of the *Wainſcot* which are before expreſs'd by the accuſtomed parts of *quarters, nails*, &c. muſt be reduced into decimal parts of a *yard*, which are as eaſie to be found by a yard ſubdivided decimally, as the common parts of quarters and nails are found by a yard vulgarly ſubdivided: but for want of a yard ſubdivided decimally, this *Reduction* may be performed by the ſeventh *Tablet* of the precedent *Table* of *Reduction*, *viz.* looking into the ſaid *Tablet*, right againſt $\frac{3}{4}$ of a yard, I find this decimal ———— } .75

Alſo the decimal correſpondent to 1 nail is ———— } .0265

And the decimal of $\frac{1}{4}$ of a nail is ———— } .015625

The ſum of thoſe three decimals is ———— } .828125

Wherefore the breath of the Wainſcot in yards and decimal parts is ———— } 3.828125

Again the decimal of half a yard is .5, wherefore the length of the Wainſcot is. ———— } 6.5

The length and breadth being multiplied one by the other produce the *ſuperficial content*, therefore the number of ſquare yards required is ———— } 24.8828125

Wherefore I conclude that 24 ſquare yards and ſomewhat more are contained in that piece of Wainſcot,

Wainscot; and it is evident by the First place of the *decimal*, that what is above 24 yards is more than $\frac{8}{10}$, but less than $\frac{9}{10}$, of a square yard; or more strictly, it is more than $\frac{88}{100}$, but less than $\frac{89}{100}$ of a square yard: but by taking all the places in the decimal you have the exact answer to this question, because the common parts of *quarters*, *nails*, and *quarters* of *nails* may be always exactly reduced into decimals, but that seldom happens in other things; nevertheless, albeit by decimal operations you cannot always hit the mark, yet you may come as near it as is possibly to be imagined, and that with much more ease than by vulgar computations in questions of this nature, as will appear by comparing the precedent operation with the common way of working here in your view, *viz.* the 3 *yards*, 3 *quarters* of a *yard*, 1 *nail*, and $\frac{1}{4}$ of a *nail* (which express the breadth before mentioned (must all be reduced into quarters of *nails* by the sixth Rule of the seventh Chapter; so there will be found 245 quarters of Nails, as you see by the operation.

y.	q.	n.	q.	n.
3	3	1	1	

```
y.  q.  n.  q.  n.
3---3---1---1
4
---
15
 4
---
61
 4
---
245 quarters of nails
```

Again the 6 yards and half which express the length aforesaid, must likewise be reduced into quarters of Nails by the aforesaid *Rule*; so there will be found 416 quarters of nails of a yard, as you see by the operation.

y.


```
y.      q.
6-------2
4
---------
26
 4
---------
104
  4
---------
416 quarters of Nails.
```

Then Multiplying the breadth and length one by the other, to wit, 245 by 416, the product will give 101920 for the superficial content of the piece of Wainscot in square quarters of nails of a yard; now these square quarters of nails of a yard must be reduced to square yards, and the readiest way to perform that, is to find first of all how many quarters of nails of a yard are contained in one yard in length, *viz.* since there are 16 nails in a yard, there are consequently 4 times 16 quarters of nails, to wit, 64 quarters of nails in a yard in length; therefore 64 multiplied by 64 produceth 4096 square quarters of nails in a yard square; lastly, I say by the Rule of three, if 4096 square quarters of nails of a yard give 1 yard square, how many yards square will 101920 square quarters of nails give? So will the answer be found 24 $\frac{3616}{4096}$ yards, which is the same with 24.8828125 before found by the decimal operation (for $\frac{3616}{4096}$ is equal to the decimal .8828125, as will appear by reducing them to a common denominator by the fourteenth

teenth *Rule* of the seventeenth *Chapter.*) Now I leave it to the Reader to judge, which of these two ways is the more expeditious, and so let him take which liketh him best.

Example 2. There is a squared piece of Timber terminated at both ends with equal long squares, *viz.* the breadth of the piece of Timber is 1 foot 5 Inches 3 quarters of an Inch, and 1 half quarter of an Inch; the depth or thickness is 1 foot 3 Inches 1 quarter of an Inch, and $\frac{1}{8}$ or half a quarter of an Inch. and the length of the piece is 11 feet 10 Inches, and 3 quarters; the question is how many solid or cubical feet are contained in that piece of Timber? The Answer may be found by *decimal Multiplication* in manner following, *viz.* Forasmuch as it is desired that the solid content may be given in feet, the parts of a foot as well in the breadth, depth, and length, which are before express'd by the accustomed parts of Inches, quarters, and half quarters, must be reduced into the decimal parts of a foot, which are as easie to be found by a foot subdivided decimally, as the other common parts by a foot vulgarly subdivided; but for want of a foot subdivided decimally, this *Reduction* may be performed by the eighth *Tablet* of the precedent *Table* of *Reduction*, viz.

The decimal correspondent to 5 inches is ——————— .416
The decimal of $\frac{3}{4}$ of an Inch is——— .062
The decimal of half a quarter of an Inch is ——————— .01
The sum of those 3 decimals is——— .488
Wherefore the breadth of the piece of Timber is ——————— 1.488

In

In like manner the common parts of inches, *&c.* in the depth or thickness of the piece of Timber, will be reduced by the said *Tablet*, into these decimals, *viz.*

The decimal correspondent to 3 inches is—— .25
The decimal of $\frac{1}{4}$ of an Inch is ——————— .02
The decimal of half a quarter of an Inch is—— .01
The sum of these 3 decimals is ——————— .28
Wherefore the depth or thickness is ——— 1.28

Again the accustomed parts of Inches, *&c.* in the length of the piece of Timber will be reduced to these decimals, *viz.*

The decimal of 10 Inches is ——————— .833
The decimal of $\frac{3}{4}$ of an Inch is——————— .062
The sum of those 2 decimals is ——————— .895
Wherefore the length of the piece is —— 11.895

Now if the breadth, depth and length be multiplied continually, the last product is the solid content required, *viz.* 1 .488 multiplied by 1 .28 produceth 1 .90464, which multiplied by 11 .895 produceth 22 .65, *&c.* Wherefore I conclude that 22 solid Feet, half a Foot, and somewhat more than half a quarter of a foot are contained in that piece of Timber.

Example 3. How many *Equinoctial Degrees* are correspondent unto 136 *days*, 21 *hours*, and 40 *minutes*? The Answer is found by multiplying the time given by 360, for as 1 *day* is to 360 *degrees*, so 136 *days*, 21 *hours*, and 40 *minutes*, to the *Equinoctial degrees* required; but first the 21 *hours* and 40 *minutes* must be deduced to decimal parts of a day, by the tenth *Tablet*, thus.

The

The decimal of 21 hours is ——————.875
The decimal of 40 minutes is ——————.02777

The sum of these 2 decimals is ——————.90277
Therefore the time propounded is—136.90277
Which being multiplied by 360 } 49284.99,&c.
produceth ——————————— }

Wherefore I conclude, that 49284 .99 or very near 49285 *Equinoctial degrees* are correspondent unto 136 *days*, 21 *hours*, and 40 *minutes*, which was required by the question.

CHAP. XXVII.

Division by Decimal Fractions.

I. IN any of the Cases which may happen in Division, if the Dividend be greater than the Divisor, the quotient will be either a whole number or else a mixt number: But when the Dividend is less than the Divisor, the quotient must necessarily be a fraction; for a lesser number is contained in a greater once at the least, but a greater is not contained once in a lesser.

II. Sometimes the Dividend, whether it be a whole number, mixt number, or decimal fraction, is to be prepared by annexing a competent number of Cyphers thereunto, to make room for the Divisor: So if 32.5 were given to be divided by 17 .325 the Dividend 32 .5 must be increased with cyphers at pleasure after this manner 32 .50000 &c. Likewise if 1 were given to be divided by 360, the Division cannot be made till the Dividend 1 be increased with cyphers, which being annexed, the Dividend will stand thus 1; .000000,&c. Here, note that the cyphers annexed in manner aforesaid do supply places of decimal parts, and will be useful in discovering the quality of the quotient according to the fourth *Rule* of this *Chapter*.

III. After the Dividend is prepared by annexing cyphers, when occasion requires (as in the last Rule,) all the places thereof must be esteemed as one whole number (to wit consisting of unities or Integers:) and so is the Divisor to be esteemed whether it be a decimal fraction or mixt number; for in all cases the Division must be performed in every respect according to the Rules of Division of whole numbers in the sixth Chapter. So if this mixt number 326 .25 were given to be divided by this mixt number 12 .3, you must divide in the same manner, as when you divide 32625 Integers by 123 Integers. Also if this decimal .8356 were given to be divided by this decimal .05, you are to divide in the same manner, as when you divide 8356 Integers by 5 Integers; and after the quotient is found the degree or place of the first figure which ariseth in the quotient must be inquired after; *viz.* you must know how far such first figure is distant from the place of units, to the end that the point or line which is used to separate between the place of unities (or first place of Integers) and the first place of decimals may be duly placed: This is the only knot in decimal Division, and may be resolved by the following Rule, *viz.*

IV.

IV. In any of the Cases which may happen in Division of decimals, the first figure which ariseth in the Quotient, will be always of the same place or degree with that figure or cypher of the Dividend, which at the first question standeth over, or at least belongeth unto the place of units in the Divisor. To illustrate this Rule I shall give examples in all the principal cases; and first let a mixt number be given to be divided by a mixt number, *viz.* Let it be required to divide 172 .5 by 3 .746. here (according to the second Rule of this Chapter) the Dividend must be encreased with cyphers at pleasure, so will it stand thus 172 .500000, &c. then Division being made according to the Rules of Division of whole Numbers in Chapter 6, the Quotient arising will be 46049, &c.

A general Rule to discover the quality of the quotient in all cases of Division by decimal Fractions.

3 .746) 172 .500000 (46049, &c.

Now it remaineth to separate the Integers in this quotient from the decimal parts; to perform which, I subscribe the Divisor 3 .746 orderly underneath.

3 .746) 172 .500000 (46, 049, &c.

3 .746

the first Dividual 172 .50 (being that part of the Dividend whereof the first question must be asked) or at least I imagine the Divisor to be so subscribed, and so I find that the figure 3 which stands in the place of Units in the Divisor will be placed under

under 7, which is the place of tens (or second place of Integers) in the Dividend; wherefore by the fourth Rule before given; I conclude that the first figure arising in the quotient must likewise stand in the place of tens (or second place of Integers) and consequently the next place on the right hand must be the place of *Units*; so it is evident that the separating point or line must be placed between the figure 6 and 0 in the quotient, that done, the true quotient is found to be 46 .049, &c. to wit, 46 Integers and $\frac{49}{1000}$ parts of an Integer, and somewhat more: for 46 $\frac{49}{1000}$ is less than the true quotient, but 46 $\frac{50}{1000}$ is greater than it, and therefore albeit, after the aforesaid Division of 172 .500000 by 3.746 is ended, their will be a remainder, to wit, 446 which seems to be greater, yet here it is less in value than $\frac{1}{1000}$ part of an unit or Integer, and if to that remainder you annex another cypher and continue the division, you will proceed nearer the truth and not miss $\frac{1}{10000}$ part of an unit of the true quotient, and in that order you may proceed infinitely near, when you cannot obtain the quotient exactly by Division of Decimals.

Example 2. Suppose this mixt number 2 .34 be given to be divided by this mixt number 52 .125 (where you may observe that the Dividend is less than the Divisor;) first (as before) annex cyphers at pleasure to the Dividend, to make room for the Divisor, then the division being prosecuted as in whole numbers, at length these figures will arise in

52 .125) 2 .3400000 (.0448, &c.

52 .125

the

the quotient, to wit, 448: and to the end the degree or quality of the firſt figure 4 may be diſcovered, I ſubſcribe the Diviſor 52.125 under the firſt dividual 2.34000 (for ſo far the firſt queſtion did extend in the Diviſion) and thereby I find that the figure 2 which ſtands in the place of units in the diviſor will be ſeated under 4, which is in the ſecond place of decimals, wherefore I conclude that the firſt figure ariſing in the quotient muſt alſo ſtand in the ſecond place of decimals, and conſequently the firſt place of decimals (which is next on the left hand to the ſecond) muſt be ſupplied with a cypher; ſo that if a cypher be prefixed on the left hand of 4, and then a point placed before that cypher, the quotient will at length be diſcovered to be .0448, &c. or $\frac{448}{10000}$, and ſomewhat more that is to ſay, $\frac{448}{10000}$ is leſs than the true quotient, but $\frac{449}{10000}$ is greater than it; and if you will proceed nearer the truth, you may continue the diviſion, as is directed in the firſt Example of this Rule.

Example 3. Where a whole number is divided by a decimal fraction, *viz.* ſuppoſe 82 Integers were given to be divided by this decimal .056; After cyphers are annexed to the dividend at pleaſure, and

.056) 82.00000 (146428, &c.

the

the diviſion proſecuted as in whole numbers (to wit, 8200000 being divided by 56) theſe figures 146428 will ariſe in the quotient: now to the end the degree or ſeat of 1, the firſt figure in the quotient may be known, I ſubſcribe the Diviſor .056 under the firſt dividual 82 (for ſo far did the firſt queſtion in the diviſion extend;) and becauſe the diviſor is leſs than unity, I ſupply the place of units by a cypher or 0 prefixed on the left hand of the point of ſeparation in the diviſor; alſo I pre-

.056) 0082.00000 (1464.28, &c.

0.056

fix cyphers before (to wit on the left hand of) the Integers in the dividend to repreſent a ſucceſſion of places of Integers (for the order of places in Integers is from the right hand towards the left;) then I find that the cypher or 0 which repreſents the place of units in the diviſor, doth ſtand under that cypher, which repreſents the fourth place of Integers in the *dividend* (as you ſee by the Example;) wherefore I conclude that the firſt figure ariſing in the quotient muſt alſo be ſeated in the fourth place of Integers, and conſequently the 4 firſt places in the quotient will be Integers, and the reſt a decimal, ſo that the true quotient is 1464 Integers, and $\frac{28}{100}$ parts of an Integer, and ſomewhat more, *viz.* 1464.28 is leſs than the true quotient, but 1464.29 is greater than it.

Example 4. Suppoſe this decimal .0125 be given to be divided by this decimal .5; after diviſion is finiſhed according to the Rules of diviſion of

.5) .0125 (25

whole

whole numbers (to wit after 125 is divided by 5) these figures 25 will arise in the quotient; now to discover the degree or seat of 2 the first figure in the quotient, I subscribe the divisor .5 under the first dividual .012, and having (as in the last Example) prefixed a cypher on the left hand of the point of separation in the divisor, to denote or represent the place of units, I find that such cypher or place of units do stand under the figure 1, which is seated in the second place of decimals in the dividend, wherefore I conclude by the Rule, that the first figure which ariseth in the quotient must also be in the second place of decimals, and therefore prefixing a cypher to supply the first place of decimals, and putting a point before that cypher, the quotient is at length discovered to be .025 or $\frac{25}{1000}$.

.5) .0125 (.025
05

Example 5. Suppose this decimal .8564 be given to be divided by this .008, first I annex cyphers to the dividend at pleasure, then prosecuting the division as in whole numbers, to wit, dividing .856400 by 8, the quotient arising is 107.050, now to discover the degree or place of 1, the first figure in the quotient, I subscribe the divisor .008 under the first dividual .8, then I prefix a cypher to set forth, or supply the place of units in the divisor, also I prefix cyphers to represent places of integers in the dividend; that done, I find that the cypher or o which supplieth

.008).856400(107.050

.008) 000.85640(107.05
0.008

plieth the place of units in the divisor, doth stand under the Cypher which is seated in the third place of Integers in the dividend; wherefore I conclude by the Rule, that the first figure arising in the quotient must be also in the third place of Integers, and consequently the three first places in the quotient will be Integers, and the rest a decimal; so that the true quotient is 107 .05 or 107 $\frac{5}{100}$.

Example 6. Let it be required to divide this decimal fraction .73952 by this .32; first dividing 73952 by 32 as if they were whole numbers, the figures arising in the quotient will be 2311. Now to discover the quality or value of the said figures I subscribe the *Divisor* .32 under the first *dividual* .73, then prefixing a Cypher as well on the left hand of the *dividend*, as of the *divisor* so subscribed (or imagined to be subscribed) as aforesaid, to represent the place of units in each of them, I find the cypher or o, which supplieth the place of units in the *Divisor*, to stand under the o which represents the place of units in the *dividend*; wherefore I conclude by the preceeding fourth *Rule*, that the first figure arising in the *quotient* will stand in the place of units, and consequently the following places of the *quotient* will be a decimal fraction, so that the true *quotient* is 2 .311 or 2 $\frac{311}{1000}$.

.32) 0.73952 (2.311
0.32

The reason of the foregoing fourth Rule will appear from the following Considerations.

P   1. If

I. If the product of the Multiplication of two numbers be divided by one of them, the quotient is the same with the other number: As, if 269.0625, the product of 14.35 multiplied by 18.75, be divided by 14.35, the quotient will give 18.75.

II. If the Divisor be multiplied by the first figure in the quotient, the Product is the first number to be subtracted from the Dividend (being the same with the last particular product in the multiplication of the two numbers that produced the Dividend;) and every particular place of that product is of the same degree with that figure or cypher of the Dividend, which stands over such particular place when the subtraction is made; For a figure of one degree (or place) cannot be subtracted from a figure of a different degree: As in the last mentioned Example, the work whereof is here in view; the Divisor 14.35 being taken as in a whole number and multiplied by 1, the first figure in the quotient produceth 1435, which must be conceived to consist of the same degrees as are in 269.0 in the Dividend, from which the said product is to be subtracted, and therefore the said product 1435 is really but 143.5, as you may see by the last particular product, in the multiplication of the mixt number 14.35 by 18.75.

14.35


```
            14.35
            18.75
          --------
             7|175
            10|045
           114|80
           143|5
         ----------
14.35)     269|0625.(18.75
           143|5
        -----------
           125|56
           114|80
         ---------
            10|762
            10|045
          ---------
              |7175
              |7175
            --------
                0
```

III. And therefore to discover the degree of the first figure in the quotient, is nothing else but to find out the degree of that figure, which multiplying the figure or cypher in any particular place of the Divisor, will produce the same degree as that figure or cypher in the Dividend is of, which stands over, or at least belongs unto such particular place of the Divisor, at the first question; because the degree produced must be subtracted from the like degree above it.

 IV. Now

IV. Now among many Rules that might be given to discover the degree of the first figure in the quotient, and consequently the degrees of all the rest, the preceeding fourth Rule of this Chapter is sufficient, namely, The first figure which ariseth in the quotient, is always of the same place or degree with that figure or cypher in the Dividend, which at the first question stands over, or at least belongs unto the place of units in the Divisor: The reason is, because if a figure standing in the units place of the Divisor be multiplied by (or doth multiply) a figure of the same degree with that degree in the Dividend, which at the first question belongs to the said units place of the Divisor, the first place in the Product shall be of that degree also, whether it be of Integers or decimal parts; and consequently the rest of the places in the said Product shall be of the same degrees with their correspondent degrees (or places) in the Dividend, as they ought to be, to the end that due Subtraction may be made (according to Observ. 2.)

So in the Example before given, the first figure 1 in the quotient, shall be of the degree or place of Tens, because if the figure 4 standing in the units place of the Divisor 14 .35, be multiplied by Ten, to wit, the degree which the figure 6 in the Dividend is of that belongs to the said 4 at the first question, it will produce four Tens, to be subtracted from the said six Tens: In like manner if a figure in the place of units be multiplied by units the first place in the Product shall be units; if by tenth parts of an unit, or Integer, the first place in the Product shall be Tenths, *&c.*

Having explained all necessary Rules in Division concerning

concerning decimal fractions, I shall give a taste of their excellent use, by the two following questions and then conclude this Chapter.

Quest. 1. A Merchant bought of Gold *Plate* 356 *ounces*, 13 *peny weight*, and 15 *grains* for 1160 *pounds sterling*, the question is what he paid for an *ounce?* *Answer* 3*l.*—5*s.*—½*d.* very near. The operation by *decimals* may be after this manner, *viz.*

By the second *Tables* of *Reduction* the *decimal* of 13 *peny weight* is——} .65

The *decimal* of 15 *grains* is——.03125

The *Sum* of those 2 *decimals* is—.68125

Wherefore the quantity of *Plate* in *ounces* and *decimal parts* of an *ounce* is ——————————} 356 .68125

Then by the *Rule* of three I say, if 356.68125 *ounces* cost 1160 *pounds*, what 1 *ounce?* Here, 'tis evident that if I divide 1160 by 356 .68125, the *quotient* will give the value of an *ounce* to wit, 3 .252, *pounds*, or 3 *pounds*, 5 *shillings* and ½ *d.* very near.

356.68125) 1160.0000000 (3 .252, &c.

Quest. 2. Suppose the length of the *Tropical year* (or the space of time wherein the Sun running through the whole *Ecliptick* circle consisting of 360 *degrees*, is returned to the same *Equinoctial* or *Solstitial* point from whence he departed) to consist of 365 *days*, 5 *hours*, and 49 *minutes*, the question is to know the Suns mean or equal motion for 1 *day*, to wit, what part of 360 *degrees* the Sun moveth in a whole day? The operation by *decimals*, thus,

 By

By the tenth *Tablet* of *Reduction* the *decimal* correspondent to 5 hours is——————————— } .2083333

The *decimal* of 49 *minutes* is————.0340277

The *sum* of those *decimals* is————.2423610

Wherefore the *time* given, in *days* and *decimal parts* of a *day* is } 365.2423610

Then by the rule of three, if 365 .242361 *days* give 360 *degrees* (or a total circumference) what will 1 *day* give? Here if I divide 360 by 365 .242361, the *quotient* will give the diurnal motion required; which will be found very near .98564, &c. or $\frac{98564}{100000}$ parts of a degree, which *decimal* being reduced into the common *Sexagenary parts* (by the fourth *Rule* of the 26 *Chapter*) will give 59′—8″, &c, and such is the Suns diurnal motion very near, according to the aforesaid supposition of the length of the *Tropical year*.

I shall here add the vulgar *Sexagenary* resolution of this question, that by comparing both ways together, the excellency of *decimal Arithmetick* in Calculations of this Nature may be the more perspicuous.

The aforesaid question being stated according to the *Rule* of three will stand thus,

days	*hours*		*degrees*	*day*
If 365 :	5 :	49 ——	360 ——	1

The first term in the Rule must be reduced into *minutes* (by the sixth *Rule* of the seventh *Chapter*;) so there will be found 525949 *minutes*.

D.	*h.*	
365 ——	5 ——	49
24		
1465		
730		
8765		
60		
525949 *minutes*.		

Likewise the third term 1 *day* must be reduced into *minutes*, which will be found to be 1440, as you see by the following operation.

1 *Day* or 24 *hours*.
60

1440 *minutes*.

Then multiply the third term by the second, to wit, 1440 by 360, the product is 518400, which being divided by the first term 525949 (according to the note in the ninth *Rule* of the 16th *Chapter*) the *quotient* will give $\frac{518400}{525949}$ parts of a degree, which fraction being reduced into the accustomed *Sexagenary* parts (by the ninth *Rule* of the seventeenth *Chapter*) will give as before 59′ 8″, &c. for the Suns mean diurnal motion; now which of these two ways is the more expeditious, I leave to him who is vers'd in both to determine.

CHAP. XXVIII.

The Rule of Three Direct in Fractions.

I. TO repeat ſuch things as have already been declared in reference to the definition of this Rule, as alſo to the due placing of the 3 given numbers, would be ſuperfluous; and if reſpect be had to the Rules of *Multiplication* and *Diviſion* in *fractions* delivered in the 20, 21, 26 and 27 Chapters, the working of the Rule of three direct in fractions as well vulgar as decimal, is the ſame with that in whole numbers, *viz.* multiply the ſecond number by the third (or the third by the ſecond,) and divide the product by the firſt number, ſo the quotient is the fourth number ſought; to wit, the anſwer of the queſtion.

Otherwiſe thus in Vulgar Fractions.

Multiply the Denominator of the firſt number by the Numerator of the ſecond, alſo multiply that product by the Numerator of the third number, and reſerve this laſt product for a new Numerator; again multiply the Numerator of the firſt number by the Denominator of the ſecond, alſo multiply this product by the Denominator of the third number, ſo ſhall this laſt product be a new Denominator; laſtly, the new fraction (whoſe Numerator and Denominator is found as aforeſaid) is the fourth number ſought, which, if it be a proper

fraction, may (if occaſion require) be reduced into the known parts of the Integer (by the ninth Rule of the ſeventeenth Chapter;) if an improper fraction, it is to be reduced into its equivalent whole number or mixt number, by the thirteenth Rule of the ſeventeenth Chapter.

Example, If $\frac{3}{4}$ of a yard of Velvet be ſold for $\frac{2}{3}$ of a pound *ſterling*, what ſhall $\frac{5}{6}$ of a yard coſt? *Anſwer* $\frac{40}{54}$ *l.* or 14 *s.* $9\frac{7}{9}$ *d.* For according to the *Rule* I multiply the Denominator 4 by the Numerator 2, and the *product* is 8, this 8 I again multiply by the Numerator 5, and the *product* gives 40 for a new Numerator: Moreover, I multiply the Numerator 3 by the Denominator 3, and the *product* which is 9 I again multiply by the Denominator 6, ſo the laſt *product* is 54 for a new Denominator; wherefore I conclude that $\frac{40}{54}$ is the fourth number ſought, which if it be reduced (according to the ninth Rule of the ſeventeenth Chapter) gives 14 *s.* $9\frac{42}{54}$ *d.* (or $9\frac{7}{9}$ *d*) for the *Anſwer* of the queſtion.

$$\begin{array}{ccccccc} y. & & l. & & y. & & l. \\ \frac{3}{4} & \text{———} & \frac{2}{3} & \text{———} & \frac{5}{6} & \text{———} & (\frac{40}{54} \end{array}$$

II. When any of the three given numbers is a whole number or mixt number, ſuch number muſt firſt of all be reduced into an improper fraction (by the tenth or eleventh *Rule* of the ſeventeenth *Chapter*) to the end that all the three given numbers may be 3 fractions: Moreover, If after ſuch Reduction, the firſt and third numbers be not fractions of Integers of the ſame particular denomination, ſuch of the ſaid numbers which is of the leſſer denomination, muſt be reduced to a fraction of the greater (by the ſixteenth *Rule* of the ſeventeenth *Chapter*;) which preparations being performed, the reſt

rest of the Work is to be prosecuted according to the first *Rule* of this *Chapter*. An *Example* of this second *Rule* here followeth. If a quantity of *Ambergreece* weighing $1\frac{5}{7}$ *lb. Troy*, be sold for 60 *l. sterling*, what are $19\frac{5}{8}$ *grains* worth at that rate? *Answers* $\frac{65940}{1552320}$*l.* or 2 *s.* $4\frac{119}{192}$ *d.*

This question being stated according to the 7 *Rule* of the 8 *Chapter* will stand thus,——

lb.	*l.*	*gr.*
$1\frac{5}{7}$ ——	60 ——	$19\frac{5}{8}$

which 3 numbers will be reduced (by the tenth and eleventh *Rules* of the seventeenth *Chapter*) into these improper *fractions.*——

lb.	*l.*	*gr.*
$\frac{12}{7}$ ——	$\frac{60}{1}$ ——	$\frac{157}{8}$

But since the third number $\frac{157}{8}$ *grains Troy* is not a fraction of an Integer of the same name with the first (which is a fraction of a pound *Troy*,) it must be reduced into a fraction of a pound *Troy*, thus, $\frac{157}{8}$ *gr.* is $\frac{157}{8}$ of $\frac{1}{24}$ of $\frac{1}{20}$ of $\frac{1}{12}$ of a pound *Troy*, which compound fraction will be reduced (by the 16 *Rule* of the 17 *Chapter*) into this single fraction, to wit, $\frac{157}{46080}$ *lb. Troy* and so the 3 numbers will at length stand thus in the *Rule.*

$$\frac{12}{7}lb. \text{——} \frac{60}{1}l \text{——} \frac{157}{46080}lb.$$

Then working as in the first *Example* of this *Chapter*, the answer will be found $\frac{65940}{1552320}$*l.* which being reduced (according to the 3 and 4 *Rules* of the 17 *Chapter*) is found equal unto 2 *s.* $4\frac{119}{192}$ *d.*

Another *Example.* When the $\frac{2}{5}$ of $\frac{3}{4}$ of a *Ship* is valued at 147 *l.*——11 *s.*——3 *d.* how much is the whole Ship worth? *Answ.* 491 *l.*——17*s.*——6*d.*

Note,

Note, when in any question whatsoever a compound fraction, to wit, a fraction of a fraction, is one of the given numbers, such compound fraction must first of all be reduced to a single fraction (by the 16 *Rule* of the 17 *Chapter*;) so here the *compound fraction* $\frac{2}{5}$ of $\frac{3}{4}$ being reduced into a single fraction gives $\frac{6}{20}$ or $\frac{3}{10}$; then say if $\frac{3}{10}$ be worth 147*l.* 11*s.* 3*d.* what is 1 or the whole *Ship* worth?

Ship	*l.*	*s.*	*d.*	*Ship*
$\frac{3}{10}$ ——	147 :-	11 :—	3 ——	1

After due reduction is made by converting the 147*l.* 11*s.* 3*d.* into pence, and that number of pence, as also the third number 1. into improper fractions, the 3 numbers will stand in the Rule thus,

Ship	*pence*	*Ship*
$\frac{3}{10}$ ——	$\frac{35415}{1}$ ——	$\frac{1}{1}$

Lastly, Proceeding as is in the first Rule of this Chapter, the fourth number will be found to be $\frac{354150}{3}$*d.* which being reduced first by the 13 *Rule* of the 17 *Chapter*, and then by the 7 *Rule* of the 7 *Chapter*, the *Answer* at length is 491*l.*——17*s.*——6*d.*

An Example of the Rule of three direct in *Decimals* may be this that follows. If 19 *ounces*, 3 *peny weight*, and 5 *grains* of Gold, be worth 62*l.*——10*s.*——6*d.* what is the value of $1\frac{1}{2}$ *ounce*? *Answ.* 4*l.*——17*s.*——$10\frac{3}{4}$*d.* very near.

By

By the 2. *Tablet* in the *Table* of *Reduction* in the 23 Chapter, the *decimal fraction* correſpondent to 3 *peny weight* is —— —— —— .15

Alſo, the *decimal* of 5 *grains* is —— .010416

The ſum of thoſe 2 *decimals* is —— .160416

Wherefore the firſt number in the Rule of three is —— —— —— oz. 19.160416

Again, by the firſt *Tablet* of the aforementioned *Table* the *decimal* of 10 *ſhillings* is —— —— —— .5

Alſo the *Decimal* of 6 *pence* is —— .025

The ſum of theſe two *decimals* is —— .525

Wherefore the ſecond number in the rule of three is —— —— —— l. 62 .525

Moreover by the ſaid *Tablet* 2. the *decimal* of $\frac{1}{2}$ of an *ounce* or 10 *peny weight* is .5, wherefore the third number in the Rule of three is —— —— oz. 1 .5

So that after the ſaid Reduction is finiſht the 3 given numbers will ſtand in the Rule thus:

oun.	*l.*	*oun.*
19.160416	62.525	1 .5

Laſtly, multiplying the ſecond number by the third, and dividing the product by the firſt number (according to the Rules of Multiplication and Diviſion of Decimals delivered in the 26 and 27 Chapters) the fourth number will be this, to wit, 4.8948, &c. that is four pound *ſterling* and $\frac{8948}{10000}$ parts of a pound, which *decimal* being reduced according to the fourth *Rule* of the 26 *Chapter*) gives 17*s.* —— 10*d.* —— 3*far.*

The

The proof of the Rule of three direct in Fractions is the ſame as in whole numbers, reſpect being had to the Rules of Multiplication in Fractions.

CHAP. XXIX.

The Inverſe Rule of Three in Fractions.

1. After a queſtion belonging to this Rule is duly ſtated (according to the ſeventh rule of the eighth Chapter) and prepared if need require, according to the ſecond Rule of the 28 Chapter; the operation will be the ſame as in the Rule of three Inverſe in whole numbers, reſpect being had to the Rules of Multiplication and Diviſion in Fractions, *viz.* multiply the firſt number by the ſecond, and divide the Product by the third; the quotient is the fourth number ſought, to wit, the anſwer of the queſtion.

Or thus, in Vulgar Fractions;

Multiply the Denominator of the third fraction by the Numerator of the ſecond, alſo multiply that Product by the Numerator of the firſt fraction, and reſerve the laſt Product for a new Numerator: again multiply the Numerator of the third fraction by the Denominator of the ſecond; alſo multiply this Product by the Denominator of the firſt fraction, ſo is the laſt Product a new Denominator; laſtly, this new fraction is the fourth number ſought, or anſwer of the queſtion.

Examples

Example, if of cloth, which is $1\frac{3}{4}$ yard in breadth $3\frac{1}{2}$ yards in length will make a Cloak, how much in length of ſtuff which is $\frac{5}{8}$ yards in breadth will make a Cloak of the ſame bigneſs with the former? *Anſwer* $9\frac{4}{5}$ yards.

The 3 numbers being duly placed will ſtand thus ———

brea.	*leng.*	*brea.*
$1\frac{3}{4}$*y.* —	$3\frac{1}{2}$*y.* —	$\frac{5}{8}$*y.*

Then (after the firſt and ſecond numbers are reduced into improper fractions) the three Numbers will ſtand thus ——— $\frac{7}{4}$ ——— $\frac{7}{2}$ ——— $\frac{5}{8}$

Laſtly, 8, 7 and 7 being multiplied continually give 392 for a numerator; alſo 5, 2 and 4 being multiplied continually give 40 for a denominator, whereby this improper fraction $\frac{392}{40}$ ariſeth, which (by the thirteenth rule of the ſeventeenth Chapter) will be found to be $9\frac{32}{40}$, or (the fraction being reduced into its leaſt terms) $9\frac{4}{5}$, which is the *Anſwer* of the queſtion.

Ex. 2. Suppoſe when Wheat is at 2*l.*—00*s.*—6*d.* the Quarter, the peny white loaf ought to weigh 8 ounces and $1\frac{5}{29}$ *peny weight* of *Troy* weight; what ought it to weigh when Wheat is at 36 ſhillings the Quarter? *Anſwer* 9 ounces and $1\frac{37}{116}$ *peny weight.*

The 3 given numbers being duly placed in the rule and reduced will ſtand thus.

pence	*p. w.*	*pence*
$\frac{486}{1}$:	$\frac{4674}{29}$:	$\frac{432}{1}$

And if the operation be proſecuted according to the rule before given, the *Anſwer* will be found $181\frac{3996}{12528}$ peny weight, or 9 ounces, $1\frac{37}{116}$ peny weight.

CHAP.

CHAP. XXX.

The double Rule of Three in Fractions.

I. THe *Double Rule of Three* is ſo called, becauſe it is compoſed of two ſingle Rules, and may either be reſolved at one Work by the Rule compound of 5 numbers, or elſe by two diſtinct ſingle Rules of three; which latter way, to ſuch as underſtand the Rule of three in fractions, is (as I conceive) leſs troubleſome in the ſtating, and (in the method whereby I intend to proſecute it) the ſame in operation with the former. This I ſhall manifeſt firſt in whole numbers, then in fractions.

Example 1. If I pay 28 ſhillings for the carriage of 3 C. weight for 50 miles, how much ought I to pay for the carriage of 17 C. for 84 miles? *Anſwer* 13*l.*——6*s.*——6*d.* $\frac{18}{25}$.

Of the 5 given numbers I make choice of three ſuch which will make a ſingle rule of three, and ſay,

	C.	*ſhil.*	*C.*
If	3 ——	28 ——	17

Which Rule I find (by the third rule of the ninth Chapter) to be direct, and therefore I multiply the third number 17 by the ſecond 28, and the product which is 476 I place as a numerator over the diviſor as denominator. Then with this fraction (whether it happen to be a proper or improper fraction) and the remaining two numbers in the queſtion, which have not yet been uſed, I form a ſecond rule of Three, and ſay, $\frac{476}{3}$

miles

miles *shill.* *miles*
If $\frac{5}{1}$ —— $\frac{476}{3}$ —— $\frac{84}{1}$

Which being a Rule of Three direct, I work as a rule of three in fractions, according to the first rule of the 28 Chapter, and so find the fourth number to be $\frac{39984}{150}$*s.* or 13*l.*—6*s.*—$6\frac{18}{25}$*d.*

Or the first single Rule being varied, the operation will be thus,

miles *C.* *miles* *C.*
1. *By a Rule inverse,* 50 —— 3 —— 84 —— ($\frac{150}{84}$

C. *sh.* *C.* *sh.*
2. *By a Rule Direct,* $\frac{150}{84}$: $\frac{28}{1}$: $\frac{17}{1}$: ($\frac{39984}{150}$

Otherwise thus,

C. *m.* *C.* *m.*
1. *By a Rule inverse,* 3 —— 50 —— 17 —— ($\frac{150}{17}$

m. *sh.* *m.* *sh.*
2. *By a rule direct,* $\frac{150}{17}$: $\frac{28}{1}$: $\frac{84}{1}$: ($\frac{39984}{150}$

Thus you see the two single rules to be varied three manner of ways in resolving the question propounded, and each way produceth the same *Answer*; the like diversity may be found in all questions resolvable by the double rule of three, or rule compound of 5 numbers.

Example 2. If $40\frac{3}{5}$*l.* in $\frac{2}{3}$ of a year, gain $2\frac{1}{2}$*l.* what will 100 *l.* gain after that rate in $\frac{7}{12}$ of a year? *Answ.* $\frac{52500}{9744}$*l.* or 5 *l.*—7 *s.*—$9\frac{3}{29}$*d.*

By

By 2 Single Rules of three, thus,

l. *l.* *l.* *l.*
1. *By a rule direct,* $\frac{203}{5}$: $\frac{5}{2}$: $\frac{100}{1}$: ($\frac{2500}{406}$

year *l.* *year* *l.*
2. *By a Rule Direct,* $\frac{2}{3}$: $\frac{2500}{406}$: $\frac{7}{12}$: ($\frac{52500}{9744}$

Or by these two single Rules,

year *l.* *year* *l.*
1. *By a Rule Direct,* $\frac{2}{3}$: $\frac{5}{2}$: $\frac{7}{12}$: ($\frac{105}{48}$

l. *l.* *l.* *l.*
2. *By a Rule direct,* $\frac{203}{5}$: $\frac{105}{48}$: $\frac{100}{1}$: ($\frac{52500}{9744}$

Otherwise thus,

l. *year* *l.* *l.*
1. *By a Rule inverse,* $\frac{203}{5}$: $\frac{2}{3}$: $\frac{100}{1}$: ($\frac{406}{1500}$

l. *year* *l.* *l.*
2. *By a Rule direct,* $\frac{406}{1500}$: $\frac{5}{2}$: $\frac{7}{12}$: $\frac{52500}{9744}$

Thus by 2 single rules of three varied three several ways, you see the *Answer* of the question to be $\frac{52500}{9744}$*l.* to wit, 5*l.*—7*s.*—$9\frac{3}{29}$*d.*

Q CHAP.

CHAP. XXXI.

The Rule of False in Fractions.

I. WHen a question propounded cannot readily be applied to the *Rule of Three*, or any of the vulgar Rules in *Arithmetick*; the best refuge for such as are not acquainted with *Algebra* is the Rule of *two False Positions*, which, for that it hath already been handled in *whole Numbers*, I shall the more briefly touch upon in *Fractions*.

II. When a number is sought by a question, you are to feign or suppose some number taken by guess to be the number sought, and to make trial whether that feigned number will answer the conditions in the question or not, by comparing the number resulting at the end of the Work, with the given number resulting from the true number sought; and if you find both those results to be the same, then is the number which you first took by guess the true number or answer of the question; but if the number resulting from the supposititious number be either greater or less than the given result, with which it ought to be compared (to see whether you have hit the mark or not) such excess or defect must be noted for the Error of the first Position, to wit, an excess must be signified by this note +; and a defect by this ——.

III. In like manner a second number must be feigned, and after trial is made therewith, to see whether it will perform the conditions prescribed in the question, by comparing the results as aforesaid,

said, the error of this second position, if too much, is to be noted by +, if too little by ——, as before.

IV. After the errors of both positions are discovered, the two numbers before supposed or feigned to be the number sought, must be multiplied by the altern errours, that is, the first Position by the second errour, and the second Position by the first errour; then if the notes of the errours be unlike, to wit, one of them +, and the other ——, the sum of the said Products is to be taken for a dividend, and the sum of the errours for a divisor; but if the notes of the errours be both alike, to wit, both of them +, or both ——, the difference of the said Products is to be taken for a dividend, and the difference of the errours for a divisor; lastly, the quotient arising from the division made by the said dividend and divisor, gives the true number sought, or answer of the question, if it be solvable by the *Rule of False*. These Rules, are the same in substance with those delivered in the 15 Chapter, and may be farther illustrated by the following Questions.

Quest. 1. A Gentleman hired a servant for a year for 6 pounds *sterling*, and a livery Cloak valued at a certain rate, but it happened that $\frac{7}{12}$ of the year being expired they fell at variance, and the Gentleman put away his Servant, giving him the Cloak together with 50 shillings in money, which was the servants full due for the time of his service, the question is to find what the Cloak was valued at? *Answ.* 2 *l.* —— 8 *s.* —— 0 *d.*

1. I suppose the Cloak to be valued at 3 pounds, and then seek how much thereof was due to the

 servant

servant, saying, if one year give 3*l.* how much $\frac{7}{12}$ of the year? *Answ.* $\frac{7}{4}$ *l.*

$$\begin{array}{llll} y. & l. & y. & \\ 1. —— & 6 —— & \frac{7}{12} —— & (\ \frac{7}{4}\,l. \end{array}$$

2. I likewise find what parts of the 6 pounds was due to the servant at the end of $\frac{7}{12}$ of the year saying, if 1 year give 6 pounds, how much $\frac{7}{12}$ of the year? *Answer,* $\frac{7}{2}$ *l.*

$$\begin{array}{llll} y. & l. & y. & \\ 1 —— & 6 —— & \frac{7}{12} —— & (\ \frac{7}{2}\,l. \end{array}$$

3. For as much as the Cloak together with the money which the servant received ought to be equal to the part of the Cloak together with the part of the 6 pounds wages due to him at the end of $\frac{7}{12}$ of the year, therefore 3 *l.* (the supposed value of the Cloak) together with 2 $\frac{1}{2}$ *l.* (the money which the servant received) should be equal to $\frac{7}{4}$ of a pound (the value of part of the Cloak due to the servant at the end of $\frac{7}{12}$ of the year) together with $\frac{7}{2}$ *l.* (the wages due for the same time) that is to say, $\frac{11}{2}$ *l.* (the sum of 3*l.* and 2 $\frac{1}{2}$*l.*) should be equal to $\frac{21}{4}$ *l.* (the sum of $\frac{7}{4}$*l.*) and $\frac{7}{2}$*l.* but it is greater by $\frac{1}{4}$, wherefore the first Position for the value of the Cloak being 3 pounds, the errour is found to be $\frac{1}{4}$ too much.

4. I make a second Supposition guessing the value of the Cloak to be 2 pounds, and Proceeding in every respect as with the first Supposition I find the *errour* to be $\frac{1}{6}$ too little, so that the two *Positions* with their *errours* will be as you see:

$$\begin{array}{ccc} Pos. & & Er. \\ 3 & + & \frac{1}{4} \\ 2 & — & \frac{1}{6} \end{array}$$

Now

Now in regard the *errours* are *fractions*, I may take in their stead whole numbers in the same proportion, to wit, multiplying the *Numerator* of the first fraction (or first *errour*) by the *Denominator* of the second, I take the Product which is 6 instead of the first *errour* $\frac{1}{4}$, likewise multiplying the *Numerator* of the second fraction by the *Denominator* of the first, I take the Product which is 4 instead of the second *errour* $\frac{1}{6}$. Or instead of the said 6 and 4 I may take 3 and 2 which are in the same proportion with 6 and 4, or with $\frac{1}{4}$ and $\frac{1}{6}$.)

$$\begin{array}{cccccc} Pos. & & Er. & & \\ 3 & + & \frac{1}{4} & 6 & 3 \\ 2 & — & \frac{1}{6} & 4 & 2 \\ \hline 6 & & & & \\ 6 & & & & \\ \hline 5) & 12 & 2\frac{2}{5} & pound & \end{array}$$

Then multiplying the *Positions* and new *errours* crosswise, and adding the Products together (because the signs are unlike) the sum is 12 for a *Dividend*, and the sum of the *errours* 3 and 2 is 5 for a *Divisor*, so the quotient will be found to be 2 $\frac{2}{5}$*l.* so much therefore was the value of the Cloak, as will easily appear if trial be made with 2 $\frac{2}{5}$*l.* in the same manner as with the first feigned number.

Quest. 2. *Vitruvius* (*in lib.* 9. *cap.* 3.) reporteth that King *Hiero* having given commandment for the making of a *Crown* of pure *Gold*, was informed that the Workman had detained part of the *Gold*, and mixt the rest with as much *Silver*, as he had stole of *Gold*; the King being much displeased at the deceit, recommended the examination of the business to the famous *Archimedes* of *Syracuse*, who without defacing the *Crown* discovered the cheat in this manner; *viz.* Experience telling him that a quantity of *Gold* would possess less room or space than the same quantity of *Sil-*

ver

ver, and consequently that a mixt mass of *Gold* and *Silver* of the same quantity would take up some mean space between the two former, he made a mass of pure *Gold* of the same weight with the *Crown*, likewise another mass of *Silver* of the same weight, then having put the *Crown*, as also the other two Masses severally into a vessel filled up to the brim with water, he diligently reserved the water flowing over into another vessel, and from those 3 several quantities of water so expell'd, he found out the quantity of *Gold* and of *Silver* in the *Crown*: But forasmuch as *Vitruvius* delivers not the practical operation, I shall here shew the same after the manner of *Cardanus*, *Gemma Frisius*, and other *Arithmeticians*.

Let us therefore suppose the weight of the *Crown* as also of the two several Masses to have been 5*l*. Suppose also, that by putting of the mass of *Gold* into the vessel, 3*l*. of water was expell'd; by putting in of the *Crown*, $3\frac{1}{4}$*l*. and by putting in of the mass of *Silver*, $4\frac{1}{2}$*l*. The question therefore is to know how much *Gold* and how much *Silver* the *Crown* was composed of. This may be resolved after this manner. Suppose 3*l*. of *Gold* to be in the *Crown*, then there remained 2*l*. of *Silver*, now say by the Rule of 3, if 5*l*. of *Gold* expel 3*l*. of water, how much 3*l*. of *Gold*? *Answer* $1\frac{4}{5}$*l*. Also if 5*l*. of *Silver* expel $4\frac{1}{2}$*l*. of water, how much 2*l*. of *Silver*? *Answer*, $1\frac{4}{5}$*l*. of water, add therefore the water of the *Silver* and of the *Gold* together, to wit, $1\frac{4}{5}$ and $1\frac{4}{5}$, so there will arise $3\frac{3}{5}$*l*. of water: This ought to have been $3\frac{1}{4}$*l*. (for so much overflowed

5 — 3 — 3 — ($1\frac{4}{5}$

5 — $4\frac{1}{2}$ — 2 — ($1\frac{4}{5}$

flowed by putting in of the *Crown*;) but it is too much by $\frac{7}{20}$, wherefore $\frac{7}{20}$ is to be noted with + for the *errour* of the *first Position* 3*l*. Again, feign another quantity of *Gold* to have been in the *Crown*, to wit, 2*l*. therefore there remained 3*l*. of *Silver*; then say if 5*l*. of *Gold* expel 3*l*. of water, how much 2*l*. of *Gold*? *Answ*. $1\frac{1}{5}$*l*. of water: Also if 5*l*. of *Silver* expel $4\frac{1}{2}$*l*. of water, how much 3*l*. of *Silver*? *Answer*, $2\frac{7}{10}$; then add $1\frac{1}{5}$ unto $2\frac{7}{10}$, the sum will be $3\frac{9}{10}$*l*. of water: this ought to have been $3\frac{1}{4}$*l*. but it is too much by $\frac{13}{20}$, wherefore $\frac{13}{20}$ is to be noted with + for the *errour* of the *second Position* 2*l*. Here because the *errours* are *fractions* having a *common Denominator*, I take their *Numerators* 7 and 13 instead of the *errours*; then multiplying cross-wise, to wit, 3 by 13 the Product is 39, also 2 by 7 the Product is 14, which subtracted from the former Product 39 (because the *errours* are like) leaves 25 for a *Dividend*; also the difference between the *errours* 7 and 13 is 6 for a *Divisor*; Lastly, dividing 25 by 6, the quotient is $4\frac{1}{6}$; so much *Gold* therefore was in the *Crown*, and consequently (because the weight of the *Crown* was 5*l*.) there was $\frac{5}{6}$*l*. of *Silver* which may be proved thus: Say, if 5*l*. of *Gold*, expel 3*l*. of water, how much $4\frac{1}{6}$*l*. of *Gold*? *Answer*, $2\frac{1}{2}$*l*. of water: Again, if 5*l*. of *Silver* expel

5 — 3 — 2 — ($1\frac{1}{5}$

5 — $4\frac{1}{2}$ — 3 — ($2\frac{7}{10}$

Pos.		*Er.*	
3	+	$\frac{7}{20}$	7
2	+	$\frac{13}{20}$	13
39			
14			

6) 25 ($4\frac{1}{6}$*lb*. of *Gold*.

pel 4 $\frac{1}{3}$ of water, how much $\frac{5}{8}$ of *Silver?* *Answer*, $\frac{3}{4}l.$ of water, which being added to 2 $\frac{1}{2}l.$ the sum is 3 $\frac{1}{4}l.$ of water, to wit, as much as flowed over when the *Crown* was put into the vessel.

Here note, that in making a trial of this nature, there is no necessity that the mass of Gold or of Silver be of the same weight with the *Crown*, or whatsoever thing is to be examined, but of what notable part of weight you please.

Note also; that for the more easie discovering of the Dividend and Divisor by the notes of + and — according to the fourth Rule of this Chapter, the following Verse may be a help to wit,

Addito dissimiles, subtrahitoque pares.

Or thus,

Notes being unlike, Addition make;
If like, lesser from greater take.

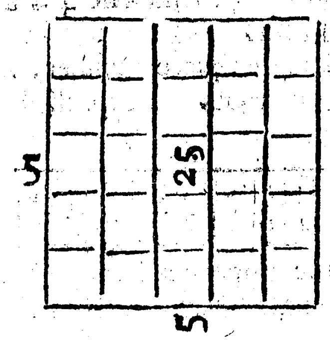
The Reader may see more questions to exercise the *Rule* of *False* in the tenth Chapter of the *Appendix*, and the demonstration thereof in the ninth Chapter of the same.

CHAP.

CHAP. XXXII.

The Extraction of the Square (or Quadrate) Root.

I. THE Extraction of the Square root is that, by which having a number given, we find out another number, which being multiplied by it self, produceth the number given.



II. In the Extraction of the Square-root, the number propounded is always conceived to be a square number, that is, a certain number of little squares comprehended within one intire great square, and the root or number required is the side of that great square, as will readily appear by this Diagram, where you see 25 little squares contained within one great square; now if the said content 25 be given, and the side or root of the square containing the said 25 little squares is required, the invention of such side or root is called the extraction of the square root; which root must

be

be such, that if it be squared, that is, multiplied by it self, the Product must be equal to the square content first given: So 5 is the square root of 25, for 5 times 5 is 25. Likewise this square number 49 being propounded, his root is 7.

III. Square numbers are either single or compound.

A single square number.

IV. A single square number is that, which being produced by the multiplication of one single figure by it self, is always less than 100: So 25 is a single square number produced by 5; likewise 4 is a square number produced by 2.

V. All the single square numbers together with their respective roots are expressed in the Table following.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed in the single square numbers of every particular figure, and in the other their respective roots; and therefore if it were demanded, What is the square root of 36, the answer will be 6. So the square root of 16 is 4; the square root of 9 is 3, &c. And contrarily the square of the root 6 is 36: Also the square of 3 is 9.

VI. When a square number is given, that exceeds not 100, and yet is none of the square numbers mentioned in the Table, for his root you are to take the root of the square number that being less, yet comes nearest unto it: So 45 being given, the root that belongs unto it is 6, and 10 being given, his correspondent root is 3.

A compound square number.

VII. A compound square number is that, which being produced by a number (that consists of more places than one) multiplied by it self, is never less than 100: So 1024 is a compound square number produced by the multiplication of 32 multiplied by it self.

VIII. To prepare any square number given for extraction, put a point over the first place thereof on the right hand (being the place of Units;) then proceeding towards the left hand, pass over the second place, and put another point over the third place; also passing over the fourth place put another point over the fifth, and so forward in such manner that between every two points which are next one to the other, one place will be intermitted: So if the square root of 1024 be required, the first point is to be placed over 4, and the second over 0 as you see, and so many points as are in that manner placed, of so many figures the root demanded will consist.

$1\dot{0}2\dot{4}$

IX. Having thus prepared your number, you may see it distributed by the points into several squares: So in the last *Example*, 10 is the first square and 24 the second; likewise if this number 144 were propounded for extraction, after points are duly placed according to the last Rule, you will see 1 to be the first square and 44 the second.

$\dot{1}4\dot{4}$

X. Having drawn a crooked line on the right hand of the number propounded for extraction (after the same manner as is usually done in Division to denote the place of the quotient,) find the

root of the first square, and place it in the quotient: so I find, by the sixth Rule aforegoing, 3 to be the correspondent root of 10; wherefore I write 3 in the quotient, and then the Work will stand as you see.

1024 (3

XI. Subscribe the square of the figure placed in the quotient under the first square of the number given, as you see in the Margent.

1024 (3
9

XII. Having drawn a line under the square (of the figure placed in the quotient, subscribed as aforesaid, subtract the same out of the first square of the number propounded, and place the remainder orderly underneath the line; so the square of 3 which is 9 being subtracted from 10, the remainder is 1, and the Work will stand as you see in the Margent.

1024 (3
9
———
1

XIII. To the said remainder bring down the next square of the number propounded, that is write down the figures or cyphers standing in the two following places of the number propounded on the right hand of the said remainder, so the square 24 being placed next to the remainder 1, there will be found this number 124 which may be called the *Resolvend.*

1024 (3
9
———
124

XIV. Double the root being the number placed in the quotient, and place the said double on the left hand of the Resolvend, like a Divisor: so the double of 3 is 6, which being placed before a crooked line on the left

1024 (3
9
———
6) 124

hand of the Resolvend 124, the work will stand as you see.

XV. Let the whole Resolvend, except the first place thereof on the right hand (being the place of units) be always esteemed as a Dividend, then demanding how often the Divisor before found, is contained in the said Dividend, and observing in that behalf the Rules before taught in Division, write the answer in the quotient, and also on the right hand of the Divisor, to wit, between the Divisor and the crooked line: So if you ask how often the Divisor 6 is found in the Dividend 12, the answer is 2, wherefore I write 2 in the quotient, and also after the Divisor 6, as you see in the Margent.

1024 (32
9
———
62) 124

XVI. Multiply all the number which standeth on the left hand of the Resolvend, (to wit, before the crooked line) by the figure last placed in the quotient, and write the Product orderly underneath the Resolvend (to wit, units under units, tens under tens, &c.) then having drawn a line under the said Product, subtract it from the Resolvend, and subscribe the remainder under the line: So 62 being multiplied by 2, the Product is 124, which if I subtract out of the Resolvend 124, the remainder is 0; and thus the whole Work being finished, the square root of 1024 (the number propounded) is found to be 32.

1024 (32
9
———
62) 124
124
———
0

Note,

Note 1. When the Product before mentioned exceeds the *Resolvend* placed above it, the work is erroneous, and then you are to reform it by placing a lesser figure in the quotient.

Note 2. For every one of the particular squares (distinguished by the points) except the first on the left hand, a *Resolvend* is to be set apart, by bringing down to the remainder the congruent particular square, as is directed in the 13 *Rule*; and as often as a *Resolvend* is set apart, so often a new *divisor* is to be found by doubling or multiplying by 2 all the root in the quotient (consisting of what number of places soever.)

Note 3. The work of the 10, 11, and 12 *Rules* for finding of the first figure in the *root*, is but once used in the extraction of the root of a number consisting of what number of places soever; but the Work of the 13, 14, 15, and 16 Rules is to be repeated for the finding of every place in the root, except the first.

The practice of these 3 *Notes* will be seen in the following *Examples*.

Example 1. Let it be required to extract the square root of 43623.

Having distributed the number propounded into several squares by points, as is directed in the eighth Rule of this Chapter, I demand the square root of 4 the first square, which I find by the 5 rule of this Chapter to be 2; wherefore placing 2 in the quotient, and the square thereof, which is 4, under the first square 4, I draw a line, and subtracting 4 from 4 the remainder is 0, which I subscribe underneath

43623 (2
4
———
0

derneath the line. This is always the first Work, which is no more repeated in the whole Extraction (as was intimated in the third *Note* aforegoing.)

Then bringing down the next square, which is 36, and placing it next after the remainder 0, the *Resolvend* is 36; and doubling the root 2 in the quotient, the Product is 4 for a *Divisor* (by the 13 and 14 Rules) and the *Dividend* will be 3 (by the 15 Rule;) wherefore I demand how often the *Divisor* 4 is contained in the *dividend* 3, and not finding it once contained in it, I place 0 in the quotient, and also next after the *Divisor* 4; and because the Product of 40 multiplied by 0 (the last Character in the quotient) is 0, the *resolvend* 36, from which the said Product ought to be deducted, remains the same without alteration; therefore I bring down 23 the next square, and place it after the remainder 36, so will 3623 be a new *resolvend*; then doubling the whole root in the quotient, which is 20, the *divisor* will be 40 (according to the second *Note* before mentioned,) and the *dividend* will be 362 (to wit, all the *resolvend* except the first place on the right-hand by Rule 15.) wherefore I demand how often the *divisor* 40 is contained in the *dividend* 362, or how often 4 in 36, and though it be 9 times in it, yet (according to the first *Note* aforegoing) I can take but 8, (for if I should take 9, and proceed according to the 15 and

43623 (20
4
———
40) 036

43623
4
———
40) 03623

and 16 Rules, an number would arise greater than the *resolvend*, from which such number arising ought to be subtracted;) wherefore I write 8 in the quotient, and also after the divisor 40; this done, I multiply 408 (the number on the left hand of the *resolvend*) by 8 the figure last placed in the quotient, and the Product, to wit, 3264 I subscribe under, and subtract from the *resolvend* 3623, so there will remain 359, thus the work being finished I find 208 to be the number of

```
 . . .
 43623 (208
 4
 ----------
408) 03623
      3264
      ----
       359
```

unities contained in the root sought; and because after the extraction is ended there happens to be a remainder, to wit, 359, I conclude that the root sought is greater than the said 208, but less than 209, yet how much it is greater than 208, no Rules of Art hitherto known will exactly discover although we may proceed infinitely near, as in the next Rule will be manifest.

XVII. To find the fractional part of the root very near, a competent number of pairs of cyphers, to wit, 00, 0000, 000000, or 00000000, &c. are to be annexed to the number first propounded, then esteeming the number propounded with the cyphers annexed to be but one entire number, the extraction is to be made according to the precedent Rules, and look how many points were placed over the number first given, so many places of Integers will be in the root, the rest of the root towards the right hand will be the Numerator of a decimal fraction, which Numerator consisteth of so many places as there were points over the cyphers

cyphers annexed: So if 43623 were given as before, to find the root thereof (according to this rule) annex cyphers in this manner, and then if you extract it according to the Rules aforegoing, you

43623.000000 (208.861, &c.

will find the root arising in the quotient to be 208.861, that is $208\frac{861}{1000}$; and because after the extraction is finisht there happens to be a remainder, I conclude that $208\frac{861}{1000}$ is less than the true or exact root, but $208\frac{862}{1000}$ is greater than it; so that by annexing three pairs of cyphers to the number propounded, you will not miss $\frac{1}{1000}$ part of an unit of the true *root*; also by annexing 4 pairs of cyphers, you will not miss $\frac{1}{10000}$ part of an unit, and in that order you may proceed infinitely near, when you cannot obtain the true root. The whole operation of the said *Example* here followeth.

```
         . .  .  . . .
         43623.000000 (208.861, &c.
         4              The root.
       -------------
 408 ) 03623
        3264
       ------
4168 )  35900
        33344
       -------
41766 ) 255600
        250596
       --------
417721 ) 500400
         417721
        --------
          82679
```


Again,

Again, if 10 were propounded to be extracted, you must prepare it thus,

10.00000000000000 (

And then the root thereof being extracted will be — $3\frac{1622776}{10000000}$, &c.

Which (according to the third Rule of the 22 Chapter) may be written thus — 3. 1622776, &c.

See here part of the Work in the extraction of the *Root* of 10, which may give you a light and understanding of the rest.

10.00000000000000 (3. 16227, &c.

9

61) 100
61

626) 3900
3756

6322) 14400
12644

63242) 175600
126484

632447) 4911600
4427129

484471

XVIII. The

XVIII. The extraction of the square root is proved by multiplying the root by it self, for that done, the Product (in such case, when there is no remainder after the extraction is finished) will be equal to the number whose square root was enquired; so in the first *Example* of this *Chapter*, the root 32 being multiplied by it self, produceth 1024 the number propounded: But when after the extraction is finished there happeneth to be a remainder, and that the root is found as near as you please, in a mixt number of Integers and Decimal parts (by annexing cyphers, as in the 17 Rule) then such mixt number being multiplied by it self must produce a mixt number less than the number first propounded for extraction, yet so near unto it, that if the figure standing in the last place of the Numerator of the Decimal fraction in the root be made greater by 1, and then the mixt number so increased be multiplied by it self, the Product must be greater than the number first propounded: So in the *Example* of the 17 *Rule*, if 208. 861 be multiplied by it self, it produceth 43622. 917, &c. which is less than the propounded number 43623, but if 208. 862 be multiplied by it self, the Product will be 43623. 335, &c. which is greater than 43623. *The Proof.*

XIX. The square root of a Fraction is found in this manner, *viz.* extract the root of the Numerator (by the precedent Rules of this Chapter) which root shall be a new Numerator. Also the root of the Denominator is to be taken for a new Denominator: So the new Fraction shall be the square root of the Fraction first propounded *To extract the square root of a Fraction.*

ed

ed, Thus the *square root* of $\frac{9}{16}$ is $\frac{3}{4}$, *viz.* the *root* of 9 is 3 for a new *numerator*, also the *root* of 16 is 4 for a new *denominator.* In like manner the *square root* of $\frac{1}{4}$ is $\frac{1}{2}$. But here note diligently, that if the *Fraction* whose *square root* is required be not in its least terms, it must first of all be reduced by the 4th *Rule* of the 17th *Chapter* before any extraction be made; for oftentimes it happens that the *Fraction* first given hath not a perfect *root*, but when such *Fraction* is reduced into its least terms, the *root thereof* may be extracted. So in this *Fraction* $\frac{8}{18}$, each term is *incommensurable* to its *square root*, *viz.* neither 8 nor 18 hath a *square root* expressible by any true or rational number; but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the *root* of this may be extracted, for the *root* of 4 is 2 for a new *Numerator*; also the *root* of 9 is 3 for a new *Denominator*; so that $\frac{2}{3}$ is found to be the *square root* of $\frac{4}{9}$, equivalent unto $\frac{8}{18}$.

XX. When either the Numerator or Denominator of a Fraction hath not a perfect square root, such root is usually exprest by prefixing this Character, $\surd$ or $\surd$ q. before the Fraction given: So the *square root* of $\frac{13}{16}$ is signified thus, $\surd\,\frac{13}{16}$, or thus $\surd$ q. $\frac{13}{16}$, because the *root* of $\frac{13}{16}$ cannot be exprest by any true or rational number whatsoever, yet it may be found very near, as in the next *Rule.*

To extract the square root near, of a fraction incommensurable to its square root.

XXI. The *square root* of a *Fraction* which is incommensurable to its *root*, may be found near, in this manner, *viz.* reduce the fraction proposed into a decimal by the third Rule of the 23 Chapter: The more places are in the decimal, the nearer will the root be found, but the decimal must consist of an even number of places, *viz.* either of two, four, six, eight, or ten, &c. places; then extract the *square root* of that decimal, as if it were a whole number, according to the Rules aforegoing, which *root* found shall be a decimal expressing near the *square root* of the fraction proposed.

So if the *square root* of $\frac{13}{16}$ be required near, reduce the said $\frac{13}{16}$ into a decimal (by the 3d *Rule* of the 23d *Chapter*) which will be found .8125000, &c. Then extracting the *square root* thereof as if it were a whole number, it will be found .9013 very near.

To extract the square root of a mixt number.

XXII. The *square root* of a mixt number commensurable too its *root* is found in the same manner as in the 19th Rule of this Chapter, the mixt number being first reduced into an improper fraction by the 10th Rule of the 17th Chapter.

So the *square root* of $34\frac{33}{64}$ will be found $5\frac{7}{8}$ *viz.* $34\frac{33}{64}$ being reduced into the improper Fraction $\frac{2209}{64}$, the *square root* of the Numerator 2209 will be 47 for a new Numerator; also the *square root* of the *Denominator* 64 is 8, for a new *Denominator*; so is found $\frac{47}{8}$, which by (the 13th Rule of the 17th Chapter) is $5\frac{7}{8}$ the *square root* sought. And here the same Caution is to be observed as in the 19th Rule of this Chapter; *viz.* The fractional part of the mixt number, or the improper fraction equivalent unto the mixt number, must be in the *least terms* before any extraction be made.

To find the square root near, of a mixt number incommensurable to its root.

XXIII. When the mixt number given is incommensurable to its *square root*, prefixing this Character before it, *viz.* √ or √ q. So the *square root*, of 7 $\frac{2}{3}$ will be thus expressed: √ 7 $\frac{2}{3}$ or √ q. 7$\frac{2}{3}$: But if you desire to find the *square root* near of a mixt number *incommensurable* to its *root*, reduce the fractional part of the mixt number into a Decimal of an even number of places, as in the 21 Rule of this Chapter, and annex the Decimal so found unto the whole part of the mixt number; then esteeming the said whole number and Decimal as one entire number, extract the *square root* thereof according to the aforegoing Rules of this Chapter, and from the root found, cut off always to the right hand, so many places as there are points over the Decimal annexed, which number so cut off shall be a Decimal, shewing the fractional part of the root, and that on the left hand shall be the whole part of the root; so the *square root* of 7 $\frac{2}{3}$ will be found 2. 7688 very near.

CHAP. XXXIII.

The Extraction of the Cube Root.

I. THE Extraction of the Cube Root is that, by which having a number given, we find another number, which being first multiplied by it self, and then by the Product, produceth the number given.

A Cubical number.

II. In the Extraction of the Cube root, the number propounded is always conceived to be a Cube number, that is a certain number of little Cubes, comprehended within one entire great Cube, and the root or number required is the side of that great Cube: what a Cube is may be well express'd by a Die, which indeed is a little Cube it self; wherefore if you place four Dice in a square form, that is, laying two and two in a rank, you shall have a square containing four Dice, upon which if you yet erect such another square of Dice, you shall have a great entire Cube comprehending two times 4, that is 8 Dice or little Cubes; and here 8 is the Cube number given, and two is the root, or number required: In like manner if you rank 25 Dice in a square form, *viz.* laying 5 in a rank, you have a square containing 25 Dice, now upon this square of Dice if you erect four other such squares one upon another, you shall have a great entire Cube comprehending 5 times 25, this is 125 little Cubes, and in this case 125 is the Cube number propounded and 5 the root or number required.

III. A Cube number is either single or compound.

A single Cube number.

IV. A single Cube number is that, which being produced by the multiplication of one single figure first by it self, and then by the product is always less than 1000. So 125 is a single Cube number produced by 5 multiplied first by it self, and then by 25 the product; for 5 times 5 is 25, and 5 times 25 is 125.

V. All the single Cube numbers, and square numbers

bers, together with their respective roots, are expressed in the Table following.

Cubes.	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost rank of the Table are placed the single Cube numbers of the particular figures 1,2,3,4,5,6,7,8,9. in the next the squares of those figures, and in the lowest rank the figures themselves being the respective roots of the Cubes and Squares in the uppermost ranks; and therefore the *Cube root* of 125 being demanded, the Answer is 5, and the *Cube root* of 216 being required, the Table will give you six, and so of the rest.

VI. When a Cube number is given, that exceeds not 1000, and yet is none of the Cube numbers mentioned in the Table, for his root you are to take the root of the Cube number that being less, yet comes nearest unto it: so 157 being given, the root that belongs unto it is 5.

A Compound Cube number.

VII. A compound Cube number is that, which being produced by a number (that consists of more places than one) first multiplied by it self, and then by the Product is never less than 1000. So 157464 is a *Compound Cube number*; being produced by 54 multiplied first by its self; and then by 2916 the Product, for 54 times, 54 is 2916, and then 54 times 2916 is 157464, *the Compound Cube number propounded.*

VIII. To

VIII. To prepare a Cube number for extraction, put a point over the first place thereof towards the right hand (to wit the place of Units;) then passing over the second and third places, put another point over the fourth, and passing over the fifth and sixth put another point over the 7th, and in that order (to wit two places being intermitted between every two adjacent points) place as many points as the number will permit: So 157464 being given, you are to place the points as in the Margent, and so many points as are in that manner placed, of so many figures the *root* demanded will consist.

$\dot{1}57\dot{4}64$

IX. Having thus prepared your number, you may see it distributed by the points into several Cubes: so in the same example 157 is the *first Cube*, and 464 the *second*. In like manner if this number 7464 were propounded for extraction, after points are duly placed as before, you will see 7 to be the *first Cube*, and 464 the *second*.

157464

7464

X. Having drawn a crooked line on the right hand of the number propounded to signifie a quotient, find the Cube root of the first Cube and place it in the quotient: So I finding (by the sixth Rule of this *Chapter*) 5 to be the correspondent root of 157, I write 5 in the quotient, and then the work will stand as you see in the Margent.

157464 (5

XI. Subscribe the Cube of the root placed in the quotient, under the first Cube of the number given. So 125 being the Cube of 5 the root (by the

157464 (5
125

fifth

fifth Rule of this Chapter) I write it under 157 the firſt Cube of the number given, as you ſee in the example.

XII. Draw a line under the Cube ſubſcribed as aforeſaid (to wit, the Cube of the root placed in the quotient) and ſubtract this Cube from the firſt Cube of the number propounded, placing the remainder orderly underneath the line: So 125 the Cube of 5 being ſubtracted from 157, the remainder is 32, and the work will ſtand as you ſee.

```
157464 (5
125
———————
 32
```

XIII. To the ſaid remainder bring down the next Cube of the number propounded (to wit, the figures or cyphers that ſtand in the 3 next places) placing the ſaid Cube next after, to wit, on the right hand of the remainder, ſo the next Cube 464 being placed after the remainder 32, there will be found this number 32464, which may be called the *Reſolvend.*

```
157464 (5
125
———————
32464 reſol.
———————
```

XIV. Having drawn a line underneath the *Reſolvend*, ſquare the root in the quotient, that is, multiply it by it ſelf, and ſubſcribe the triple of the ſaid ſquare or product under the reſolvend in ſuch manner, that the firſt place (to wit, the place of units) of the ſaid triple ſquare may ſtand directly under the third place (or place of hundreds) in the reſolvend: So the ſquare of the root 5 is 25, the triple whereof is 75, which I ſubſcribe under the Reſolvend

```
157464 (5
125
———————
 32464 reſol.
———————
 75
```


vend in ſuch manner, that the figure 5 which is in the firſt place (to wit, the place of units) in the triple Product 75, may ſtand under 4, which is ſeated in the third place of the reſolvend, as you ſee in the Margent.

XV. Triple the root or number in the quotient, and ſubſcribe this triple number in ſuch manner, that the firſt place thereof (to wit, the place of units) may ſtand directly under the ſecond place (to wit the place of tens) in the Reſolvend: ſo the triple of the root 5 is 15, which I ſubſcribe in ſuch manner, that the figure 5 which is in the firſt place (to wit the place of units) in the ſaid triple number, doth ſtand directly under 6, which is ſeated in the ſecond place of the reſolvend, and the Work will ſtand as in the Margent.

```
157464 (5
125
————————
 32464 Reſolv.
————————
 75
  15
```

XVI. The triple ſquare of the root, and the triple of the root being placed one under the other, as is directed in the 14 and 15 Rules aforegoing, draw a line underneath, and add them together in ſuch order as they are ſeated, and let the ſum be eſteemed as a diviſor: So the triple ſquare 75, and the triple number 15, being added together as they are ranked in the Work, the ſum will be 765 for a Diviſor.

```
157464 ( 5
125
————————
 32464 Reſolv.
————————
 75
  15
————————
 765 Diviſor.
————————
```

XVIII. Let

XVII. Let the whole Resolvend, except the first place thereof towards the right hand (to wit, the place of Units) be esteemed as a Dividend, then demanding how often the first figure (towards the left hand) of the Divisor is contained in the correspondent part of the dividend, and observing in that behalf the Rules before taught in Division, write the Answer in the quotient: So if I ask how often 7 (the first figure of the Divisor towards the left hand) is contained in 32 (the correspodent part of the Dividend placed above) the Answer will be 4; wherefore I write 4 in the quotient, as you see in the Example.

157464 (54
125

32464 *Resolv.*

75
15

765 *Divisor.*

XVIII. Having drawn another line under the Work, multiply the triple square before subscribed (as is directed in the fourteenth Rule) by the figure last placed in the quotient, and subscribe this Product under the said triple square; (to wit, units under units, tens under tens, &c.) So 75 being multiplied by 4, the Product is 300, which I subscribe under 75 (the triple square) and the work will stand as you see in the Margent.

157464 (54
125

32464 *Resolv.*

75
15

765 *Divisor.*

300

XIX. Multiply

XIX. Multiply the figure last placed in the quotient first by it self, and then the Product by the triple number before subscribed (as is directed in the 15th Rule of this Chapter;) this done, subscribe the last Product under the said triple number (to wit; units under units, tens under tens, &c.) so 4 being squared or multiplied by it self, the Product is 16, which being multiplied by the triple number 15, the Product is 240, this therefore I subscribe under the aforesaid triple number 15, and the Work will stand as you see.

157464 (54
125

32464 *Resolven.*

75
15

765 *Divisor.*

300
240

XX. Subscribe the Cube of the figure last placed in the quotient, under the resolvend, in such manner that the first place of this Cube (to wit, the place of units,) may stand under the place of units in the resolvend: So 64 being the Cube of 4, I write it under the resolvend 32464, in such manner that the figure 4, which is in the place of units in the *Cube* 64, may stand under the figure 4 which is seated in the place of units of the resolvend: Observe the Work in the Margent,

157464 (54
125

32464 *Resolvend.*

75
15

765 *Divisor.*

300
240
64

XXI.

XXI. Drawing yet another line under the work, add the three last numbers together in the same order as they are seated, and subtract the sum of them from the resolvend, placing the remainder orderly underneath: So the sum of the three last numbers, as they are ranked in the Work, is 32464, which if you subtract out of the resolvend 32464, the remainder is 0. Thus the whole Work being finished, the Cube root of 157464 (the number propounded) is found to be 54.

```
 . .
157464 (54
125
------------
 32464 Resolvend.
------------
 75
  15
------------
 765 Divisor.
------------
300
 240
  64
------------
32464
------------
    0
```

Note 1. When the sum of the three last numbers before mentioned is greater than the resolvend, the Work is erroneous, and then you are to reform it by placing a lesser figure in the quotient.

Note 2. For every one of the particular Cubes (distinguished by the points) except the first Cube on the left hand, a resolvend is to be set apart, by bringing down to the remainder the next Cube (as is directed in the 13 Rule.) And as often as a resolvend is set apart, so often is a new Divisor to be found, by adding the *triple* of all the *root* in the *quotient* (consisting of what number of places soever) to the *triple* of the *square* of such *root*, after they are orderly placed according to the 14 and 15 *Rules*.

Note

Note 3. The Work of the 10, 11, and 12 *Rules* for finding of the first figure in the *root* is but once used in the extraction of the root of any number whatsoever, but the Work of all the following *Rules* is to be used for the finding of every place in the *root*, except the first.

The practice of these 3 *Notes* will be seen in the following *Examples.*

Example 2, Let it be required to extract the *Cube root* of 8302348.

Having distributed the number given into several *Cubes* by points, as is directed in the eighth *Rule* of this *Chapter*, I demand the *Cube root* of 8 (the first *Cube* on the left hand) which I find by the fifth *Rule* of this *Chapter* to be 2, wherefore placing 2 in the quotient, and 8 the Cube thereof under 8 the first Cube, I draw a line, and subtracting 8 out of 8 the remainder is 0, which I subscribe under the line. This is always the first Work, and is no more repeated in the whole extraction (as was intimated in the 3 Note aforegoing;) then bringing down the next Cube (to wit, the figures standing in the three following places of the number propounded) which is 302, I place it after the remainder 0, so is 302, the *resolvend*; this done, having drawn a line underneath the *resolvend*, I seek for the triple of the square of the root, *viz.* the *root* in the *quotient*, is 2, which multiplied by it self produceth the square 4, the triple whereof is 12, this I subscribe under the *resolvend*, in such manner that the figure 2 in

```
. . .
8302348 (2
8
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0
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in the units place of this triple square 12, may stand directly under the figure 3, which is seated in the third place of the *resolvend*, (to wit, the place of hundreds) according to the 14th Rule aforegoing; Again, I triple the root 2, which produceth 6, and subscribe this triple number 6 under the second place (or place of tens) in the *resolvend*, to wit, under 0 (according to the 15th *Rule* of this *Chapter*;) then drawing a line under the Work, and adding together the said two numbers last subscribed, as they are ranked, the sum of them is 126 for a divisor (according to the 16th *Rule* aforegoing.

```
8302348 (2
8

0302 Resolvend.

 12
 06

126 Divisor
```

That done, esteeming 30, to wit, all the places except the first or place of units in the *resolvend*, as a *Dividend*, I demand how often the *divisor* 126 is contained in 30, and not finding it once contained therein, I write 0 in the *quotient*; and now because the sum of the three numbers which ought to have been Produced (according to the 18, 19, and 20 *Rules* of this *Chapter*) by the multiplication of 0 (which was last placed in the *quotient*) amounts to 0; the *resolvend* 302, out of which the said sum should have been subtracted, remains the same without alteration; wherefore having drawn a line under the Work, I write down a new the old *resolvend* 302, and bringing down the next *Cube* 348, I annex it to the said

302

302; so there will be a new *resolvend*, to wit, 302348.

Then squaring the root 20 (that is, multiplying of it by it self) the Product is 400, which I triple or multiply by 3, and subscribe the Product 1200 underneath the new resolvend in such manner, that the place of units in this triple quadrate 1200 may stand under the place of hundreds, or third place of the resolvend 302348, to wit, under 3 (according to the 14th *Rule*.) Again, I subscribe the triple of the *root* 20, which is 60, in such manner that the place of units in this triple root 60 may stand under the place of tens or second place of the resolvend, to wit, under 4, then adding together the two numbers last subscribed, to wit, 1200 and 60, in such order as they are ranked in the Work, the sum is 12060 for a Divisor.

```
8302348 (202
8

0302 Resolvend

12
 06

126 Divisor

302348 Resolvend

1200
  60

12060 Divisor

2400
 240
  08

242408 Ablatitium

 59940
```


Again,

Again, esteeming the whole resolvend, except the first place (or place of units) as a dividend, to wit, 30234, I demand how often 1 (the first figure of the divisor towards the left hand) is contained in 3 the correspondent part of the Dividend; and though it be three times contained in it, yet (according to the first *Note* at the end of the 21 *Rule* of this *Chapter*) I dare take but 2, for if I should take 3, and proceed according to the 18, 19, 20, and 21 *Rules* of this *Chapter*, a number would arise greater than the resolvend (from which such number arising ought to be subtracted) wherefore I write 2 in the *quotient*.

Then multiplying the triple square 1200 before subscribed, by 2 (the figure last placed in the quotient,) the Product is 2400, which I subscribe under the said 1200 (to wit, units under units, and tens under tens, &c.) Also multiplying the triple root 60 before subscribed, by 4 (the quadrate of 2 the figure last placed in the quotient) the Product is 240, which I subscribe under the said triple root 60; last of all I subscribe 8 the *Cube* of the said new *root* 2, under the place of units or first place of the *resolvend*, to wit, under 8, and having added together those three numbers last subscribed, to wit 2400, 240 and 8 as they stand in ranks in the Work, the sum of them is 242408, which being subducted from the *resolvend* 302348, there will remain 59940. Wherefore the Work being finished, I find 202 to be the number of unities contained in the *Cube root* of 8302348 the number propounded: And because after the extraction is ended there happens to

to be a remainder, to wit 59940, I conclude that the *Cube root* sought is greater than the said 202, but less than 203; yet how much it is greater than 202, no Rules of Art hitherto known will exactly discover, although we may proceed infinitely near, as by the next *Rule* will be manifest.

XXII. To find the fractional part of the root very near ternaries of Cyphers, to wit, 000, 000000, or 000000000, &c. are to be annexed to the number first propounded; then esteeming the number propounded with the cyphers annexed to be but one entire number, the Extraction is to be made according to the preceeding *Rules* of this *Chapter*, and look how many points were placed over the number first given, so many of the foremost places in the Quotient are the Integers or unities contained in the Cube root sought, and the rest of the places in the quotient are to be esteem'd as the Numerator of a Decimal fraction, which Numerator consisteth of so many places as there were points over the cyphers first annexed: So if 8302348 were given as before, to find the *Cube root* thereof (according to this Rule) annex cyphers in this manner.

$$\dot{8}30\dot{2}34\dot{8},00\dot{0}00\dot{0}($$

And then if you prosecute the extraction according to the Rules foregoing, you shall find the *Cube root* sought to be 202. 48, &c. that is, $202\frac{48}{100}$ and more; wherefore you may conclude that $202\frac{48}{100}$ is less than the true *root*, but $202\frac{49}{100}$ is

 greater

greater than it: So that by annexing two ternaries of cyphers, to wit, 6 cyphers, to the number propounded, you will not miſs $\frac{1}{100}$ part of an unit of the true *root*, alſe by annexing 3 ternaries of cyphers, to wit, 9 cyphers, you will not miſs $\frac{1}{1000}$ part of an unit of the true *root*, and in that order you may proceed infinitely near, when you cannot obtain the true *root*. The whole operation of the ſaid Example here followeth, where you may obſerve, that for the more certain and eaſie placing, as well of the numbers which conſtitute the ſeveral Diviſors, as of thoſe which conſtitute the Ablatitious numbers to be ſubtracted form the ſeveral and reſpective Reſolvends, down-right lines are drawn between the particular *Cubes* of the number propounded, firſt diſtinguiſhed by points as before.

8	302	348	000	000	(202. 48, &c.
8					
0	302				Reſolvend
1	2				
	06				
1	26				Diviſor
	302	348			Reſolvend
	1200	0			
		60			
	120	60			Diviſor
	2400	0			
	2	40			
		08			
	242	408			Ablatitium
	59	940	000		Reſolvend
	12	241	2		
		6	06		
	12	247	26		Diviſor
	48	964	8		
		96	96		
			64		
	49	061	824		Ablatitium
10	878	176	000		Reſolvend
1	228	972	8		
		60	72		
1	229	033	52		Diviſor
9	831	782	4		
	3	886	08		
			512		
9	835	668	992		Ablatitium
1	042	507	008		

In like manner the *Cube root* of 2 will be found to be near equal to 1. 25992, &c. that is, 1 $\frac{25992}{100000}$ and more.

XXIII. The extraction of the Cube root is proved by multiplying the root cubically, to wit, the root being first multiplied by it self, and then the product multiplied by the root, the number arising or last Product (in case there be no remainder after the extraction is finished) will be equal to the number propounded: so in the first Example of this *Chapter*, the *Cube root* 54 being multiplied first by it self produceth 2916, which being multiplied again by 54 produceth 157464, to wit, the number whose *Cube root* was inquired. But when after the Extraction is finished, there happeneth to be a remainder, and that the *root* is found as near as you please in *Integers* and *decimal parts* (by annexing cyphers as in the 22 *Rule* of this *Chapter*,) then such mixt number expressing the *root*, being multiplied cubically, must produce a mixt number less than the number first propounded, yet so near unto it, that if the figure standing in the last place of the *decimal fraction* in the *root* be made greater by 1, and the mixt number so increased be multiplied cubically, the Product must be greater than the number first propounded: So in the Example of the 22 *rule* of this *Chapter*, if 202.48 be multiplied cubically, it produceth 8301305 .49, &c. which is less than the propounded number 8302348, but if 202.49 be multiplied cubically, there will arise 8302535 .49, &c. which is greater than the said given number.

The Proof.

XXIV. The Cube root of a Fraction is found in this manner, *viz.* extract the Cube root of the Numerator

Numerator (according to the aforegoing Rules,) which root reserve for a new Numerator; also the Cube root of the Denominator shall be a new Denominator; lastly this new Fraction shall be the Cube root of the Fraction first propounded: so the *cube root* of $\frac{8}{27}$ is $\frac{2}{3}$, for the *cube root* of 8 is 2 for a new *Numerator*, also the *cube root* of 27 is 3 for a new *Denominator*. In like manner the *cube root* of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently, that the *fraction* whose *cube root* is required, must be in its least terms before any Extraction be made; for oftentimes it happens that the *fraction* first given hath not a perfect root, albeit, when such *fraction* is reduced into its least terms, the root thereof may be extracted: so in this *Fraction* $\frac{16}{54}$ neither the *numerator* nor *denominator* hath a perfect *cube root*, yet the said $\frac{16}{54}$ being reduced to its least terms $\frac{8}{27}$, (by the fourth Rule of the 17 Chapter) the *cube root* of this may be extracted, for the *cube root* of 8 is 2 for a new *numerator*, also the *cube root* of 27 is 3 for a new *denominator*, so that the *cube root* of $\frac{8}{27}$ (which is equal to $\frac{16}{54}$) is found to be $\frac{2}{3}$.

To extract the Cube root of a fraction.

XXV. The Cube root of a fraction which hath not a perfect Cube root may be found near in this manner, *viz.* reduce the Fraction given into a Decimal fraction, by the third Rule of the 23 Chapter, the more places are in the Decimal, the nearer will the root be found, but the Decimal must consist of ternaries of places, to wit, either of three, six, nine, or twelve, &c. places; then extract the Cube root of the Numerator of that Decimal, as if it were a whole number (according to the Rules before given,) which root found shall be a Decimal

expressing near the Cube root of the Fraction propounded.

So if the *cube root* of $\frac{2}{3}$ were required, I reduce the said $\frac{2}{3}$ into a *decimal*, whose *numerator* may consist of *ternaries* of places, to wit into this, 666666666666 &c. then extracting the *cube root* thereof, I find 8735, which is very near the *cube root* of $\frac{2}{3}$.

XXVI. The Cube root of a mixt number commensurable to its root may be found in the same manner as in the 24 Rule of this Chapter, the mixt number being first reduced into an improper fraction (by the 10 Rule of the 17 Chapter.)

So the *cube root* of $12\frac{19}{27}$ will be found to be $2\frac{1}{3}$, *viz.* reducing $12\frac{19}{27}$ into this improper fraction $\frac{343}{27}$ the cube root of $\frac{343}{27}$ will be found $\frac{7}{3}$ or $2\frac{1}{3}$. And here the same caution is to be observed as in the 24 *Rule* of this *Chapter*; *viz.* the fractional part of the mixt number, or the improper fraction equivalent unto the mixt number, must be expressed by a *Numerator* and *Denominator* in the least terms before any extraction be made.

XXVII. When the mixt number, whose Cube root is required, hath not a perfect cube root, this character, √ c. is usually prefixed before such mixt number; so the *cube root* of $2\frac{3}{8}$ is thus expressed, √ c. $2\frac{3}{8}$. Likewise √ c. $\frac{5}{8}$ denotes the *cube root* of $\frac{5}{8}$, which is a fraction, whose *cube root* is inexpressible by any true or rational number: But if you desire to know the *cube root* near of a mixt number which hath not a perfect *cube root*, reduce the fractional part of the mixt number into a *decimal* (as in the 25 *Rule* of this *Chapter*) and annex the *decimal* so found unto the Integers of the mixt number; then esteeming the said Integers with the *decimal* so annexed as one entire number, extract the *cube root* thereof and from the *root* found cut off always to the right hand so many places as there were points over the said *decimal* annexed, which places so cut off shall be the fractional part of the *root*, and those remaining on the left hand shall be the Integers of the *root*: So the *cube root* of $2\frac{3}{8}$ will be found 1 .334, and more.

XXVIII. I might here proceed to shew the extraction of the *roots* of the *Biquadrate* (or fourth *Power*) the fifth *Power*, &c. but their operations being exceeding tedious, and hardly intelligible without the knowledge of *Algebra*, I shall only in this place touch upon the Extraction of the *Biquadrate root*, because it may be extracted by the Rules delivered in the 32 *Chapter*, and referr the more curious Arithmetician for further satisfaction in this matter, to my Treatise of the Elements of *Algebra*.

To extract the Biquadrate root.

XXIX. A quadrate or square number miltiplied by it self produceth a *Biquadrate* number: So 4 multiplied by it self produceth the *Biquadrate* 16. Therefore if a number be propounded and the *Biquadrate root* thereof be required, first extract the *quadrate* or *square root* of the number propounded, and then extract the *square root* of that *root* for the *Biquadrate root* sought. Thus if 20736 be a number propounded, the *Biquadrate root* thereof will be found 12: For the *square root* of 20736 is 144, and the *square root* of 144 is 12. When the number given hath not a perfect *Biquadrate root*, you are to annex *quaternaries* of *Cyphers*, to wit, either 4, 8, 12, or 16, &c. *cyphers*, and then proceed as before; so will you find the *root* near, whose fractional part will be a dicimal. Thus the *Biquadrate root* of 7 will be found near 1 .62.

CHAP.

CHAP. XXXIV.

The Relation of Numbers in Quantity.

I. THus far single Arithmetick: Comparative Arithmetick insues, which is wrought by numbers, as they are considered to have Relation one to another.

Boetius Ar. l. 1. cap. 21. II. This Relation consists in quantity, or quality.

III. Relation in quantity is the reference or respect that the numbers themselves have one unto another: As when the comparison is made between 6 and 2, or 2 and 6: 5 and 3, or 3 and 5.

IV. Here the Terms or Numbers propounded are always two, whereof the first is called the Antecedent, and the other the Consequent: So in the first Example, 6 is the Antecedent, and 2 the Consequent: and in the second, 2 is the Antecedent, and 6 the Consequent.

V. Relation in Quantity consists either in the difference, or else in the rate or reason that is found betwixt the Terms propounded.

VI. The difference of two numbers is the remainder, which is left after subtraction of the less out of the greater: So 6 and 2 being the terms propounded, 4 is the difference betwixt them: for if you subtract 2 out of 6, the remainder is 4. *Difference.*

VII. The rate or reason betwixt two numbers is the quotient of the Antecedent divided by the Consequent: So if it be demanded what rate or reason 6 hath to 2, I answer, Triple reason! For if you divide 6 the Antecedent, by 2 the Consequent, the quotient is 3, 2 being contained just 3 times in 6. In like manner is there subtriple reason betwixt 2 and 6, for if you divide 2 by 6, the quotient is $\frac{2}{6}$, or (which is all one) $\frac{1}{3}$, because 6 being not once found in 2, there remains 2 for the Numerator, 6 the Divisor being the Denominator of the Fraction given you in the Quotient, according to the *9 Rule* of the 16 *Chapter* aforegoing. *Rate or Reason.*

VIII. This rate or reason of numbers is either equal or unequal.

IX. Equal reason is the Relation that equal numbers have unto one another: as 5 to 5, 6 to 6, 7 to 7, &c. *Equal Reason.*

X. Here the one being divided by the other, the quotient is always an Unit: For if it be demanded how often 5 is in 5, the answer is 1.

XI. Unequal reason is the relation that enequal numbers have one unto another: and this is either of the greater to the less, or of the less to the greater. *Unequal Reason.*

XII. Unequal reason of the greater to the less, is when the greater Term is Antecedent: as of 6 to 2, 5 to 3, and the like.

XIII. Here the quotient of the Antecedent divided by the Consequent is always greater than an Unit: So 6 divided by 2, the Quotient is

3.

3, and 5 divided by 3, the quotient is $1\frac{2}{3}$.

XIV. Unequal reaſon of the leſs to the greater, is when the leſſer Term is Antecedent: as of 2 to 6, 3 to 5, &c.

XV. Here the quotient of the Antecedent divided by the conſequent is always leſs than an unit: So 2 divided by 6, the quotient is $\frac{2}{6}$ or $\frac{1}{3}$; and 3 divided by 5, the quotient is $\frac{3}{5}$.

XVI. Each of theſe kinds of unequal reaſon is again ſubdivided into five other kinds or varieties, whereof the three firſt are ſimple, and the other two are mixt.

XVII. The ſimple kinds of unequal reaſon are, 1. Manifold. 2. Superparticular. 3. Superpartient.

XVIII. Manifold reaſon of the greater to the leſs is, when the Conſequent is contained in the Antecedent divers times without any part remaining: As 4 to 2, 8 to 4, 16 to 8, which is called Double reaſon, becauſe the leſs is contained twice in the greater; ſo 6 to 2 is triple reaſon, 8 to 2 fourfold reaſon, &c.

Manifold Reaſon.

XIX. Here the quotient of the Antecedent divided by the conſequent is always a whole number: So 8 divided by 2, the quotient is 4.

XX. The oppoſite of this kind, *viz.* of the leſs to the greater, is called ſubmanifold: Examples hereof are 2 to 4, 4 to 8, 8 to 16, &c. Likewiſe 2 to 6, 2 to 8, 2 to 10, &c.

Submanifold.

XXI. Superparticular is, when the Antecedent contains the conſequent once, and beſides an aliquot part of the conſequent; that is, an half, a third, a fourth, or a fifth part, &c. of the conſequent, as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here three divided by 2, the quotient is $1\frac{1}{2}$, and 4 being divided by 3 the quotient is $1\frac{1}{3}$. In like manner 5 divided by 4, the quotient is $1\frac{1}{4}$, and 6 divided by 5 the quotient is $1\frac{1}{5}$; wherefore I ſay 2 and half 2 (that is 1) conſtitute 3: So likewiſe 3 and one third part of 3 (*viz.* 1.) conſtitute 4, and ſo of the reſt.

Superparticular.

XXII. Here the quotient of the Antecedent divided by the Conſequent is a mixt number, whoſe whole part, as alſo the Numerator of the fraction annexed, is always an unit: As is obſervable in the examples laſt mentioned.

XXIII. The oppoſite reaſon of this kind is Subſuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, &c.

Subſuperparticular.

XXIV. Superpartient is, when the Antecedent contains the Conſequent once, and beſides divers parts of the conſequent: As 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, &c: here 5 divided by 3, the quotient is $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3 (*viz.* 2) conſtitute 5.

Superpartient.

XXV. Here the quotient of the Antecedent divided by the conſequent is a mixt number, whoſe whole part being an unit, hath always for the Numerator of the fraction annexed unto it a number compoſed of more units than one: So the conference being made betwixt 5 and 3, and 5 the Antecedent being divided by 3 the conſquent, the quotient is $1\frac{2}{3}$.

XXVI. The

XXVI. The opposite of this reason is Subsuperpartient: Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like.

Subsuperpartient.

XXVII. The mixt kinds of unequal reason are Manifold Superparticular, and manifold Superpartient.

XXVIII. Manifold Superparticular reason is, when the Antecedent contains the consequent divers times, and besides an aliquot part of the consequent: as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

Manifold Superparticular.

XXIX. Here the quotient of the Antecedent divided by the consequent is a mixt number, whose whole part consisting of more units than one, hath always an unit for the Numerator of the Fraction annexed unto it; so 5 divided by 2 the quotient is $2\frac{1}{2}$, and 21 divided by 5, the quotient is $4\frac{1}{5}$.

XXX. The opposite of this Reason is Submanifold Superparticular; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, &c.

Submanifold Superparticular.

XXXI. Manifold Superpartient is, when the antecedent contains the consequent divers times, and besides divers parts of the consequent; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, &c.

Manifold Superpartient.

XXXII. Here the quotient of the Antecedent divided by the Consequent is a mixt Number, whose whole part as also the Numerator of the Fraction annexed unto it, is always a Number composed of more units than one: So 8 divided by 3, the quotient is $2\frac{2}{3}$, and 28 divided by 5, the quotient is $5\frac{3}{5}$.

Submanifold Superpartient.

XXXIII. The

XXXIII. The opposite here is Submanifold Superpartient: as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

And these are the several kinds of varieties of the Rates or Reasons that are found amongst Numbers, so that no two Numbers whatsoever can be named, but the Rate or Reason betwixt them is comprehended under one of these five kinds.

CHAP. XXXV.

The Relation of Numbers in Quality, where of Arithmetical and Geometrical Proportion.

I. RElation in quality (otherwise called Proportion) is either the reference or respect that the Reasons of Numbers have one unto another, or else which the differences of numbers have one to another.

Vide Euclid. l. 3. d. 5. & Alsted. Arith. c. 5.

II. Therefore here the Terms propounded ought always to be more than two, for otherwise there cannot be a comparison of Reasons or Differences in the Plural number.

III. This proportion is either Arithmetical, or Geometrical.

IV. Arith-

IV. Arithmetical proportion is, when divers numbers differ according to an equal difference, as 2, 4, 6, 8, 10, &c. here 2 is the common difference betwixt 2 and 4, 4 and 6, 6 and 8, 8 and 10, &c. So 1, 2, 3, 4, 5, 6, 7, &c. differ by *Arithmetical* Proportion, 1 being the common difference betwixt them.

Arithmetical Proportion.

V. Arithmetical Proportion is either continued or interrupted.

VI. Arithmetical Proportion continued is, when divers numbers are linked together by a continual progression of equal differences. Such are the examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, &c. And 100000, 200000, 300000, 400000, &c.

1. *Continued.*

VII. In a rank of numbers that differ by Arithmetical Proportion continued, the sum of the first and last Terms being multiplied by half the number of the Terms, the Product is the total sum of all the Terms: So it being demanded, how many strokes the Clock strikes betwixt midnight and noon; the Terms of the Progression in this question are Twelve, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. for in that order the Clock strikes, wherefore if I multiply 13 the sum of 12, and 1 (the first and last Terms) by 6 (being half the number of the Terms) the Product is 78, which is the total sum of all the Terms propounded being added together.

VIII. Or thus, Multiply the number of the Terms by the half sum of the first and last Terms, and then likewise the Product will give you the total of all the Terms: So 13, 11, 9, 7, 5, 3, being given their total is 48, for 8 the half sum of 13 and 3, the first and last Terms being multiplied by 6, the number of the terms, the product is 48.

IX. Three numbers being given, that differ by Arithmetical proportion continued, the mean being doubled, is equal to the sum of the extreams: so 5, 6, 7, being given, 6 being doubled is equal to the sum of 5 and 7 the two extreams.

X. Arithmetical Proportion may be continued either upwards or downwards.

Upwards.

XI. Upwards, when the Terms of the Progression increase, as these, 2, 4, 6, 8, 10, 12, &c. or these, 1, 2, 3, 4, 5, 6, &c. And this last rank is more particularly termed *Natural Progression.*

XII. Here when the first term is also the common difference of the terms, the last term being divided by the number of the terms, the quotient will give you the first term of the rank: Again in this case the first term multiplied by the number of the terms produceth the last term: So this rank 3, 6, 9, 12, 15, 18, 21, being propounded, wherein 3 is both the first term as also the common difference of the terms; I say 21 the last term being divided by 7 the Number of the terms, the quotient is 3 the first term; contrariwise 3 the first term multiplied by 7 produceth 21 the last term.

XIII. Arithmetical proportion continued downwards is, when the terms of the progression decrease: Such as are 35, 32, 29, 26, 23, 20: And 40, 35, 30, 25, 20, 15, 10, 5.

Downwards.

XIV. Here when the laſt term is alſo the common difference of the terms, the firſt term being divided by the Number of the terms, the quotient will give you the laſt term: Again, the laſt term multiplied by the number of the terms, produceth the firſt term of the rank.

This Rule is in the inverſe of the 12th Rule aforegoing.

For Example, this rank 40, 35, 30, 25, 20, 15, 10, 5, being propounded, in which 5 is both the laſt term, and likewiſe the common difference of the terms, I ſay, 40 the firſt term being divided by 8 the number of the terms, the quotient is 5 the laſt term: On the other ſide 5 the laſt term being multiplied by 8, the product is 40 the firſt term.

XV. Arithmetical Proportion interrupted is, when the Progreſſion is diſcontinued: as in theſe numbers 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10 differ according to Arithmetical proportion, but ſo do not 4 and 8 differ, for 2 is the common difference betwixt 2 and 4, 8 and 10, whereas the difference betwixt 4 and 8 is 4. In like manner 8, 14, 17, 23, differ by Arithmetical proportion interrupted.

2. Interrupted.

XVI. Four numbers being given, that differ by Arithmetical proportion either continued or interrupted, the ſum of the two means is equal to the ſum of the two extreams: So 5, 6, 7, 8, being given, the ſum of 6 and 7, the two mean numbers, is equal to the ſum of 5 and 8, the two extreams: and 8, 14, 17, and 23, being propounded the ſum of 14 and 17 being added together is equal to the ſum of 8 and 23.

XVII. Geo-

XVII. Geometrical proportion is, when divers numbers differ according to like Rate or reaſon: that is, when the reaſons of numbers, being compared together are equal. So 1, 2, 4, 8, 16, 32, &c. which differ one from another by double reaſon, are ſaid to differ by *Geometrical* proportion, for as 1 is half 2, ſo 2 is half 4, 4 half 8, 8 half 16, 16 half 32, &c.

Geometrical Proportion.

XVIII. Geometrical proportion is either continued or interrupted.

1. Continued.

XIX. Geometrical proportion continued is, when divers numbers are linked together by a continued progreſſion of the like reaſon: Of this ſort is the example laſt given: For as 1 is to 2, ſo is 2 to 4, 4 to 8, 8 to 16, 16 to 32, &c. So likewiſe the numbers 3, 9, 27, 81, 243, 729, &c. differ by *Geometrical* proportion continued, *viz.* by triple reaſon, each of them being contained three times in the next number that follows it.

XX. In numbers continually proportional from 1, the firſt number from 1 is the root or firſt power, the ſecond is the ſquare or ſecond power, the third the cube or third power, the fourth the Biquadrate or fourth power, the fifth the fifth power, the ſixth the ſixth power &c. So in this rank of numbers, 1, 3, 9, 27, 81, 243, 729, &c. 3 is the root, 9 the ſquare, 27 the cube, 81 the Biquadrate, 243 the fifth power, 729 the ſixth power, &c.

XXI. The root being multiplied by it ſelf produceth the ſquare, which being again multiplied by the root produceth the cube, and ſo each proportional being multiplied by the root, produceth the propor-

Mean proportionals.

proportional next above it, and then the numbers comprehended betwixt 1, and the last number produced are called mean proportionals: So in this rank of proportional numbers, 1, 2, 4, 8, 16, 32, &c. 2 the root being multiplied by it self produceth 4 the square, which being again multiplied by 2 produceth 8 the cube, then 8 being multiplied by 2, the product is 16 the biquadrate, and so of the rest in their order, and here 2, 4, 8, and 16, are the mean proportionals in the rank propounded.

XXII. If you multiply the root by it self, and consequently the subsequent numbers by themselves, the numbers intercepted betwixt 1 and the number last produced may not unfitly be called continual means: So 2 being given for the root, multiplied by it self, the product is 4, which being again multipled by it self produceth 16, then 16 in like manner squared produceth 256, which likewise multipled by it self produceth 65536, I say then that 2, 4, 16, and 256 are continual means betwixt 1 and 65536.

Continual means. Briggius Arith. Log. c. 6.

XXIII. The continual means comprehended betwixt any number given and 1, are discovered by a continued extraction of the square roots; for example, 65536 being given, the root thereof extracted is 256, whose root is 16, then the root of 16 is 4, and the root of 4 is 2; so that at last I find 256, 16, 4, and 2 to be continual means intercepted betwixt 65536 and 1 as before.

XXIV. In numbers that increase by Geometrical proportion continued, if you multiply the last term by the quotient of any one of the terms divided

divided by another term, which being less is next unto it, and then deducting the first term out of that product, divide the remainder by a number that is an unit less than the quotient, the last quotient will give you the total of all the terms propounded in the progression; so this rank 2, 6, 18, 54, 162, 486, 1458, being propounded, wherein the proportionals differ by subtriple proportion, I first take 2 and 6 the two first terms, and dividing 6 by 2, I find the quotient 3, wherefore multiplying 1458 the last term, by 3 the quotient, the product is 4374, out of which if I deduct 2 the first term, the remainder is 4372, which being divided by 2 (*viz.* a number which is an unit less than three the quotient) the last quotient gives me 2186, which is the total sum of the proportionals propounded,

XXV. Three proportionals being given, the square of the mean is equal to the product of the extreams: So 4, 8, and 16 being propounded, 8 times 8 being 64, is equal to 4 times 16, which is likewise 64.

XXVI. Geometrical Proportion interrupted is, when the progression of like reason is discontinued, in such sort that four numbers being given, the like reason is not found betwixt the second and third, that is betwixt the first and second, and the third and fourth; of this sort are these numbers 2, 4, 16, 32. here as 2 is to 4, so is 16 to 32, for they differ by double reason; but as 2 is to 4, so is not 4 to 16, for 4 and 16 differ by fourfold reason, 4 being contained 4 times in 16: So likewise 4, 8, 8, 16, differ according to Geometrical proportion interrupted.

2. Interrupted.

XXVII. The numbers of Multiplication and Division are proportional; for in Multiplication, as 1 is to the Multiplicator, so is the Multiplicand to the product, or as 1 is to the Multiplicand, so is the Multiplicator to the product: Again, in Division, as the Divisor is to 1, so is the Dividend to the Quotient: Or as the Divisor is to the Dividned, so is 1 to the Quotient.

XXVIII. Four proportional Numbers whatsoever being given, the product of the two means is equal to the product of the two extreams: So 2, 4, 16, 32, being propounded, 4 times 16 (which is 64) is equal to 2 times 32, which is likewise 64.

Here endeth the first Book, which containeth all that is absolutely necessary, for the full understanding of *common or practical Arithmetick.* Such as desire to see how the same is performed by artificial, or borrowed numbers, called *Logarithms*, may peruse Mr. *Wingate's Second Book*, being a distinct *Treatise* of *Artificial Arithmetick.*

AN

AN APPENDIX,

CONTAINING

Choice Knowledge in *Arithmetick*, both *Practical* and *Theoretical*; the Contents whereof are express'd in the following Page.

Composed by *John Kersey*,

Teacher of the

MATHEMATICKS,

At the Sign of the *Globe* in *Shandois-street* in *Covent-Garden.*

Vox audita perit, litera scripta manet.

The CONTENTS OF THE APPENDIX.

CHAP.

An Explication of such Notes or Characters, which for brevity's sake are used in this APPENDIX.

THis x is a note of *Addition*, signifying that the number which followeth such sign is to be added to the number preceding it; so 3 + 4 implieth that 4 is to be added to 3: Sometimes also when no number is placed next after the said note, it implieth that the number preceding is not exactly exprest; so the *square root* of two is 1.414 + or 1.414, &c. that is, 1 $\frac{414}{1000}$ and somewhat more.

This — is a sign of *Subtraction*, signifying that the number which followeth such sign is to be subtracted from the number preceding it; so 6 — 2 signifieth the difference between 6 and 2, or 2 to be subtracted from 6.

This x is a sign of *Multiplication*, signifying that the number which precedeth such sign is to be multiplied into, or by the number following the sign; So 3 x 4 implieth that 3 is to be multiplied by 4; likewise by 3 x 4 x 8 is understood the *continual multiplication* of the numbers 3, 4, and 8; *viz.* 3 is to be multiplied by 4, and the product is to be multiplied by 8. Sometimes also the said sign hath reference to as many of the preceding or following numbers as have a little line placed over them; so 3 x 2 x 6 2 x 6 x 3 signifieth that 3 is to be multiplied by the *sum* of 2 and 6. Likewise

wise 8——5 x 3, or 3 x 8——5 implieth that 3 is to be multiplied by the *difference* between 8 and 5: Moreover if A and B represent two numbers, then A x B or A B implieth the product of the multiplication of those numbers: Likewise $\overline{B-C}$ x A signifieth the product arising form the multiplication of the excess of the number B above the number C, by (or into) the number A. Again, if A B and A C represent two lines, then ▭ A B x A C implieth a rectangular Figure or long square made of the lines A B and A C.

Numbers placed as you see in the Margent denote a *Divisor*, a *Dividend*, and a *Quotient*, to wit, 3 the *Divisor*, 18 the *Dividend*, and 6 the *Quotient*; the like is to be understood of other numbers so placed. 3) 18 (6

Numbers placed after the manner of a *fraction* denote a *quotient*, which ariseth from dividing the *Numerator* by the *Denominator*; so $\frac{3 \times 5 \times 6}{3 \times 4}$ is equal to the *Quotient*, which ariseth from dividing the *product* of the *continual multiplication* of 2, 5 and 6 by the *product* of 3 multiplied by 4.

Four numbers placed as you see in the Margent are *Geometrical proportionals*, *viz.* As 2 is to 4, so is 6 to 12: or if 2 give 4, then 6 will give 12. Sometimes also they are placed thus, 2....4....6....12. 2 . 4 :: 6 . 12

This = is a note of *equality* or *equation*; so by 3 + 4 = 5 + 2 is signified that the sum of 3 and 4 is equal to the sum of 5 and 2: Also 7 — 3 = 9 — 5 signifieth that the *difference* between 7 and 3 is equal to the *difference* between 9 and 5; that is, 7 lessened

leſſened by 3 leaves the ſame remainder, as 9 leſſened by 5. Alſo $4 \times 3 = 12$ implieth that the *product* of the multiplication of 4 by 3 is equal to 12.

$>$ This is a ſign of *majority*, ſignifying that the number on the left hand of ſuch ſign is *greater* than the number on the right hand thereof, ſo $5 > 3$ implieth that 5 is *greater* than 3.

$<$ This is a ſign of *minority*, ſignifying that the number on the left hand of ſuch ſign is *leſs* than the number on the right hand thereof; ſo $3 < 5$ implieth that 3 is *leſs* than 5.

This Character $\surd$ or $\surd$ q. ſignifies the ſquare root of the number which follows it, ſo $\surd\ 144$ implies the ſquare root of 144, to wit 12.

Alſo this $\surd$ c. ſignifies the cube root of the number which follows it: So $\surd$ c. 1728 ſignifies the cube root of 1728, which cube root will be found to be 12.

A N

AN APPENDIX.

CHAP. I.

Of Contractions in the Rule of Three.

SUch as are well verſ'd in the Parts of *Arithmetick*, which have been fully laid open in the precedent Book, and are mindfull of the *Notes* or *Symbols* before explained, will find no difficulty in the 1, 2, 3, 4, 5, and 10 Chapters of this *Appendix*, wherein divers compendious operations no leſs delightfull than uſefull are methodically handled, and the reſt will be as eaſie to ſuch as are but meanly acquainted with *Geometrical demonſtration*.

II. To repeat the brief ways of *Multiplication* ſet forth in the 10, 11, and 12 *Rules* of the fifth *Chapter*, or thoſe of *Diviſion*, in the 11, 15, and 16 *Rules* of the

the ſixth *Chapter* aforegoing, would be a ſuperfluous work, and therefore I ſhall preſuppoſe the *Reader* to be throughly acquainted with them, as alſo with competent knowledge in the operations of fractions both *vulgar* and *decimal*.

III It will be no ſmall advantage to the Practical *Arithmetician*, to have by heart not only the common Table of *Multiplication*, but this alſo in the *Margent*, to the end that when a number is given to be multiplied or divided by 12, (which happens in the *Reduction* of *ſhillings* to *pence* and the converſe) the *product* or *quotient* may be written down in one line only, as in the Examples following.

2				24
3				36
4				48
5				60
6	+	12	=	72
7				84
8				96
9				108

```
  3472              4736
    12                12
-----------       -------------
12) 41664 (3472   12) 56832 (4736
```

IV. When a whole number is given to be divided by a Diviſor, which is equal to the product of the Multiplication of two ſingle figures, inſtead of dividing by that Diviſor you may firſt divide by one of thoſe ſingle figures, and then divide the quotient by the other, ſo will the laſt quotient be the ſame as if the Diviſion had been finiſht by the Diviſor firſt given: Thus if 3466 *farthings* be given to be reduced to *ſhillings*, becauſe 8 x 6 = 48, I firſt divide 3466 by 8,

8, ſo there will ariſe 433 for a new *Dividend*, and 2 *farthings* remain; then I divide the ſaid 433 by 6, ſo there will ariſe--72 $\frac{1}{6}$, or 72 *ſhillings*, 2 *pence*, which with the 2 *farthings* remaining of the firſt *Diviſion* make in all 72 *s.* : 2 $\frac{1}{2}$ *d.* which is the very *quotient*, when 3466 farthings are divided by 48. Note that you are to reſerve a farthing for every unit remaining of the firſt Diviſion by 8, and 2 pence for every unit remaining of the ſecond Diviſion by 6. The reaſon of the operation is evident, for $\frac{1}{6}$ of $\frac{1}{8} = \frac{1}{48}$.

```
8) 3466
           s.  d.
6) 433 ( 72::2 ½
```

In like manner, if 7136 pence are given to be reduced into pounds, becauſe 240 *d.* = 1 *l.* alſo 6 x 40 = 240, therefore if 7136 pence be firſt divided by 6, the quotient will give 1189 ſix pences, and 2 pence remain; then if 1189 be divided by 40, (that is by 4, after 9 the laſt place of the Dividned towards the right hand is cut off) the quotient will be 29 *l.* and there will remain 29 ſix pences, or 14 *s.* 6 *d.* which together with the 2 pence remaining of the firſt Diviſion, and the ſaid 29 *l.* makes in all 29 *l.* : 14 *s.* : 8 *d.* which is the ſame with the quotient, when 7136 pence are divided by 240, for $\frac{1}{40}$ of $\frac{1}{6} = \frac{1}{240}$.

```
 6) 7136
             l.   s.  d.
40) 118|9 (29 : 14 : 8
```

Again, ſuppoſe 3463 pence are given to be reduced into ſhillings, foraſmuch as 4 + 3 = 12, I firſt divide 3463 by 4, ſo there will ariſe 865 for a new Dividend and 3 pence remain: Then I divide the ſaid 865 by 3, ſo there will ariſe 288 $\frac{1}{3}$ or 288 *s.* 4 *d.*

4) 3463
s. d.
3) 865 (288 .. 7

4 *d.* which with the 3 pence before remaining make 288 *s.* 7 *d.* which is the same with the quotient, when 3463 pence are divided by 12, for $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{12}$.

V. In the Rule of Three as well direct as inverse, when the Divisor with either of the other two given numbers may be severally divided by some common measure, without leaving any remainder, the quotients may be taken for new terms, and proceeding in like manner as often as is possible, the operation according to the tenth Rule of the eighth Chapter, or the second Rule of the ninth Chapter, will be much contracted: So if it be demanded what 52 yards of Cloth will cost at the rate of 21 *l.* for 14 yards; the *Answer* will be found 78 pounds, in manner following.

y. ... *l.* ... *y.*
14 ... 21 ... 52
2 ... 3 ... 52
1 ... 3 ... 26 .. (78

In the first rank you may observe, that the Divisor 14 and the second term 21, being severally divided by their common measure 7, the three new terms (in the second rank) will be 2, 3, 52. Again in the second rank the Divisor 2 and the third term 52 being severally divided by their common measure 2, the three new terms (in the third rank) will be 1, 3, 26. Lastly, working with these according to the *Rule of Three direct*, the *Answer* to the question (or fourth term) will be found to be 78.

Another

Another Example, If 21 men will finish a work in 16 days, what time must be allowed to 12 men for the finishing of such a work? *Answer*, 28 days.

men ... *days* ... *men*
21 ... 16 ... 12
7 ... 16 ... 4
7 ... 4 ... 1 (28 *days.*

In the first rank you may observe, that the Divisor 12 (for the Rule is inverse) and the first term 21 being severally divided by their common measure 3, the three new terms (in the second rank) will be 7, 16, 4. Again in the second rank, the Divisor 4, and the second term 16, being severally divided by their common measure 4, the three new terms in the third rank will be 7, 4, 1. Lastly, working with these as the Rule of three inverse requires, the *Answer* to the question (or fourth term) will be found 28.

VI. In the Rule of three, as well direct as inverse, when the Divisor and either of the other two terms are fractions having a common denominator, the said denominators may be rejected, and their numerators retained as new terms: So if it be demanded what is the value of $\frac{7}{8}$ of an *Ell*, when $\frac{3}{8}$ of an *Ell* are worth 66 pence, the *Answer* will be found 154 pence, and the work will stand as you see.

$\frac{3}{8}$ .. 66 .. $\frac{7}{8}$
3 .. 66 .. 7
1 .. 22 .. 7 (154

Another

Another Example. If $3\frac{3}{4}$ *yards* of Scarlet cloth cost 8 *l.* 15 *s.* what is the price of one *yard* at that rate? *Answer* 2 *l.* 6 *s.* 8 *d.*

$\frac{15}{4} \ldots \frac{35}{4} \ldots 1$

$15 \ldots 35 \ldots 1$

$3 \ldots 7 \ldots 1 \ldots (2\frac{1}{3}\,l.$

VII. In the Rule of Three as well direct as inverse when the Divisor only is a fraction, either of the other two terms may be reduced to a fraction of the same Denominator, and then the Denominators may be rejected, as before in the sixth Rule, also when one of the three given terms is a fraction, and is not the Divisor, the Divisor may be reduced to a fraction of the same Denominator with the fraction first given, and then the common Denominators may be likewise cancelled.

An *Example* of the first Case may be this, if $\frac{7}{8}$ of a *yard* cost 14 *s.* what is the price of 1 *yard*? *Answer* 16 *shillings.*

yard	*shill.*	*yard.*
$\frac{7}{8}$ ...	14 ...	1
$\frac{7}{8}$ ...	14 ...	$\frac{8}{8}$
7 ...	14 ...	8 .. (16 *shill.*

An *Example* of the second Case; if of stuff which is $\frac{3}{4}$ of a yard in breadth, 7 yards in length will make a Garment; how much of that stuff which is one yard in breadth will be sufficient for the same purpose? *Answer* $5\frac{1}{4}$ *yards.*

Rules

Rules of 3 Inverse. $\begin{cases} \frac{3}{4} \ldots 7 \ldots 1 \\ \frac{3}{4} \ldots 7 \ldots \frac{4}{4} \\ 3 \ldots 7 \ldots 4 \;(\ldots \frac{21}{4} \text{ or } 5\frac{1}{4} \end{cases}$

CHAP. II.

Rules of Practice by Aliquot parts.

I. AN *Aliquot part* takes its name from the *Latine* word *aliquoties*, for (according to *Euclid*) an *aliquot part* is of a greater number such a part, which being taken (*aliquoties* or) certain times doth precisely constitute that greater number; so 3 is an *aliquot part* of 12, for 3 taken 4 times doth exactly make 12, without any excess or defect; in like manner 4 is an *aliquot part* of 20, because 4 taken 5 times doth precisely make 20; but 7 is not an *aliquot part* of 20, for 7 taken twice doth want of 20, and being taken thrice doth exceed 20; this kind of *part* last mentioned is by *Euclid* called *pars aliquanta*, of which there will be no use in this place.

II. When the *Rule of Three direct* hath 1 or an *Integer* for the first time, it is commonly called a *Rule of Practice*, either from the great use and practice thereof in common affairs, or else for that questions of this nature, may be resolved by operations more speedy and practical than those of the *Rule of Three.*

III. The

III. The choicest of these *Rules* of *Practice* may be reduced to 5 Cases, *viz.*

When the price of 1 or an Integer consists,
1. *Of shillings under 20.*
2. *Of pounds and shillings.*
3. *Of pence under 12.*
4. *Of shillings and pence.*
5. *Of pounds, shillings, pence, with parts of a peny.*

All which cases with others of the like nature are handled in their order.

IV. Any even number of shillings is either $\frac{1}{10}$ of a pound (that is 2 *shillings,*) or else is composed of $\frac{1}{10}$ *l.* (to wit 2 *s.*) taken certain times: So 8 *s.* is composed of $\frac{1}{10}$ *l.* (or 2 *shillings*) taken four times, in like manner 18 *s.* is composed of $\frac{1}{10}$ *l.* taken nine times.

V. When the price of 1, or an integer of what name soever, is 2 shillings, the price of as many Integers as one will of that name is discoverable at first sight, to wit by accounting the double of the figure which stands in the first place (towards the right hand) of the said number of Integers, as shillings, and the rest of the said number as pounds: so 345 yards at two shillings the yard will cost 34 *l.* 10. *s.* for the double of 5 is 10, which I write down apart as shillings, then esteeming the remaining figures towards the left hand, to wit 34, as an entire number of pounds, the *Answer* will be 34 *l.* 10. *s.* This contraction is nothing else, but dividing the number

yard	*shill.*	*yards*
1 ..	2 ..	345

Answer 34 *l.* 10 *s.*

ber of Integers, whose price is required by 10. More *examples* hereof are these;

oz.	*shill.*	*oz.*
1 ...	2 ...	2057

	l.	*s.*
Answ.	205 ..	14

yard	*shill.*	*yards*
1 ...	2 ...	120

	l.	*s.*
Answ.	12 ..	0

VI. When the given price of 1 or an Integer is any even number of shillings greater than two shillings, multiply the number of Integers, whose price is required, by half the given number of shillings, with this caution, that the double of the figure which ariseth in the first place of the product be written apart as shillings, and the rest of the product as pounds: So if it be demanded what 218 yards at 8 shillings the yard will amount unto, the *Answer* will be found 87 *l.* 4 *s.* for I multiply 218 by 4 (which is half 8 the given number of shillings) saying, 4 times 8 is 32, here the double of 2 (to wit, of that figure which is to possess the first place in the product) is 4, which I set apart as shillings, keeping 3 in mind for the three tens, again 4 times 1 is 4, which

y.	*s.*	*y.*
1 ..	8 ..	218
		4
		87 : 4

U 3 with

with 3 in mind makes 7; lastly, 4 times 2 makes 8, so I conclude that the *Answer* to the question is 87 *l.* 4 *s.* The reason of this contraction is evident from the fourth and fifth Rules aforegoing. More examples of this Rule are these following.

yard	*s.*	*yards*
1 ...	14 ...	436
	l.	*s.*
Answ.	305 ..	4

yard	*s.*	*yards*
1 ...	18 ...	230
	l.	*s.*
Answ.	207 ..	0

VII. Any odd number of shillings is either compos'd of $\frac{1}{10}$ *l.* (or 2 *s.*) and of $\frac{1}{20}$ *l.* (or 1 *s.*) or else it is composed of $\frac{1}{10}$ *l.* (or 2 *s.*) taken certain times, and of $\frac{1}{20}$ *l.* (or 1 *s.*) So 3 *s.* is compos'd of 2 *s.* and 1 *s.* Also 7 *s.* is composed of 2 *s.* taken three times and of 1 *s.* Likewise 13 *s.* is composed of 2 *s.* taken six times and of 1 *s.*

VIII. When the given price of 1 or an Integer is an odd number of shillings, work for the greatest even number of shillings contained in that odd number, according to the fifth or sixth Rule aforegoing; then for the odd shilling remaining, take $\frac{1}{20}$ of the number of Integers, whose price is required (by the 16th Rule of the sixth Chapter of the preceding Book.) These two results added together give the Answer to the question:

question: So if it be demanded what 2344 ounces at 13 *s.* the ounce will cost, the answer will be found 1523 *l.* 12 *s.* For if (according to the sixth Rule of this Chapter) I multiply 2344 by 6, (to wit, by half the remainder, when one is abated from 13 the given number of shillings) there will arise 1406 *l.* 8 *s.* Then taking $\frac{1}{20}$ of 2344, there will arise 117 *l.* 4 *s.* which being added to the former product gives 1523 *l.* 12 *s.* for the *answer* to the question.

oz.	*shill.*	*oz.*	
1 ..	13 ..	2344	
		6	
		l.	*s.*
		1406 ..	8
		117 ..	4
		l.	*s.*
	Answ.	1523 ..	12

Note, When 5 shillings is the given price of 1 or an Integer, the briefest way will be to take $\frac{1}{4}$ of the number of Integers, whose value is required, for such quotient will give the pounds and shillings, which answer the question: so 2347 ounces at 5 *s.* the ounce amount unto 586 *l.* 15 *s.* for $\frac{1}{4}$ of 2347 is 586 $\frac{3}{4}$ or 586 *l.* 15 *s.* But when the given price of 1 is any other odd number of shillings, this eighth Rule will be as compendious as any other whatsoever.

More Examples of this Rule are these following.

yard	*shill.*	*yards*
1 ...	19 ...	739
	l.	*s.*
	665 ...	2
	36 ...	19
Answ.	702 ...	1

 yard

yard	shill.	yards.
1 ...	17 .. :	345

l.	s.
276 ..	0
17 ..	5
Answ. 293 ...	5

XI When the given price of 1 or an Integer consists of pounds and shillings, first multiply the number of Integers whose price is required, by the number of pounds in the said given price, and subscribe the product as pounds; then proceed with the shillings in the said given price, according to the sixth or eighth Rule of this Chapter, and having subscribed that which ariseth under the aforesaid product of pounds, add them all together for the answer of the question: So if it be demanded what 328 hundred weight will amount untoat 2 *l.* 17 *s. per C.* (or one hundred weight) the *answer* will be found to be 934 *l.* 16 *s.* as by the operation is evident.

C.	l.	s.	C.
1 ...	2 :	17 ...	328

l.	s.
656 ..	0
262 ..	8
16 ..	8
Answ. 934 :	16

More

More *Examples* to illustrate this *Rule* are these following:

C.	l.	s.	C.
1 ...	7 :	12 ... :	504

l.	s.
3528 ..	
302 ..	8
Answ. 3830 ..	8

C.	l.	s.	C.
1 ...	5 :	7 ...	129

l.	s.
645 ..	
38 ..	14
6 ..	9
Answ. 690 ..	3

X. Any number of pence under 12 is either an Aliquot part of a shilling, or else compos'd of Aliquot parts thereof; so 3 pence is an Aliquot part, to wit, $\frac{1}{4}$ of a shilling. Likewise 4 is $\frac{1}{3}$ of 12; moreover 5 pence are compos'd of 2 Aliquot parts, to wit, of 3 pence; which is $\frac{1}{4}$ of a shilling, and of 2 pence which is $\frac{1}{6}$ of a shilling; all which will readily appear by the following Table.

Pence

Pence.	*Aliquot parts of a Shilling.*
1	$\frac{1}{12}$ (or $\frac{1}{3}$ of $\frac{1}{4}$)
$1\frac{1}{7}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4}+\frac{1}{6}$
6	$\frac{1}{2}$
7	$\frac{1}{4}+\frac{1}{3}$
8	$\frac{1}{3}+\frac{1}{3}$
9	$\frac{1}{2}+\frac{1}{4}$
10	$\frac{1}{2}+\frac{1}{3}$
11	$\frac{1}{3}+\frac{1}{3}+\frac{1}{4}$

XI. When the given price of 1 or an Integer is an Aliquot part of a ſhilling, divide the number of Integers whoſe value is required by the denominator of ſuch aliquot part; ſo will the quotient be the number of ſhillings which anſwer the queſtion, which number of ſhillings (when there is occaſion) may be reduced to pounds by the brief way of dividing by 20: So if it be required to know what 2686 ounces at 4 pence the ounce will amount unto; the anſwer will be found 44 *l.* 15 *s.* 4 *d.* for ſince 4 *d.* is an aliquot part, to wit, $\frac{1}{3}$ of a ſhilling, I divide 2686 by 3, ſo will the quotient be $895\frac{1}{3}$ *s.* or 895 *s.* 4 *d.* which ſhillings being divided by 20, give 44 *l.* 15 *s.* 4 *d.* for the anſwer to the queſtion, as you ſee by the following operation,

	oz.	*d.*	*oz.*
	1 ...	4	2686
		s.	*d.*
	20) 89|5	.. 4	
Anſw.	44 .. 15 .. 4		

More Examples of this Rule are theſe following.

	yard	*d.*	*yards.*
	1 ...	6 ...	759
		s.	*d.*
	20) 37|9	: . 6	
Anſw.	18 .. 19 .. 6		

	yard	*d.*	*yards.*
	1 ...	1 ...	204
Anſw.		17 *ſhillings.*	

XII. When the given price of an Integer is compos'd of aliquot parts of a ſhilling, divide the number of Integers, whoſe price is required, by the ſeveral denominators of the aliquot parts contained in the given number of pence, then add the quotients

ents together, and the sum shall be the number of shillings which answer the question: So if it be demanded what 2347 yards of linen cloth will cost at 9 pence the yard, the answer will be found 88 *l.* 0 *s.* 3 *d.* For since 9 *d.* is compos'd of 6 *d.* and 3 *d.* to wit, of aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a shilling, I first divide 2347 by 2 (the denominator of the aliquot part $\frac{1}{2}$) so there ariseth $1173\frac{1}{2}$, or 1173 *s.* 6 *d.* Again, dividing the said 2347 by 4 (the denominator of the other aliquot part) there will arise $586\frac{3}{4}$, or 586 *s.* 9 *d.* which two quotients being added together give 1760 *s.* 3 *d.* or 88 *l.* 0 *s.* 3 *d.* which is the answer of the question. More Examples to illustrate this Rule are these.

yard	*d.*	*yards.*
1	9 ...	2347

	s.	*d.*
	1173 :	6
	586 :	9
20)	176|0 :	3
	l. *s.*	*d.*
Answ.	88 : 0 :	3

yard	*d.*	*yards*
1 ...	8 ...	782

	s.	*d.*
	260 ...	8
	260 ...	8
20)	52|1 ...	4
	l. *s.*	*d.*
Answ.	26 .. 1 ..	4

oz.	*d.*	*oz.*
1 ...	11 ...	540

	180	
	180	
	135	
20)	49|4	*s.* *d.*
Answ.	24 ...	15 : 0

XIII. When the given price of an Integer consists of shillings and pence, first multiply the number of Integers whose value is required by the said given number of shillings, and subscribe the product as shillings, then divide the said number of Integers by the several denominators which are correspondent to the aliquot parts contained in the given number of pence, and subscribe the quotient or quotients underneath the aforesaid product of shillings, all which being added together give the number of shillings which answers the question: So if it be demanded what 347 yards of cloth will cost at the rate of 7 *s.* 10 *d.* the yard, the answer will be found 135 *l.* 18 *s.* 2 *d.* for first 347 being multiplied by 7 (the given number of shillings) produceth 2429 shillings, then dividing 347 by 2 and 3 severally, (because 10 *d.* is compos'd

yard	*s.*	*d.*	*yards*
1 ..	7 :	10 ..	347

	s.	*d.*
$7 \times 347 =$	2429 :	
2) 347(..	173 :	6
3) 347(..	115 :	8
20)	271|8 :	2
	l. *s.*	*d.*
Answ.	135 : 18 :	2

pos'd of $\frac{1}{2}$ and $\frac{1}{3}$ of a shilling) the quotients will be $173\frac{1}{2}$ and $115\frac{2}{3}$, that is 173 *s.* 6 *d.* and 115 *s.* 8 *d.* Lastly, the sum of all is 2718 *s.* 2 *d.* or 135 *l.* 18 *s.* 2 *d.*

More Examples of this kind are these.

```
yard   s.   d.   yards.
 1 ... 17 : 9 ... 540

                    s.
                 { 3780
17 × 540 =       {  540.
 2 ) 540( ..        270
 4 ) 540( ..        135
              ----------
             20) 958|5
                  l.  s.  d.
      Answ.      479 : 5 : 0
```

```
 y.   s.   d.   y.
 1 ... 14 : 6 ... 313

                 s.
             { 1252
14 × 313 =   {  313.
 2 ) 313( .     156 : 6
            ----------
            20) 453|8
Answ.       226 .. 18 : 6
```

XIV. When the price of an Integer consists of shillings and pence, and that such shillings and pence jointly considered do make an aliquot part of a pound, it will oftentimes be a briefer way than that in the last Rule, to divide the number of Integers, whose value is required, by the denominator of such aliquot part, so will the quotient give the answer

answer to the question in pounds and known parts of a pound. Thus if it be demanded what 767 yards will cost at the rate of 6 *s.* 8 *d.* the yard, the answer will be found 255 *l.* 13 *s.* 4 *d.* For since 6 *s.* 8 *d.* is an aliquot part, to wit, $\frac{1}{3}$ of a pound, I divide 767 by 3. so there ariseth in the quotient $255\frac{2}{3}$, or 255 *l.* : 13 *s.* : 4 *d.* which is the answer of the question.

```
 y.    s.    d.
 1 ... 6  :  8 .. 767
               l.    s.   d.
3 ) 767 ( 255 .. 13 : 4
```

Note that the *Aliquot parts* of a pound convenient for this Rule are these exprest in the following *Table.*

sh.	*d.*	*Aliquot parts of a pound.*
6 ..	8	$\frac{1}{3}$
3 ..	4	$\frac{1}{6}$
2 ..	6	$\frac{1}{8}$
1 ..	8	$\frac{1}{12}$
1 ..	4	$\frac{1}{15}$
1 ..	3	$\frac{1}{16}$

XV. When the given price of 1 or an Integer consists of pounds, shillings and pence, reduce the said pounds and shillings all into shillings, then proceed according to the 13 Rule of this Chapter: So 517 *C.* at 3 *l.* : 17 *s.* 5 *d. per C.* will be found to amount unto 2001 *l.* 4 *s.* 5 *d.* for having reduced 3 *l.* 17 *s.* into 77 *s.* I multiply 517 by 77, and write down the particular

particular products; then for the 5 pence which is compos'd of the aliquot parts $\frac{1}{4}$ and $\frac{1}{6}$ of a shilling, I take $\frac{1}{4}$ and $\frac{1}{6}$ of 517, and subscribe the quotients orderly underneath the aforesaid products: Lastly, adding all together the sum is 4002 4 *s.* 5 *d.* or 2001 *l.* 4 *s.* 5 *d.* for the answer of the question.

```
C.      l.    s.    d.    C.
1 ... 3 : 17 : 5 .. 517
-------------------------
77 x 517 = { 3619
           { 3619.
  4) 517 (..   129 : 3 d.
  6) 517 (..    86 : 2
-------------------------
     20) 4002|4 : 5
          l.     s.  d.
   Answ. 2001 : 4 : 5
```

More *Examples* of this *Rule* are these following.

```
C.      l.    s.    d.    C.
1 ... 5 : 13 : 8 ... 108
-------------------------
                      s.
              {  324
113 x 108 =   {  108.
              { 808 ..
  3) 108 (..  36
              36
-------------------------
     20) 1227|6
          l.   s.  d.
   Answ. 613 : 16 : 0
```



```
C.      l.    s.    d.    C.
1 ... 2 : 10 : 6 ... 84
-------------------------
   50 x 84 = 4200.
               42
   ---------------
                l.    s.   d.
   20) 424|2 (212 : 2 : 0
```

```
C.      l.    s.    d.    C.
1 ... 1 : 12 : 4¼ ... 306
-------------------------
                   s.   d.
32 x 306 = { |  612
           { | 918.
  3) 306(..  |  102
 48) 306(..  |    6 : 4½
             ------------
       20) 990|0 : 4½
            l.   s.  d.
   Answ. 495 : 0 : 4½
```

Note when the given price of an Integer consists of certain pence together with $\frac{1}{2}$ *d.* or $\frac{3}{4}$ *d.* it will be convenient to take due *aliquot parts* of the number of Integers propounded for all the given price of an Integer except 1 *d.* and the said $\frac{1}{2}$ *d.* or $\frac{3}{4}$ *d.* then for that peny, and $\frac{1}{2}$ *d.* take $\frac{1}{8}$ of the said Integers propounded, and if there be yet a farthing, take $\frac{1}{6}$ of the said *quotient* which ariseth by taking $\frac{1}{8}$; both which quotients give the value in shillings correspondent to 1 $\frac{3}{4}$ *d.* this will be evident by the following *Examples.*

yard	d.	yards
1 ...	$8\frac{3}{4}$ ...	326

	s.	d.
3) 326(..	108 ..	8
4) 326(..	81 ..	6
8) 326(..	40 ..	9
6) 40(..	6 ..	8
6) 9(..	0 ..	$1\frac{1}{2}$

20) 23|7 .. $8\frac{1}{2}$

	l.	s.	d.
Answ.	11 :	17 :	$8\frac{1}{2}$

	s.	d.	
1 ...	3 :	$6\frac{1}{2}$ ...	720

	s.
3 x 720=	2160
4) 720(..	180
6) 720(..	120
8) 720(..	90

	l.	s.	d.
20) 255|0 (	127 :	10 :	0

XVI. When the price of an Integer is given, and the price of many Integers of the ſame name together with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an Integer is required, the value of thoſe Integers may be firſt found by ſome of the precedent Rules, and then for the price of $\frac{1}{2}$ of an Integer, take $\frac{1}{2}$ of the given price of an Integer; likewiſe for $\frac{1}{4}$ of an Integer, take $\frac{1}{4}$ of

of the ſaid given price, alſo for $\frac{3}{4}$ of an Integer take the compoſed of $\frac{1}{2}$ and $\frac{1}{4}$ of the ſaid given price: So if it be demanded what 34 *C.* 3 *qu.* (to wit, 34 hundred weight, and $\frac{3}{4}$ of an hundred weight) of Sugar will coſt at 4 *l.* 16 *s.* 3 *d.* *per C.* the *Anſwer* will be found 167 *l.* 4 *s.* $8\frac{1}{4}$ *d.* as by the ſubſequent operation is manifeſt.

C.	l.	s.	d.	C.	q.
1 ...	4 :	16 :	3 ...	34 :	3.

	s.	d.
96 x 34=	204	
	306.	
4) 34(..	8 ..	6
the quotients for $\frac{1}{2}$ C.	48 ...	$1\frac{1}{2}$
$\frac{1}{4}$ C.	24 ...	$0\frac{3}{4}$

20) 334|4 ... $8\frac{1}{4}$

	l.	s.	d.
Anſw.	167 ...	4 ..	$8\frac{1}{4}$

An example of *Averdupois* greater weight, where the quantity whoſe price is ſought conſiſts of entire hundred weights, quarters of an hundred, and of ſome number of pounds, which is not an aliquot part of 28 or $\frac{1}{4}$ *C.*

C.	l.	s.	d.		C.	q.	lb.
1 ..	5 :	15 :	$7\frac{3}{4}$	....	218 :	3 :	24

$115 \times 218 =$	1090			
	218			
	218			
2) 218 (..	109	*d.*		*far.*
8) 218 (..	27 :	3 :		0
$\frac{1}{8}$ of 27 *s.* 3 *d.* ..	4 :	6 :		2
The quotients arising for $\frac{1}{2}$ C.	57 :	9 :		$3\frac{1}{2}$
$\frac{1}{4}$ C.	28 :	10 :		$3\frac{3}{4}$
14 *lb.*	14 :	5 :		$1\frac{3}{4}$ +
7 *lb.*	7 :	2 :		$2\frac{3}{4}$ +
3 *lb.*	3 :	1 :		0 +
	20) 2532|2 :	3 :		2 +
	l.	*s.*		*d.*
Answ.	1266 :	2 :		$3\frac{1}{2}$ +

The example last mentioned being (of those questions which ordinarily happen in trade) one of the hardest to be resolved by the *Rule of Practice*, I shall touch upon the aforegoing operation, where you may observe the price of 218 C. 3 *qu.* to be found after the manner of former Examples; then for 14 *lb.* part of the 24 *lb.* in the question, I take $\frac{1}{2}$ of the price of $\frac{1}{4}$ C. Likewise for 7 *lb.* I take half the price of 14 *lb.* and so there yet remains 3 *lb.* whose price is found by taking $\frac{3}{7}$ of the price of 7 *lb. viz.* the price of 7 *lb.* being very near 7 *s.* $2\frac{1}{2}$ *d.* or $86\frac{1}{2}$ *d.* I multiply $86\frac{1}{2}$ by 3, and divide the quotient by 7, so there ariseth 37 *d.* or 3 *s.* 1 *d*, very near; lastly, all being added together, the sum is found to be

be very near 25322 *s.* $3\frac{1}{2}$ *d.* or 1266 *l.* 2 *s.* $3\frac{1}{2}$ *d.*

Note, That a quarter of a farthing or $\frac{1}{16}$ of a peny) is the smallest money exprest in the example, and where any thing ariseth less than a quarter of a farthing it is omitted, but it is supposed to follow this note +, for which surplusages some respect ought to be had in adding all together: Now albeit, in resolving questions after this practical manner there will be some error, yet the loss for the most part will be less than a farthing, which is inconsiderable.

XVII. When the price of 1 or an Integer consists of divers denominations, as pounds, shillings, pence; and the price of a certain number of Integers, which exceeds not a single figure, is required, work as in the following Example, *viz.* If it be required to find what 8 C. must cost at 3 *l.* 13 *s.* $7\frac{1}{2}$ *d.* *per* C. it is evident that 8 C. must cost 8 times 3 *l.*

C.	l.	s.	d.	C.
1 ..	3 :	13 :	$7\frac{1}{2}$ ..	8
			8	
Answ.	29 :	9 :	0	

13 *s.* $7\frac{1}{2}$ *d.* therefore I multiply $\frac{1}{2}$ by 8, saying, 8 half pence make 4 pence, which I reserve in mind; again, 8 times 7 pence make 4 *s.* 8 *d.* (to wit, 8 six pences make 4 *s.* and there are 8 pence besides) to which adding 4 pence in mind, there will arise 5 *s.* which I reserve in mind, and subscribe a cypher under the place of pence; again, I say 8 times 13 shillings make 5 *l.* 4 *s.* (to wit, 8 Angels make 4 *l.* and 8 times 3 *s.*, make 1 *l.* 4 *s.*) to which adding 5 *s.* in

in mind, the ſum will be 5 *l.* 9 *s.* wherefore I ſubſcribe 9 *s.* (the exceſs above the pounds) under the ſhillings, and keep 5 *l.* in mind; laſtly, I ſay 8 times 3 pounds make 24 pounds, which with 5 pounds in mind make 29 pounds; ſo that the total product or anſwer of the queſtion is found to be 29 *l.* 9 *s.*

More Examples of this kind are theſe.

C.	l.	s.	d.	C.
1 ..	17 :	15 :	$5\frac{1}{4}$ ...	7
			7	
Anſw.	124 :	8 :	$0\frac{3}{4}$	

C.	l.	s.	d.	C.
1 ...	18 :	12 :	$6\frac{3}{4}$ ...	8
			8	
Anſw.	149 :	00 :	6	

XVIII. When the price of 1 *lb.* weight is known, and the price or value of 1 *C.* (to wit 112 *lb.*) is required, the anſwer may ſometimes be given more ſpeedily than by any of the former Rules, by this Rule which follows, *viz.* Find the number of farthings contained in the given price of 1 *lb.* weight, then take twice that number of ſhillings, and once that number of groats, and having added them together the ſum will give the value of 1 C. to wit 112 *lb.* weight: So if it be demanded what 1 C. or 112 *lb.* weight of Cheeſe will coſt at the rate of $3\frac{1}{4}$ pence the pound weight, the anſwer will be 1 *l.* 10 *s.* 4 *d.*

For

For according to the ſaid Rule, the number of farthings contained in $3\frac{1}{4}$d. (the price of 1 pound weight is 13, therefore the double of 13 ſhillings is ..

	l.	s.	d.
therefore the double of 13 ſhillings is ..	1 :	6 :	0
13 Groats make ..	0 :	4 :	4
Therefore the ſum (which is the price of 1 *C.* or 112 *lb.* weight) is ...	1 :	10 :	4

The reaſon of this Rule is evident, for if 1 *lb.* weight coſt 13 farthings, then 112 *lb.* muſt neceſſarily coſt 112 times 13 farthing, or (which is the ſame) 13 times 112 farthings; but 13 times 112 farthings are equal to twice thirteen ſhillings together with once thirteen groats, becauſe 112 farthings are compoſed of twice 48 farthings (or two ſhillings) and of 16 farthings (or one groat;) wherefore the truth of the ſaid Rule is evident.

Another Example, when Sugar is at $5\frac{1}{2}$ *d.* the pound weight, what is the value of 1 C. (or 112 *lb.* weight?) *Anſw.* 2 *l.* 11 *s.* 4 *d.* For in $5\frac{1}{2}$ *d.* are contained 22 *farthings,* therefore the double of 22 *ſhillings* is ..

	l.	s.	d.
therefore the double of 22 *ſhillings* is ..	2 :	4 :	0
22 Groats make ..	0 :	7 :	4
Which added together give the price of 1 *C.* or 112 *lb.* to wit ..	2 :	11 :	4

Compendious ways of computing Intereſt and Factors allowances.

XIX. When the gain of (or allowance for) 100 Integers conſiſt of ſome number of pounds not exceeding 10, the gain of as many like Integers and known parts of an Integer as one will, may be found very briefly by the following method, *viz.* If 100 *l.* gain 3 *l.* what is the gain

gain of 246 *l.* 18 *s.* 10 *d.*) *Answer* 7 *l.* 8 *s.* 1 $\frac{98}{100}$ *d.* First, I multiply 246 *l.* 18 *s.* 10 *d.* by 3 (the second term) after the manner delivered in the 17 Rule of this Chapter, and write down the product which is 740 *l.* 16 *s.* 6 *d.* Then I divide the said product by 100 (the first term in this *Rule of Three*) in this manner, *viz.* I divide 740 pounds by 100, which is performed by cutting off towards the right hand

```
 l.    l.    l.      s.    d.
100 .. 3 .. 246  :  18  :  10
                            3
-----------------------------
      l. 7|40 : 16 : 06
          |20
      -----------
      s. 8|16
          |12
      -----------
      d. 1|98
```

the two last places of 740, so the quotient gives 7 poundsand there will be a remainder of 40 pounds, which 40 pounds I reduce into shillings, so there will arise 800 *s.* to which adding the 16 *s.* which stand in the place of shillings, the sum will be 816 shillings; these are all to be divided by 100 (by cutting offtwo places as before,) so the quotient will give 8 shillings, and there will remain 16 shillings, which being reduced to pence, and unto them 6 pence being added (to wit the 6 pence which stands in the place of pence) there will arise 198 pence; these also are to be divided by 100 (by cutting off two places to the right hand as before,) so the quotient

tient gives 1 peny, and there will remain 98 pence; so the exact quotient or answer of the question is found to be 7 *l.* 8 *s.* 1 $\frac{89}{100}$ *d.*

More Examples of this Rule are these following.

```
 l.     l.     l.     s.    d.
100 ... 6 ... 793  :  12  :  7
                             6
------------------------------
      l. 47|61 : 15 : 6
           |20
      -----------
      s. 12|35
           |12
      -----------
      d.  4|26
```

```
 l.     l.     l.     s.    d.
100 ... 8 ... 43  :  14  :  3
                            8
------------------------------
      l.  3|49 : 14 : 0
           |20
      -----------
      s.  9|94
           |12
      -----------
      d. 11|28
```

After the same manner may this following question and such like be resolved, *viz.* When 100 Ells of Linen cloth cost 30 *l.* 18 *s.* 9 *d.* what is the price of 1 Ell? *Answer* 6 *s.* 2 *d.* 1 *farth.*

Ells

Ells	l.	s.	d.	Ell.
100 :	30 :	18 :	9 : . .	1
	20			

Shil. 6|18
|12

Pence 2|25
| 4

Farth. 1|00

XX. When the given gain of (or allowance for) 100 Integers consists of some number of pounds not exceeding 10, together with some Aliquot part or parts of a pound, the operation will be little different from the last mentioned Examples, as may appear by the resolution of the subsequent question, *viz.* What must be allowed for 2156 *l.* 13 *s.* 4 *d.* at the rate of 6 *l.* 15 *s.* for 100 *l.*? *Answ.* 145 *l.* 11 *s.* 6 *d.* thus found; first I multiply the said 2156 *l.* 13 *s.* 4 *d.* by 6 (the number of pounds in the given allowance 6 *l.* 15 *s.*) after the manner of the last examples, and subscribe the product which is 12940 *l.* underneath the line as you see, then since 15 *s.* are equal to $\frac{1}{2}$ *l.* together with $\frac{1}{4}$ *l.* I take $\frac{1}{2}$ of 2156 *l.* 13 *s.* 4 *d.* which is 1078 *l.* 6 *s.* 8 *d.* likewise $\frac{1}{4}$ of the said 2156 *l.* 13 *s.* 4 *d.* to wit, 539 *l.* 3 *s.* 4 *d.* and having subscribed these quotients underneath the product first found, and added them all together, I find 14557 *l.* 10 *s.* 0 *d.* for the total product, with which I proceed as in the former Examples; and so at length the *Answer* is found to by 145 *l.* 11 *s.* 6 *d.* View diligently the operation

l. l. l. s. d.
100 .. $6\frac{3}{4}$ .. 2156 : 13 : 4
$6\frac{3}{4}$

12940 : 00 : 0
1078 : 06 : 8
539 : 03 : 4

l. 145|57 : 10 : 0
|20

s. 11|50
|12

d. 6|00

CHAP. III.

Concerning Exchanges of Coins, Weights, and Measures.

I. THe rate or proportion between *Coins*, *Weights*, &c. of different kinds being known, either from some good Author, or rather by experience; it will not be difficult, to such as understand the *Rule of Three*, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind. But since in some cases the common way of working may be much contracted,

tracted, I ſhall endeaver to ſhew the moſt compendious ways to perform this buſineſs.

II. In exchanging of things of different kinds (whether they be *Coins* or *Weights*, &c.) when two things of different kinds are compared together, the queſtion may be reſolved by one ſingle *Rule of Three*, as will be evident by the ſubſequent Examples, *viz.*

Queſt. 1. How many *Riders* at 21 *s.* 2 $\frac{1}{2}$ *d. ſterling* the piece, ought to be received for 251 *l.* 6 *s.* 4 $\frac{1}{2}$ *d.* of *ſterling* money? *Anſwer*, 237 *Riders.* For the firſt and third terms in the *Rule of Three*, which ariſe from this queſtion, being converted into half pence, the proportion will be this,

$$509 . 1 :: 120633 . 237$$

Queſt. 2. If 100 *Ells* of *Antwerp* make 75 *yards* of *London*, how many yards of *London* meaſure will 27 *Ells* of *Antwerp* make? *Anſwer* 20 $\frac{1}{4}$ *yards.*

$$100 . 75 :: 27 . 20\tfrac{1}{4}.$$

III. When more than two different *Coins*, *Weights*, *Meaſures*, &c. are compared together, *viz.* when one kind of *Coin* is compared with a ſecond of another kind; that ſecond with a third; the third with a fourth; the fourth with a fifth, *&c.* two different caſes are ordinarily raiſed from ſuch compariſon, *viz.*

It may be required to know.
1. *How many pieces of the firſt Coin are equal in value to a given number of pieces of the laſt Coin:* Or,
2. *How many pieces of the laſt Coin are equal in value to a given number of pieces of the firſt kind of Coin.*

An

An Example of the firſt caſe.

If 35 ells of *Vienna* make 24 ells at *Lyons*; 3 ells of *Lyons* 5 ells of *Antwerp*; and 100 ells of *Antwerp* 125 ells at *Franckfort*; how many ells of *Vienna* are equal unto 50 ells at *Franckfort*? *Anſwer*, 35 ells of *Vienna.*

For the more eaſie underſtanding of the reſolution of this queſtion and others of like nature, Let *a* repreſent an ell at *Vienna*; *b* an ell at *Lyons*; *c.* an ell at *Antwerp*, and *d* an ell at *Franckfort*; then may the given terms in the queſtion be ſtated in the following order.

$$\text{Suppoſitions}\begin{cases} 35\ a = 24\ b \\ 3\ b = \ \ 5\ c \\ 100\ c = 125\ d \end{cases}$$
$$\text{The queſtion}\quad 50\ d = \ ?a$$

Which order of placing the ſaid given numbers (or terms) being obſerved, it appears that if 35 *a* be accounted to ſtand in the firſt place: 24 *b* in the ſecond; 3 *b* in the third; 5 *c* in the fourth; 100 *c* in the fifth, *&c.* then all the terms which ſtand in odd places, to wit, in the firſt, third, fifth, and ſeventh places, will neceſſarily fall under the firſt row or column on the left hand, and all the terms which ſtand in even places, to wit, in the ſecond, fourth, and ſixth places, will fall under the latter column.

Theſe things premiſed, all queſtions which fall under Caſe 1. before-mentioned may be reſolved by this Rule, *viz.*

Rule

Rule I.

Multiply all the given terms which ſtand in odd places (to wit in the firſt column) according to the rule of continual multiplication, and reſerve the laſt product for a dividend: Again multiply continually all the terms which ſtand in even places, ſo ſhall the product be a diviſor, and the quotient ariſing from the ſaid Dividend and Diviſor ſhall be the anſwer of the queſtion.

So in the laſt mentioned queſtion, if all the numbers in the firſt column, to wit, 35, 3, 100, and 50 be multiplied continually, the product will be 525000 for a Dividend; alſo if all the numbers in the latter column, *viz.* 24, 5 and 125 be multiplied continually, the laſt product will be 15000 for a Diviſor, and the quotient ariſing from the ſaid Dividend and Diviſor will be 35, which is the number of ells of *Vienna* required.

35	24
3	5
100	125
50	

525000 : 15000) 525000 (35

The reaſon of the ſaid Rule I. will be manifeſt by ſolving the queſtion propounded by three ſingle Rules of three, thus,

I. 24 *b.* 35 *a* :: 3 *b.* $\frac{35 \times 3}{24}$ *a* (= 5 *c.*

II. $\frac{5\,c}{1}\,\frac{35 \times 3}{24}\,a :: \frac{100}{1}\,c.\ \frac{35 \times 3 \times 100}{5 \times 24}\,a$ (= 125 *d.*

III. $\frac{125}{1}\,d.\ \frac{35 \times 3 \times 100}{5 \times 24}\,a :: \frac{50}{1}\,d.\ \frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}\,a.$

which fourth proportional laſt found, to wit, $\frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}$ being well viewed and compared with the before mentioned order of placing the terms given in the queſtion gives the very Rule I. before expreſt in words.

An Example of the latter of the two Caſes before mentioned.

If 10 *lb.* of *Averdupois* weight at *London* be equal to 9 *lb.* of *Amſterdam*; 45 *lb.* at *Amſterdam.* 49 *lb.* at *Bruges*; and 98 *lb.* at *Bruges* equal to 116 *lb.* at *Dantzick*; how many *lb.* of *Dantzick* are equal to 112 *lb* of *Averdupois* weight at *London*? *Anſwer*, 129. 92 *lb.* of *Dantzick.*

That the operation may be the more clear, let *a* repreſent one pound of *Averdupois* weight; *b* one *lb.* of *Amſterdam*; *c* one *lb.* of *Bruges*, and *d.* one *lb.* of *Dantzick*; then let the queſtion be ſtated after the order in the firſt Caſe, *viz.*

$$\textit{Suppositions}\begin{cases}10\,a = 9\,b\\ 45\,b = 49\,c\\ 98\,c = 116\,d\end{cases}$$

$$\textit{The question}\ 112\,a = \ ?\,d$$

These things premised, all questions which fall under Case 2. before mentioned may be solved by this Rule, *viz.*

Rule II.

Multiply all the given terms which stand in even place (to wit in the latter column) and the last odd term in the first column according to the rule of continual multiplication, and reserve the last product for a Dividend; again, multiply continually the rest of the terms which stand in odd places (to wit in the first column) for a Divisor, so shall the quotient arising be the answer of the question.

Or in this latter case if you place the last of the given terms in the same column with the even terms, the rule for solving questions, which fall under the latter case will be this which followeth, *viz.*

Multiply continually all the numbers in the latter column for a Dividend; also multiply continually all the numbers in the first column for a Divisor, so shall the quotient arising be the answer of the question. Thus the answer of the last mentioned question will be found 129.92, to wit, $129\frac{92}{100}$ *lb.* of *Dantzick*, as is evident by the subsequent operation.

$$\begin{array}{r|l} 10 & 9\\ 45 & 49\\ 98 & 116\\ & 112\\ \hline 44100 & 5729472\ (129.92 \end{array}$$

The reason of the said Rule II will be manifest by solving the question propounded, by three single Rules of three, thus,

$$I.\ 9\,b.\ 10\,a :: 45\,b..\ \frac{45 \times 10}{9}\,a\ (=49\,c.$$

$$II.\ \frac{49}{1}\,c.\ \frac{45 \times 10}{9}\,a :: \frac{98}{1}\,c.\ \frac{45 \times 10 \times 98}{49 \times 9}\,a\ (=116\,d.$$

$$III.\ \frac{45 \times 10 \times 98}{49 \times 9}\,a.\ \frac{116}{1}\,d :: \frac{112}{1}\,a.\ \frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98}\,d.$$

which fourth proportional last found, to wit, $\frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98}$ being well viewed and compared with the forementioned order of placing the terms given in the question discovers the very Rule II. before exprest in words.

Note when the same numbers happen to be Multiplicators in the Dividend, and also in the Divisor, such Multiplicators may be cancelled in both, and thereby much labor will oftentimes be spared.

Such which have much practice in calculating *Exchanges*, and do exactly know the rate or proportion between two different weights or measures or coins, which they would compare together, may by the *Rule three* frame Tables of proportions for the more speedy reducing of a given quantity of one kind of Weight, measure, *&c.* into a quantity of the same value in another kind of weight, &c. In the expressing of which proportions it will be very convenient that the first number or Antecedent of each proportion be made 1 or unity, and the second term or consequent a Decimal, or else a mixt number whose Fractional part is a Decimal, for then the Coin, weight, *&c.* of the one place (whose term is 1) may be reduced into that of the other place, by help of those Tables and of Multiplication of Decimals without sensible error: For Example, It hath been observed by some ingenious Merchants that 100 *lb.* of *Averdupois* weight at *London*, are equal unto 89 *lb.* in *Paris* by the King's beam and consequently 1 *lb. Averdupois* is equal to $\frac{89}{100}$ *lb.* or .89 *lb.* at *Paris*; (for if 100 give 89, then 1 will give .89;) therefore any number of pounds *Averdupois* being multiplied by .89 (with respect unto Multiplication of Decimals, explained in the 26 Chapter of the preceeding Book) will produce pounds of *Paris*: Again, if 89 *lb.* of *Paris* be equal to 100 *lb. Averdupois*, then 1 *lb.* of *Paris* will be near equal to 1.1235 *lb.* of *Averdupois*; therefore any number of pounds of *Paris* being multiplied by 1.1235 will produce pounds *Averdupois* very near.

Upon this ground I have collected the proportions in the following Tables, wherein I would not have

have any to confide further than they shall know them to be agreeable to truth, for I have only derived them from those delivered by Mr. *Lewis Roberts* Merchant, in his *Map of Commerce* Printed at *London*, *Anno* 1638. and do herein only aim at the instruction of ingenious *Merchants* and *Factors* in the briefest ways of calculating their exchanges, the rate or proportion being truly known; in which practice, *Decimal Arithmetick* (which hath no Enemy but the Ignorant) will be very serviceable.

Tho: Jones

A Table for the Reduction of Averdupois *Weight at* London, *to the Weights of divers Foreign Cities and remarkable Places.*

		lb.
One pound of *Averdupois* weight at *London*, makes at	*Antwerp*	.9615
	Amsterdam	.9
	Abeville	.91
	Ancona .	1 .282
	Avignon	1 .12
	Burdeaux	.91
	Burgoigne	.91
	Bollonia	1 .25
	Bridges	.98
	Callabria	1 .3698
	Callais	1 .07
	Constantinople	.8474 *Loder*;
	Deep	.91
	Dantzick	1 .16
	Ferrara	1 .3333
	Florence	1 .282
	Flanders in general	1 .06
	Geneva	.9345

Genoa

		lb.
One pound of *Averdupois* weight at *London*, makes at	*Genoa*	1 .4084 *suttle*
		1 .4285 *gross*
	Hamburg	.92
	Holland	.95
	Lixborn	.881
	Lyons	1 .07 *common weight.*
		.98 *silk weight.*
		.9 *customers weight.*
	Leghorn	1 .3333
	Millan	1 .4285
	Mirandola	1 .3333
	Norimberg	.88
	Naples	1 .4084
	Paris	.89
	Prague	.83
	Placentia	1 . 3888
	Rotchel	1 . 12
	Rome	1 . 27
	Rouan	.875 *by vicont.*
		.9017 *com. weight.*
	Sevil	1 .08
	Tholousa	1 .12
	Turin	1 .2195
	Venetia	1 .5625 *subtle*
		.9433 *gross*
	Vienna	.813

The use of the preceding Table will be manifest by the subsequent *example, viz.*

How much weight at *Dantzick* do 320 *lb. Averdupois* make? *Answer*, 371.2 *lb.* Seek in the precedent Table for *Dantzick*, and right against it you shall find 1.16 which shew that 1 *lb. Averdupois* is equal to 1.16 *lb.* at *Dantzick*, therefore multiply 320 by 1.16, so will the product be 371.2 *lb.* of *Dantzick*, as by the Operation is manifest.

Aver.		*Dantz.*		*Aver.*		*Dantz.*
1	:	1.16	::	320	:	371.2

$$\begin{array}{r} 1.16 \\ \hline 1920 \\ 320 \\ 320 \\ \hline 371|20 \end{array}$$

A Table for the Reduction of the Weights of divers Foreign Cities and remarkable Places to Averdupois *Weight at* London.

One pound weight in	makes at *London* of *Averdupois* weight lb.
Antwerp	1.04
Amsterdam	1.1111
Abbeville	1.0989
Ancona	.78
Avignon	.8928
Burdeaux	1.0989
Burgoyne	1.0989
Bollonia	.8
Bridges	1.0204
Callabria	.73
Callais	.9345
Deep	1.0989
Dantzick	.862
Ferrara	.75
Florence	.78
Flanders in general	.9433
Geneva	1.07
Genoa subtle,	.71
Genoa gross,	.7

One pound weight in		makes at *London* of *Averdupois* weight lb.
Hamburg		1 .0865
Holland		1 .0526
Lixborn		1 .135
Lyons	common weight.	.9345
	ſilk weight.	1 .0204
	cuſtom weight.	1 .1111
Legborn		.75
Millain		.7
Mirandola		.75
Norimberg		1 .1363
Naples		.71
Paris		1. 1235
Prague		1. 2048
Placentia		.72
Rotchel		.8928
Rome		.7874
Rouan	by Vicont.	1 .1428
	common weight	1 .1089
Sevil		.9259
Tholouſa		.8928
Turin		.82
Venetia	ſuttle,	.64
	groſs,	1 .06
Vienna		1 .23

The

The uſe of the laſt mentioned Table, will be manifeſt by this example, *viz.*

In 224 *lb.* weight at *Hamburg*, how many pounds *Averdupois*?

Anſw. 243.376 *lb.*

Seek in the Table for *Hamburg*, and right againſt it you wll find 1.0865, which ſheweth that 1 *lb.* of *Hamburg* makes 1.0865 *lb. Averdupois*; therefore if 1.0865 be multiplied by 224 the product will be pounds *Averdupois*.

1 . . . 1 .0865 . . . 224

224

43460

21730

21730

243|3760

A

A Table for the Reduction of English Ells to the Measures of divers Foreign Cities, and remarkable places.

One Ell at *London* makes at

Place	Measure	
Amsterdam	1.6949	Ells
Antwerp	1.6666	Ells
Bridges	1.64	Ells
Arras	1.65	Ells
Norimberg	1.74	Ells
Colen	2.08	Ells
Lisle	1.66	Ells
Mastrich	1.57	Ells
Frankford	2.0866	Ells
Dantzick	1.3833	Ells
Vienna	1.45	Ells
Paris	.95	Ells
Rouan	1.03	Aulnes
Lions	1.0166	Aulnes
Callais	1.57	Aulnes
Venice linen,	1.8	Braces
Venice silk,	1.96	Braces
Lucques	2.	Braces
Florence	2.04	Braces
Milan	2.3	Braces
Leghorn	2.	Braces
Madera Isles	1.0328	Braces

Sevil

One Ell at *London* makes at

Place	Measure	
Sevil	1.35	Vares
Lisbone	1.	Vares
Castilia	1.3875	Vares
Andoluzia	1.3625	Vares
Granado	1.3625	Vares
Genoa	4.8083	Palmes
Saragosa	.55	Canes
Rome	.56	Canes
Barselona	.7125	Canes
Valentia	1.2125	Canes

The use of the aforesaid Table will be manifest by the subsequent example, *viz.*

In 325 ells of *London*, how many ells at *Antwerp?*

Answ. 541.645 ells: Seek in the Table for *Antwerp*, and right against it you shall find 1.6666 which being multiplied by 325 produceth 541.645 ells of *Antwerp*, as by the operation is manifest.

```
1 . . . 1.6666 . . . 325
           325
    ----------
         83330
        33332
       49998
    ----------
       541 | 6450
```

A

A Table for the Reduction of the Measures of divers Foreign Cities and remarkable places to English Ells.

One Ell at	*Amsterdam*	makes at *London*	.59	Ells
	Antwerp		.6	
	Bridges		.6097	
	Arras		.606	
	Norimberg		.5747	
	Colen		.4807	
	Lisle		.6024	
	Mastrich		.6369	
	Frankford		.4792	
	Dantzick		.7228	
	Vienna		.6896	
One Auln at	*Paris*		1.0526	
	Rouan		.9708	
	Lions		.9836	
One Brace at	*Callais*		.6369	
	Venice linen,		.5555	
	Venice silk :		.5102	
	Lucques		.5	
	Florence		.4901	
	Milan		.4347	
	Leghorn		.5	
	Madera Isles		.9681	

Sevil

One Vare at	*Sevil*	makes at *London*	.7407	Ells
	Lisbone		1.	
	Castilia		.7207	
	Andoluzia		.7339	
	Granado		.7339	
One Palm at *Genoa*			.2079	
One Cane at	*Saragosa*		1.8181	
	Rome		1.7857	
	Barselona		1.4035	
	Valentia		.8247	

The use of the said Table will be manifest by the subsequent example, *viz.*

In 730 Aulness at *Lions*, how many ells at *London.*

Answ. 718.028. Seek in the Table for *Lions*, and right against it you shall find .9836, which being multiplied by 730 produceth 718.028 ells of *London*, as by the operation is manifest.

```
1 . . . .9836 . : . 730
         730
      ---------
       295080
      68852
      ---------
      718.028
```

Note,

Note, that one and the same kind of Weight or Measure doth seldom or never alter from its peculiar quantity, in the Kingdom or Common-wealth, where such weight or measure was first established; but one and the same kind of money doth often rise and fall in its value in foreign parts: For which cause I have spared the pains of calculating *Decimal Tables* for *Coins*, yet to give some light to such as read modern relations, and want experimental knowledge in this matter I shall here insert a *Table*, in the same estate as I find it in the aforesaid *Map* of *Commerce*, and refer the Reader, for further satisfaction, to the *Table* in *Rider's Dictionary*, concerning *Coins*, *Weights*, and *Measures*, both ancient and modern.

Of

Of Exchanges of London, *with divers foreign Cities.*

London doth exchange with

	Pence			
Placentia sterl.	64	for	1	*Crown*
Lyons	64	for	1	*Crown*
Rome	66	for	1	*Ducat*
Genoa	65	for	1	*Crown*
Milan	64 ¾	for	1	*Crown*
Venice	50	for	1	*Ducat*
Florence	53 ½	for	1	*Ducaton*
Naples	50	for	1	*Ducat*
Lecchia in *Calabria*	50	for	1	*Ducat*
Barri	51	for	1	*Ducat*
Palermo	57 ½	for	1	*Ducat*
Mesina	56 ½	for	1	*Ducat*
Antwerp & *Colin*	1 *l. sterl.*	for	34½	*shill. flem.*
Valentia	57 ½	for	1	*Ducat*
Saragosa	59	for	1	*Ducat*
Barselona	64	for	1	*Ducat*
Lexborn	53 ½	for	1	*Ducat*
Bollonia	53 ½	for	1	*Ducaton*
Bergamo	52	for	1	*Ducaton*
Frankfort	59 ½	for	1	*Florin*
Genoa	83	for	1	*Crown*

London

London exchangeth in the denomination of *pence sterling* with all other Countries, *Antwerp* and those neighbouring Countries of *Flanders* and *Holland* excepted, with which it exchangeth by the entire *pound* of 20 *shillings English* (or *sterling*.)

CHAP. IV.

Practical Questions about various things: viz. *Tare, Tret, Loss, Gain, Barter, Factorship, and Measuring of Tapestry.*

Of abatements and allowances in Traffick, viz. I. *Of Tare.*

IN the trade of Merchandice there are in use various allowances, and abatements, known by the names of *Tare, Tret, &c.* concerning which I shall give a few examples, whereby the practical *Arithmetician* will easily see, that there is more difficulty in the name than in the thing; for the rate, or proportion agreed upon, in any allowance or abatement (be it called by what name soever) being once known, the Arithmetical work will quickly be dispatcht by the *Rule of Three*, or else by that and some of the former rules mixtly used, as will partly appear by the following questions.

Gross weight is composed of the neat weight of the commodity, and also the Tare, to wit, the Chest, Bag, Butt, &c. which containeth the commodity.

Quest. 1 A Factor buyeth 4 Chests of Sugar marked A. B. C. D The gross weight of each Chest in *Averdupois* greater weight is as followeth.

	C.	q.	lb.
A.	11 ...	1 ...	19
B.	10 ...	3 ...	20
C.	11 ...	2 ...	13
D.	10 ...	1 ...	17
The total gross weight	44 ...	1 ...	13

Now supposing the *Tare* or weight of each Chest, when it is empty, to be 37 *lb.* the question is what neat weight of Sugar will remain, when the total *Tare* is subtracted? *Answ.* 43 *C.* 0 q. 4 *lb.*

	C.	q.	lb.
from	44 ..	1 ..	13
Subtr.	1 ..	1 ..	08
Rem.	43 ..	0 ..	05 *the neat weight of Sug.*

Quest. 2. If from 990 *C.* 3 *qu.* 21 *lb.* *gross weight*, *Tare* is to be subtracted after the rate of 14 *lb. per C.* (or 112 *lb.*) of *gross weight*, how many C. neat will remain? *Answ.* 867 *C.* 0 *qu.* 7 $\frac{7}{8}$ *lb.*

I. The gross weight being converted into pounds by the sixth Rule of the 7th Chapter of the preceding Book, will give 110985 *lb.*

II. Then by the *Rule of Three.*

112 . 14 :: 110985 . 13873 $\frac{1}{8}$
or 8 . 1 :: 110985 . 13873 $\frac{1}{8}$

lb.
III. From 110985 *the gross weight:*
Subtr. 13873$\frac{3}{8}$ *the total Tare.*

C. *qu.* *lb.*
Rest neat 97111$\frac{7}{8}$ = 867 .. 0 .. 7$\frac{7}{8}$

Note, when the number of *lb.* to be abated *per C.* for *Tare* is an *aliquot part* of 112, as in the last mentioned example, where 14 = $\frac{1}{8}$ of 112, the operation may be thus;

C.		C.		C.	q.	lb.		C.	qu.	lb.
1	.	$\frac{1}{8}$	::	990 :	3 :	21	:	(123 :	3 :	13$\frac{1}{8}$

$\frac{1}{8}$ of	990 *c.* =	123 :	3 :	00
	3 *q* =	00 :	0 :	10$\frac{4}{8}$
	21 *lb.* =	00 :	0 :	02$\frac{5}{8}$
Total Tare		123 :	3 :	13$\frac{1}{8}$
Rest neat		867 :	0 :	07$\frac{7}{8}$

Quest. 3. Suppose at some City, there is a Custom in selling of certain Merchandice by weight, to allow or cast in as an overplus to the buyer, 4 *lb.* weight for every 100 *lb.* weight that is bought, and in that proportion for a greater or lesser quantity. Now if a Merchant buy 1175 *lb.* weight of some commodity, and is to be allowed thereupon after the aforesaid rate, the question is, how many *lb.* weight ought he to receive in all? *Answ.* 1222 *lb.* weight. *Of Tret.*

100. 104 :: 1175 . 1222

This kind of allowance is commonly called *Tret.*

Quest. 4. Suppose a Merchant hath 1222 *lb.* weight of a certain commodity, part whereof he bought at a certain rate *per lb.* and the rest was allowed to him or cast in as an overplus, after the rate of 4 *lb.* weight for every 100 *lb.* weight which he bought; the question is, to know how many pounds neat weight he bought? *Answ.* 1175 *lb.* weight.

104. 100 :: 1222. 1175.

This question is the converse of the former, and sheweth how to make abatement for *Tret.*

Quest. 5. If from 55 *C.* 1 *qu.* of gross weight *Tare* is to be subtracted after the rate of 16 *lb. per C.* and from the remainder *Tret* is to be abated after the rate of 4 *lb. per* 104 *lb.* the question is, what the neat weight is worth in money after the rate of 8 *l.* 8 *s.* for every *C.* (or 112 *lb.*?) *Answ.* 382 $\frac{1}{2}$ *l.*

I. The gross weight in *lb.* is 6188 *l.*
II. 112 . 16 :: 6188 . 884
or 7 . 1 :: 6188 . 884
III. 6188 - 884 = 5304
IV. 104 . 100 :: 5304 . 5100
V. 112 . 8$\frac{2}{5}$ :: 5100 . 382$\frac{1}{2}$

Quest. 6. A Merchant hath bought Linen cloth at 11 *s. per* ell, which proving worse than he expected, he is willing to sell it at such a price that he may lose precisely after the rate of 1 $\frac{2}{3}$ *l.* for every 20 *l.* that he laid out; the question is to know at what price he ought to sell the ell, that the proportion in the said *Of Loss and Gain.*

said loss may be observed? *Answ.* 10 *s.* 1 *d. per* ell.

I. $20 - 1\frac{2}{3} = 18\frac{1}{3}$
II. $20 \,.\, 18\frac{1}{3} :: 11 : 10\frac{1}{2}$ *pence*

Otherwise,

I. $20 \,.\, 1\frac{2}{3} :: 11 \,.\, \frac{11}{12}$
II. $11 - \frac{11}{12} = 10\frac{1}{12}$

Quest. 7. If 100 *lb.* weight of any commodity cost 30 *s.* at what price must 1 *lb.* weight of that commodity be sold to gain after the rate of 10 *l.* for every 100 laid out? *Answ.* $3\frac{24}{25}$ *d. per lb.* weight.

I. $100 \,.\, 110 :: 30 \,.\, 33$
II. $100 \,.\, 33 :: 1 \,.\, \frac{33}{100}$ *s.* (or $3\frac{24}{25}$ *d.*

Quest. 8. A Merchant selleth a parcel of Jewels which cost him 250 *l.* ready money, for 559 *l.* payable at the end of 6 months; the question is (his security being supposed to be good) what his gain was worth in ready money upon rebate of interest at the rate of 6 *l.* for 100 *l.* for a year? *Answ.* 300 *l.*

$559 - 250 = 309$
$103 \,.\, 100 :: 309 \,.\, 300$

Of Barter.

Quest. 9. How much Sugar at 8 *d. per lb.* weight may be bought for 20 C. of Tabacco at 3 *l. per* C.? *Answ.* 1800 *lb.* weight of Sugar.

$1 \,.\, 3 :: 20 \,.\, 60$
$\frac{1}{30} \,.\, 1 :: 60 \,.\, 1800$

Quest. 10. A. hath 100 pieces of Silks, which are worth but 3 *l. per* piece in ready money, yet he barters them with B. at 4 *lb. per* piece, and at that rate takes their value of B. in Wools at 7 *l.* 10 *s. per* C. which are worth but 6 *l. per* C. in ready money, the question is to know what quantity of Wools pays for the Silks, and which of the two A. or B. is the gainer, and how much? *Answ.* $53\frac{1}{3}$ C. of Wools pays for the Silks, and A. gaineth 20 *l.* by the barter.

I. $7\frac{1}{2} \,.\, 1 :: 400 \,.\, 53\frac{1}{3}$
II. $\begin{cases} 1 \,.\, 6 :: 53\frac{1}{3} \,.\, 320 \\ \text{or } 7\frac{1}{2} \,.\, 6 :: 400 \,.\, 320 \end{cases}$

So it is evident that the true worth of the Wool which B. delivered was 320 *l.* for which he received only of A. the worth of 300 *l.* in Silks, and therefore B. loseth 20 *l.* by the barter.

Of Factorship. See brief rules for computing of Factors allowances in the 19, and 20 Rules of the second Chap. of this Appendix.

Quest. 11. A Merchant delivered to his Factor 600 *l.* upon condition that if the Factor add to it 250 *l.* of his own money, and bestow his pains in managing the whole stock, he shall then have $\frac{2}{5}$ parts of the total gain. The question is to know what stock the Factor's service was estimated at? *Answ.* 150 *l.*

I. The Factor's part of the gain being $\frac{2}{5}$ the Merchant must necessarily have the remainder, which is $\frac{3}{5}$.
II. $\frac{3}{4} \,.\, \frac{2}{5} :: 600 \,.\, 400$
III. $400 - 250 = 150$

Quest. 12. A Merchant delivereth to his Factor 320 *l.* and permitteth him to add to it 64 *l.* of his own money, to be employed in traffick, and by agreement between them the Factor's ſervice is eſtimated equivalent to a certain ſtock; which is ſuch, that if the total gain be divided proportionably according to thoſe three ſtocks, the Factor is to receive $\frac{1}{5}$ of the total gain, in conſideration of the ſaid imaginary ſtock (being the value of his ſervice;) the queſtion is to know the full part of the gain belonging to each, aud what ſtock the Factor's ſervice was valued at? *Anſw.* The Merchant $\frac{2}{3}$ of the gain, and the Factor $\frac{1}{3}$, whoſe ſervice was valued at 96 *l. ſtock.*

I. $320 + 64 = 384$

II. $\frac{4}{5} \,.\, \frac{1}{5} :: 384 \,.\, 96$

III.

$$\left.\begin{array}{r} 320 \\ 64 \\ 96 \\ \hline 480 \,.\, 1 :: \end{array}\right. \qquad \left\{\begin{array}{l} 320 \,.\, \frac{2}{3} \\ 160 \,.\, \frac{1}{3} \end{array}\right.$$

Quest. 13. If a piece of Arras Hangings, in the form of a long ſquare, hath for its length $6\frac{1}{4}$ yards *Engliſh*, and breadth 4 yards; how many ſquare ells, or ſticks *Flemiſh* are contained in that piece, when the length of a *Flemiſh* ell is equal to $\frac{3}{4}$ yard *Engliſh*? *Anſwer*, $44\frac{4}{9}$ ſquare ells or ſticks *Flemiſh*.

Of Meaſuring of Tapeſtry.

Foraſmuch as by ſuppoſition, a *Flemiſh* ell in length, hath ſuch proportion to an *Engliſh* yard in length, as 3 to 4, and conſequently the ſquare of the one to the ſquare of the other, as 9 to 16. Therefore in a direct proportion, as 9 is to 16; ſo is any given number of ſquare yards *Engliſh* to a number of ſquare ells *Flemiſh*, which will take up equal ſpace with the ſaid ſquare ells *Engliſh*. Alſo in a direct proportion, as 16 is to 9, ſo is any given number of ſquare ells *Flemiſh* to a number of ſquare yards *Engliſh*, which will take up an equal ſpace with the ſaid *Flemiſh* ells: Therefore to reſolve the aforeſaid queſtion, firſt find the number of ſquare yards *Engliſh* contained in the ſaid piece of Arras, by multiplying the length and breadth in yards mutually one by the other, then proceed according to the aforeſaid proportion; ſo the work will ſtand thus,

I. $6\frac{1}{4} \times 4 = 25$ ſquare yards *Engliſh*.

II. $9 \,.\, 16 :: 25 \,.\, 44\frac{4}{9}$ ſquare ells *Flemiſh*.

Otherwiſe.

$6\frac{1}{4}$ yards *Engliſh* in length give by the *Rule of Three* in *Flemiſh* ells } $8\frac{1}{3}$ *length.*

Alſo 4 yards *Engliſh* give in *Flemiſh* ells ———————— } $5\frac{1}{3}$ *breadth.*

Therefore the product of the ſaid $8\frac{1}{3}$ multiplyed by $5\frac{1}{3}$, gives for the ſuperficial content as before ..... } $44\frac{4}{9}$

Queſt. 14. If a piece of Tapeſtry in the form of a long ſquare be in length $15\frac{1}{4}$ ells *Flemiſh*, and in breadth $4\frac{1}{3}$ ells *Flemiſh*, how many ſquare yards *Engliſh* are contained in that piece, when 4 ells *Flemiſh* in length are equal to 3 yards *Engliſh*? *Anſw.* $37\frac{11}{64}$ ſquare yards *Engliſh*.

I. $15\frac{1}{4} \times 4\frac{1}{3} = 66\frac{1}{12}$.

II. $16 \,.\, 9 :: 66\frac{1}{12} \,.\, 37\frac{11}{64}$.

CHAP. V.

Concerning the Interest of Money, and the Construction of Tables to that purpose.

I. IN resolving questions concerning interest of money, four things are to be well observed, to wit, First, the Principal, or money lent for gain or interest; Secondly, the time for which the said Principal is lent; Thirdly, the rate or proportion which the Principal bears to the sum of the Principal and Interest; and Fourthly, the Interest it self: So if 100 *l.* be lent upon condition that 106 *l.* shall be repaid at the end of a year, the said 100 *l.* is called Principal; the time for which the said principal is lent is one year; the proportion which the principal bears to the sum of the principal and interest is such as 100 hath to 106; Lastly, the interest it self is 6 *l.*

II. Interest is either Simple or Compound.

III. Simple Interest is that which ariseth or is computed from the principal onely: So if 100 *l.* be lent for two years, the simple Interest thereof after the rate of 6. pounds for 100 pounds for 1 year will be 12 pounds, *viz.* 6 pounds due at the first years end, and 6 pounds due at the second years end.

IV. Compound Interest is that which ariseth from the principal, and also from the interest thereof, and therefore it is called interest upon interest: So if 100 pounds be lent and forborn 3 years, and compound interest thereof is to be computed

puted after the rate of 6 pounds for 100 *l.* for one year; there will arise besides the simple interest of the principal for three years, the interest of 6 pounds (due at the first years end) for 2 years, and the interest of 6 pound (due at the second years end) for one year following.

V. Rebate or discompt of money is, when a sum of money due at any time to come, is satisfied by the payment of so much present money, which if it were put forth at a certain rate of interest for the said time, would become equal to the sum first due: So if 100 pounds be due at the end of two years, and is to be satisfied by the payment of present money upon rebate, after the rate of 6 pounds *per centum, per annum, simple interest*, there ought to be so much ready money paid, which in two years after the said rate of interest would be augmented unto 100 *l.* In like manner if the rebate or discompt were to be made after any rate of compound interest, so much ready money ought to be paid, which at such rate of compound interest, for the time agreed on, would become equal to the sum first due. *Examples* of the manner of computation by rebate may be seen in the tenth and fourteenth Rules of this Chapter.

The foundation upon which the Rules for computing simple interest are grounded.

VI. In the taking of interest, or use-money, for the loan or forbearance of money lent, respect must be had to the rate limited by Act of Parliament, which now restraineth all persons from taking more than 6 *l.* for the interest or use of 100 *l.* lent for a year, but what part of 6 *l.* may be taken for the interest of 100 *l.* lent for half a year, a quarter of

of a year, a month, or any other part of a year, is not expreſt in the Act; In this caſe therefore we muſt obſerve cuſtom and daily practice, ſo we ſhall find that 3 *l.* is uſually taken for half a years intereſt of 100 *l.* and 30 *s.* for a quarter of a year, *&c.* by which practice, this following Analogy (which is the ground or reaſon of the common rules for computing ſimple intereſt) ſeems to be aſſumed for a ſafe expoſition of the Statute, *viz.* That ſuch proportion as the whole year (ſuppoſed to conſiſt of 365 days) hath to any propounded ſpace of time more or leſs than a year, ſuch proportion any intereſt (not exceeding the rate limited by the Act) for any Principal lent for a year, ought to have to the intereſt of the ſame Principal for the time propounded: This Analogy being granted, the manner of computing ſimple intereſt, for any Principal lent and forborn any time propounded, will be ſuch as is expreſt in the two next Sections.

VII. The intereſt or gain of 100 *l.* principal money forborn for a year being known, the intereſt of any other principal money for the ſame time may be found out by one ſingle *Rule of Three*; for as 100 *l.* principal is in proportion to the intereſt thereof, ſo is any other principal to its intereſt: So if it be demanded what 270 *l.* will gain in a year at the rate of 6 *l.* for 100 *l.* for one year, the *Anſwer* will be found to be 16 *l.* 4 *s.* For,

```
 l.   l.    l.    l.      l.   s.   d.
100 : 6 : : 270 . 16,2 (or 16 : 4 : 0
```

A ſecond Example. What is the intereſt of 175 *l.* 18 *s.* 11 *d.* for a year, at the rate of 6 *l.* for 100 *l.* for a year? *Anſw.* 10 *l.* 11 *s.* 1 $\frac{62}{100}$ *d.* as by the following operation (which is performed after the practical manner delivered in the nineteenth Rule of the ſecond Chapter of this *Appendix*) is evident.


```
 l.    l.    l.    s.   d.     l.   s.   d.
100 . 6 : : 175 : 18 : 11 ( 10 : 11 : 1 62/100
        multiply by . . 6
 ----------------------------------
l. . : . . . . 10|55 : 13 : 6
                 |20
 ----------------------------------
s. . . . . . . 11|13
                 |12
 ----------------------------------
d. . . . .      1|62
```

VIII. If the intereſt of 100 *l.* principal for one whole year, or 365 days be known, the ſimple intereſt of any other principal, for any number of days more or leſs than 365, may be found out by the following Rule, *viz.*

A Rule for computing ſimple intereſt for any number of days.

Multiply theſe three numbers according to the Rule of continual Multiplication, to wit, the given intereſt of 100 *l.* for a year, the principal, whoſe intereſt is required, and the number of days preſcribed, reſerving the laſt product for a Dividend: Alſo multiply 365 by 100, and reſerve this product for a Diviſor; Laſtly finiſh Diviſion, ſo ſhall the quotient be the intereſt or gain ſought.

Note here, that the two principals, to wit 100 *l.* and the other propounded, are ſuppoſed to be of one and the ſame denomination: Alſo the intereſt required

required will be of the ſame denomination with the given intereſt of 100 *l.*

For an example of this Rule, let it be required to find out the intereſt of 400 *l.* for a week, or 7 days at the rate of 6 *l.* for 100 *l.* for a year, or 365 days; Firſt multiplying theſe three numbers 6, 400, and 7 continually (*viz.* multiplying 6 by 400, and the product thence ariſing by 7) the laſt product will be 16800 for a Dividend; alſo multiplying 365 by 100, the product is 36500 for a Diviſor; Laſtly, dividing 16800 by 36500 (after cyphers at pleaſure are added to 16800) the quotient (according to the fourth Rule of the 27th Chapter of the preceding Book) will be diſcovered to be this decimal .4602, which is equal to 9 *s.* 2 *d.* 1 *farth.* (as will appear by the brief way of valuing a decimal fraction in the fourth Rule of the 26th Chapter.

The reaſon of the above mentioned rule for the computing of intereſt for days, will be manifeſt by this following way of ſolving the ſame queſtion by two ſingle *Rules of Three*, viz.

$$\text{I. } 100 \,.\, 6 :: 400 \,.\, \frac{6 \times 400}{100}$$

$$\text{II. } \frac{365}{1} \,.\, \frac{6 \times 400}{100} :: \frac{7}{1} \,.\, \frac{6 \times 400 \times 7}{365 \times 100}$$

Which fourth proportional in the latter *Rule of Three*, to wit, $\frac{6 \times 400 \times 7}{365 \times 100}$, being well viewed, the truth of the rule before delivered will be manifeſt.

Hence one vulgar errour in computing intereſt is

is diſcovered, for ſome argue thus, 6 *l.* is the intereſt of 100 *l.* for a year, therefore 10 *s.* (or $\frac{1}{12}$ of 6*l.*) is the intereſt for a month, and conſequently 2*s.* 6*d.* for a week or ſeven days, and ſo the intereſt of 400 *l.* for 7 days, computed after that manner would be 10 *s.* which exceeds the anſwer found by the preceding Rule by 9 $\frac{3}{4}$ *d.* very near, which fallacy hath its riſe from the taking, (or rather miſtaking) of 28 days for $\frac{1}{12}$ part of the number of days in a year, when indeed the juſt $\frac{1}{12}$ part of 365 days conſiſts of 30 $\frac{5}{12}$.

Another Rule for computing ſimple Intereſt for days.

Moreover, by the help of this decimal fraction of a pound, to wit, .000164383, which is very near the intereſt of one pound for a day at the rate of 6 *per cent. per annum* (as will appear by the preceding rule) the intereſt of any principal (ſuppoſed to be pounds or decimal parts of a pound) for any number of days propounded at the ſaid rate of intereſt, may be found out by multiplication only, *viz.* Firſt multiply the ſaid decimal .000164383 by the principal whoſe intereſt is required, then multiply that product by the number of days propounded, ſo ſhall this laſt product be the intereſt required; (but in theſe multiplications reſpect muſt be had to the cutting off of places in the products, according to the ſecond and third Rules of the 26th Chapter of the preceding Book;) for example, if it be required to find the intereſt of 1000 *l.* for 131 days, at the rate of 6 *per cent. per annum*, the *Anſw.* will be found 21.534 +, or 21 *l.* 10 *s.* 8 *d.* + for according to the rule laſt given.

.000164383

$.000164383 \times 1000 \times 131 = 21.534+$

But at another rate of interest, a peculiar decimal instead of the said .000164383 (which serves only for 6 *per cen. per annum*) must be found out by the first rule aforegoing, before the latter rule can take place, the reason of which latter rule doth also evidently arise from two single rules of three.

The manner of summing up Annuities in arrear with allowances of simple interest.

IX. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, simple interest being computed for each particular yearly payment, from the time it became due, until the end of the term of years, the work will be as in this following example, *viz.* If an Annuity, or yearly rent of 134 *l.* 10 *s.* 6 *d.* be all forborn till the end of 4 years, what will it then amount unto, simple interest being allowed at the rate of 6 *per cent. per annum* for each years rent, from the time on which it was due, until the end of the said term of four years? *Answ.* 586 *l.* 10 *s.* 6 $\frac{96}{100}$ *d.*

It is evident by the question, that at the rate of interest propounded, there must be computed the interest of 134 *l.* 10 *s.* 6 *d.* (due at the third years end) for one year (to wit, the fourth year;) also the interest of the like sum due at the second years end, for two years (to wit, the third and fourth years;) likewise the interest of the same sum due at the first years end, for three years (to wit, the second, third and fourth years:) all which interest being added to the sum of the four years rent, the total sum will shew what the said Annuity will amount unto at the end of the said term of 4 years.

Explication.

	years	*l.*	*s.*	*d.*
The interest of 134 *l.* 10 *s.* 6 *d.* at 6 *per cent. per annum*, for	1 is...	8	1	5.16
	2 is...	16	2	10.32
	3 is...	24	4	3.48
The sum of the 4 years rent (to wit, 4 times 134 *l.* 10 *s.* 6.	is...	538	2	0
All which added together give the *Answer* of the question, to wit,	...	586	10	6.96

Of rebate or discompt of money at simple interest.

X. When it is required to find out how much ready money will satisfie a Debt due at the end of any space of time to come, by rebating or discompting at a given rate of simple interest, it may be effected by this rule, *viz.* First, find out the interest of 100 *l.* at the given rate of interest, for the time which the ready mony is to be paid before-hand, then adding the interest so found to 100 *l.* make always the sum of that addition the first term in a rule of Three; 100 *l.* the second term; and the debt propounded to be satisfied the third term; lastly, the fourth proportional found out by the said *Rule of Three* shall be the ready mony which ought to be paid in satisfaction of the debt propounded.

Example 1. If a debt of 100 *l.* be payable at the end of a year to come, how much ready money will discharge that debt by rebating or discompting at the rate of 6 *per cent. per annum?* *Answ.* 94 *l.* 6 *s.*

6 *s.* 9 *d.* 2 *f.* very near; for by the *Rule of Three*,

106 . 100 :: 100, 94 3396 +

That is to say, if 106 *l.* (which is compos'd of 100*l.* principal and 6 *l.* interest) proceeds from 100 *l.* principal forborn for a year, from what principal forborn for a year doth 100 *l.* (compos'd of principal and interest) proceed from? *Answ.* 94.3396 *l.* + (or 94 *l.* 6 *s.* 9 ½ *d.* very near) principal money; therefore 94 *l.* 6 *s.* 9 ½ *d.* in ready money, is of equal value with 100 *l.* due at the end of the year to come; for if the said 94 *l.* 6 *s.* 9 ½ *d.* be put forth at interest for a year, at the rate of 6 *per cent. per ann.* it will gain 5 *l.* 13 *s.* 2 ½ *d.* very near, which together with the said 94 *l.* 6 *s.* 9 ½ *d.* makes the 100 *l.* the debt first propounded to be discharged by rebate.

Example 2. If 150 *l.* 10 *s.* be payable at the end of 73 days to come, how much present money will discharge the said debt, by rebating after the rate of 6 *per cent. per annum*? *Answ.* 148 *l.* 14 *s.* 3 ½ *d.* + as by the following operation is manifest.

	days	l.	days	l.
I.	365 .	6 ::	73 .	1.2

	l.	l.	l.	l.
II.	101.2 .	100 ::	150.5	148.7154 +

That is to say, First, I seek by a single *Rule of Three* the interest of 100 *l.* for 73 days, at the rate of interest propounded, saying, if 365 days (or a year) gain 6 *l.* what will 73 days gain? *Answ.* $1\frac{2}{10}$ *l.* or 1.2 *l.* Then adding the said 1.2 to 100, I say, by

by a second *Rule of Three*, if 101. 2 *l.* principal and interest, payable at the end of 73 days to come, be equivalent to 100 *l.* ready money, what ready money is 150 *l.* 10 *s.* (or 150. 5) payable at the end of 73 days to come equivalent unto? So by multiplying and dividing (according to the rules of Decimal Multiplication and Division explained in Chapter 26 and 27 of the preceding Book) the quotient or answer of the question will be found 148.7154 +, that is, 148 *l.* 14 *s.* 3½ *d.* + for the decimal .7154 being valued according to the brief way at the end of the fourth rule of the 26th Chapter, will by inspexion only be discovered to be 14 *s.* 3½ *d.* which rule I shall here once for all, advise the Learner to be well acquainted with.

The Proof.

Seek (by the *Rule of Three*) what the ready money found as aforesaid will gain, in so much time as it is paid before-hand at the rate of interest propounded; then having added this gain to the said ready money, if the sum be equal to the debt first propounded to be satisfied by rebate, the ready money was rightly found out. So the last example will be thus proved,

l.	l.	l.	l.
100.	1 . 2 ::	148.7154 .	(1.7845

Which fourth proportional 1.7845 being added to 148.7154, the sum will be 150.4999 +, which doth not want a farthing of 150 *l.* 10 *s.* the debt first propounded.

Of the present worth of Annuities by discompting at simple interest.

XI. When it is required to find the present worth of an annuity, by rebating or discompting at a given rate of simple interest, the operation will be as in the following example, *viz.* How much present money is equivalent to an annuity or rent of 100 *l. per annum* to continue five years, rebate being made at the rate of 6 *l.* for 100 *l.* for one year, at simple interest? *Answ.* 425 *l.* 18 *s.* 9 ½ *d.* very near.

It is manifest that there must be computed the present worth of 100 *l.* due at the first years end; also the present worth of 100 *l.* due at the second years end, and in like manner for the third, fourth and fifth years; all which particular present worths being added together, the aggregate or sum will be the totall present Worth of the Annuity, to wit in the example above propounded, $425\frac{8286150}{8821267}$ *l.* that is, 425 *l.* 18 *s.* 9½ *d.* very near.

The operation by decimals (which will come near enough to the truth) will be as followeth, *viz.*

	l.	*l.*	*l.*	*l.*
1.	106 .	100 ::	100 .	94,33962 +
2.	112 .	100 ::	100 .	89,28571 +
3.	118 .	100 ::	100 .	84,74576 +
4.	124 .	100 ::	100 .	80,64516 +
5.	130 .	100 ::	100 .	76,92307 +
			Answ.	425,93933 +

Here

Here by the way, from the manner of resolving the last mentioned question, that *Rule* commonly called *Equation of Payments*, which is insisted on by divers *Arithmetical Writers*, will be found erroneous, which I thus prove.

1. Since that rule aims at the reducing of several days of payment, upon which particular sums of money are due, unto a mean time upon which the aggregate or total of those particular sums ought to be paid, without damage to the *Debitor* or *Creditor*, there must be necessarily some rate of interest implied; for otherwise why may not any day at pleasure be assigned for one intire payment?

2. If some rate of interest be implied, then equity requires that the present worth of the total sum payable at one entire payment, rebate or discompt being made according to that rate of interest, may be equal to the sum of the present worths of the particular sums of money, rebate being made at the same rate of interest.

3. In regard the said *Rule* doth mention no particular rate of interest, it ought to be true at any rate of interest whatsoever.

4. Let us therefore examine the said *Rule* according to the rate of 6 *per cent. per annum*, simple interest, by taking the last mentioned question for an example, which (according to the accustomed manner) will be thus stated, *viz.* If 500 *l.* ought to be paid by five equal yearly payments, to wit, 100 *l.* at each years end, what time ought to be given for the payment of the said 500 *l.* at one entire payment, without loss either to the *Debitor* or *Creditor*.

5. By proceeding according to the said rule of *Equation of Payments* (which saith, if the sum of the

products

products, arising from the multiplication of each particular sum of money by its respective time, be divided by the sum or aggregate of the said particular sums of money, the quotient will be the mean time to be assigned for one entire payment) there will be found three years, which time (according to the said rule) ought to be given for the payment of the whole 500 *l.*

6. Now if 500 *l.* due at the end of three years to come be worth as much in present money, as is the present worth of an *Annuity* of 100 *l.* to continue five years, then the said *Rule of Equation* is true; otherwise false; but the present worth of 500 *l.* due at the end of three years to come, rebate being made at the rate of 6 *per centum, per annum, simple interest*, will be found (by the tenth rule of this Chapter) to be 423 *l.* 14 *s.* 6 *d.* 3 *f.* very near; also the present worth of the said *Annuity*, rebate being made as before, is found (as appeareth by the resolution of the last mentioned question) to be 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near; wherefore it is evident that the *Creditor* loseth 2 *l.* 4 *s.* $2\frac{3}{4}$ *d.* very near, by receiving the whole 500 *l.* at three years end: Moreover at 6 *per cent. per annum, compound interest*, he would lose 1 *l.* 8 *s.* 6 *d.* very near, as will be manifest by the *Tables of compound interest* hereafter expressed: So that the loss will be either more or less according as the rate of interest doth differ: And therefore I conclude the said Rule (as also all other rules or resolutions of questions which have dependance thereon) to be erroneous.

Although questions of this nature seldom come into practice, yet he that will take the pains, may find out such a mean time as is required by the said *Rule*

Rule of Equation of *payments*, at any rate of simple interest by this following rule, *viz.*

First, By the preceding tenth Rule of this Chapter find out the present worth of every particular sum in the question payable at a time to come, by rebating at the rate of interest agreed on; then find in what time the sum of those present worths will be augmented unto the total of all the particular sums payable at times to come, according to the first agreement, so shall the time found out be the mean time for the payment of the whole debt: thus the mean or equated time in the last example will be found to be 2.8979, *&c.* years (not three years, as the said *Rule* of *Equation* of *payments* would have it) for by rebating at 6 *per cen. per annum*, simple interest, 500 *l.* payable at the end of 2.8979, *&c.* years to come (that is 2 years and 328 days very near) is worth in ready money 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near, and the same ready money is also the present value of 100 *l.* Annuity for 5 years, at the same rate of interest, as before hath been manifested. But to return to the path from which I have made a digression.

From the preceding tenth rule of this Chapter the following *Tables* I. and II. are deduced, whose construction and use are afterwards declared.

Years	*Table* I. Which sheweth in decimal parts of a pound, the prsent worth of one pound due at the end of any number of years to come, not exceeding 7 years, at the rate of 6 *per centum*, *per annum*, simple interest.	Years	*Table* II. Which sheweth in pounds and decimal parts of a pound, the present worth of one pound Annuity, to continue any number of years not exceeding 7, at the rate of 6 *per centum*, *per annum*, simple interest.
1	.943396	1	. 943396
2	.892857	2	1 . 836253
3	.847457	3	2 . 683710
4	.806451	4	3 . 490162
5	.769230	5	4 . 259393
6	.735294	6	4 . 994687
7	.704225	7	5 . 698912

The Construction of Table I.

The numbers in the first Table which are placed right against the numbers of years 1, 2, 3, 4, 5, 6, and 7, are decimal fractions, one pound of English money being the Integer, and are thus found (according to the preceding tenth Rule of this Chapter) *viz.*

106 . 100 : : 1 . ,943396 +
112 . 100 : : 1 . ,892857 +
118 . 100 : : 1 . ,847457 +

whereby

whereby it appears, that 1 *l.* due at the end of a year to come, is worth in ready money .943396 +, that is, 18 *s.* 10 *d.* 1 *f.* and somewhat more. Also 1 *l.* due at the end of two years to come, is worth in ready money .892857 +, or 17 *s.* $10\frac{1}{4}$ *d.* rebate being made at the rate of 6 *per centum*, *per annum*, simple interest, the like is to be understood of the rest of the numbers in *Table* I. which may be continued to more years, and other *Tables* also of rebate may be framed upon the same ground, for months, or days, by the ingenious Artist.

The use of Table I.

The practical use of the said first Table will be manifest by solving this following question; *viz.* How much ready money will discharge 345 *l.* 15 *s.* 6 *d.* due at the end of five years to come, by rebating simple interest at the rate of 6 *per centum*, *per annum*? *Answer*, 265 *l.* 19 *s.* $7\frac{1}{4}$ *d.* which is thus found out; *viz.* In the preceding 5 *Table* I. right against 5 years, I find the decimal .76923, which shews that 1 *l.* due at the end of five years to come is worth in ready money .76923 (that is, 15 *s.* $4\frac{1}{2}$ *d.*) then instead of 15 *s.* 6 *d.* mentioned in the question propounded, taking the decimal .775 which is equal to 15 *s.* 6 *d.* (the same being reduced according to the fifth Rule of the 23 Chapter of the preceding Book) I say, by the *Rule of Three*,

1 . ,76923 : : 345.775 . (265.9805 +

That is to say, if 1 *l.* give .76923 *l.* what will 345 .775 *l.* give? *Answ.* 265.9805 *l.* for multiplying 345 .775 by .76923, according to the second Rule of the 26 Chapter of the preceding Book, the product will be 265.9805, that is, 265 *l.* 19 . . $7\frac{1}{4}$ *d.*

 The

The Construction of Table II.

The numbers in the second *Table* are found out by the addition of those in the first, *viz.* the first number in the latter *Table* is the same with the first number in the former, the second in the latter is the sum of the first and second in the former; the third in the latter is the sum of the first, second and third in the former, and in that manner the rest are found; (the reason of which composition is manifest from the example of the eleventh rule aforegoing;) otherwise the numbers in *Table* II. may be found more easily thus, *viz.* the first number in the said *Table* II. is the same with the first number in *Table* I. the second number in the latter *Table* is compos'd of the second number in the former and the first in the latter, the third number in the latter *Table* is compos'd of the third number in the former and the second in the latter, the fourth in the latter is compos'd of the fourth in the former and the third in the latter; the like is to be understood of the rest of the numbers in *Table* II. which might be continued to more years, and fitted to other rates of interest, but I shall spare that labour, in regard a more equal way of finding out the present worth of an Annuity, agreeable to the accustomed and practical rates of buying and selling Annuities of Rents, for terms of years, is grounded upon a computation of interest upon interest as will hereafter be made manifest, for at simple interest an Annuity will be overvalued.

The use of Table II.

The use of *Table* II. will appear by this following example; *viz.* What is the present worth of an Annuity of 100 *l.* *per annum* payable yearly during the term of five years, discompt or rebate being made at the rate of 6 *per centum*, *per annum*, simple interest? *Answer*, 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near, which is thus found out, *viz.* In the preceding *Table* II. right against five years, I find this number 4.259393, which shews that an Annuity of 1 *l.* payable yearly during five years, is worth in ready money 4.259393 *l.* (that is 4 *l.* 5 *s.* 2 *d.* and somewhat more) therefore, I say, by the *Rule of Three*;

l.		*l.*		*l.*		*l.*
1	.	4.259393	: :	100	.	(425.9393

That is to say, if 1 *l.* give 4.259393 *l.* what will 100 *l.* give? *Answer* 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near, for by multiplying 4.259393 by 100, the product (according to the second rule of the 26 Chapter of the preceding Book) is 425.9393, that is, 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near. Which operation being compared with the manner of solving the same question before mentioned in the eleventh Rule of this Chapter, the great benefit of Tables of this kind in point of expedition will be apparent.

Of the forbearance of Money at compound interest.

XII. When it is required to know, unto what sum of money any propounded principal forborn any number of years will at the end of such term be augmented unto, interest upon interest being computed at a given rate, there must be found a rank of continual proportionals, more in number by one than is the number of years in the question; of which proportionals the first is the principal assigned, the second must increase or

or proceed from the first, the third from the second, &c. in such manner or rate, as 106 proceeds from 100 (or as 108 from 100, if the rate of interest be 8 *per centum*) then will the last proportional be the Answer of the question: So if 300 pounds principal money be put forth at interest upon interest, at the rate of 6 *l.* for 100 *l.* for one year, and all forborn until the end of 4 years, there will then be due 378.743088, or 378 *l.* 14 *s.* $10\frac{1}{2}$ *d.* very near, as by the four following *Rules of Three* is manifest.

$$100 . 106 :: \begin{cases} 300 . 318 \\ 318 . 337.08 \\ 337.08 . 357.3048 \\ 357.3048 . 378.743088 \end{cases}$$

For the said 300 *l.* will at the first years end be augmented unto 318 *l.* which 318 *l.* being put forth as a *principal* for 1 year, will (at the second years end) be augmented unto 337.08, again this 337.08 being put forth as a *principal* for 1 year, will (at the third years end) be augmented unto 357.3048, in like manner 357.3048 being put forth as a *principal* for 1 year, will (at the fourth years end) be augmented unto 378.743088, which is the number required by the question. And if the work be well examined, it will appear (as was before declared) that the *principal* first assigned, to wit 300 *l.* and the numbers resulting successively at the ends of the several years are *continual proportionals, viz.* these five numbers are so qualified, that if the second be mul-

300 | 318 | 337.08 | 357.3048 | 378.743088

tiplied

tiplied by it self, the product will be equal to the product of the first and third; also if the third be multiplyed by it self, the product will be equal to the product of the second and fourth; in like manner, if there were more *continual proportionals* in a rank, if any one proportional which is placed between two next on each side of such one, be multiplied by it self, the product will be equal to the product of those two extreams (which is a property peculiar to continual proportionals.)

Two numbers being given to find a third, a fourth, a fifth, &c. in continual proportion.

Note here by the way, that if any two numbers be propounded, suppose 300 and 318, and it be required to find to them a third, a fourth, a fifth, &c. in continual proportion, multiply the second proportional 318 by it self, and divide the product 101.124 by the first proportional 300, so shall the quotient 337.08 be a third in continual proportion; In like manner if you multiply the third proportional 337.08 by it self, and divide the product 113622.9264 by the second proportional 318 the quotient 357.3048 shall be a fourth in continual proportion, and after the same manner a fifth, a sixth, or as many as you please may be found out.

From what hath been said by way of explication of the preceding twelfth Rule, the following *Table* III. is deduced, the construction and use whereof is afterwards declared.

TABLE

TABLE III.

Which sheweth what one Pound will amount unto, being forborn unto the end of any term of years under 31, compound interest being computed yearly, at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	1.08160	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.23216	1.25440
3	1.12486	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100	1.36763	1.40492
4	1.16985	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410	1.51807	1.57351
5	1.21665	1.27628	1.33823	1.40255	1.46932	1.53862	1.61051	1.68505	1.76234
6	1.26531	1.34009	1.41851	1.50073	1.58087	1.67710	1.77156	1.87041	1.97382
7	1.31593	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871	2.07616	2.21068
8	1.36856	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358	2.30453	2.47596
9	1.42331	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794	2.55803	2.77307
10	1.48024	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374	2.83942	3.10584
11	1.53945	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311	3.15175	3.47854
12	1.60103	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842	3.49849	3.89597
13	1.66507	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227	3.88328	4.36349
14	1.73167	1.97993	2.26090	2.57853	2.93719	3.34172	3.79749	4.31044	4.88711
15	1.80094	2.07892	2.39655	2.75903	3.17216	3.64248	4.17724	4.78458	5.47356

A continuation of the preceding Table III.

Years	4	5	6	7	8	9	10	11	12
16	1.87298	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497	5.31089	6.13039
17	1.94790	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447	5.89509	6.86604
18	2.02581	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991	6.54355	7.68996
19	2.10684	2.52695	3.02559	3.61652	4.31570	5.14166	6.11590	7.26234	8.61276
20	2.19112	2.65329	3.20713	3.86968	4.66095	5.60441	6.72749	8.06231	9.64629
21	2.27876	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024	8.94916	10.80384
22	2.36991	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027	9.93357	12.10031
23	2.46471	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430	11.02626	13.55234
24	2.56330	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973	12.23915	15.17862
25	2.66583	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470	13.58546	17.00006
26	2.77246	3.55567	4.54938	5.80735	7.39635	9.39915	11.91817	15.07986	19.04007
27	2.88336	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999	16.73864	21.32488
28	2.99870	3.92012	5.11168	6.64883	8.62710	11.16713	14.42099	18.57990	23.88386
29	3.11865	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309	20.62369	26.74993
30	3.24330	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940	22.89229	29.55992

The Construction of the preceding Table III.

The numbers 1, 2, 3, 4, &c. to 30, in the first column on the left hand signifie years; the numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, placed at the head of the rest of the columns signifie rates of interest, for 100*l.* lent for a year, and the numbers placed in the several columns underneath those rates of interest, are found out by the *Rule of Three* in decimals, in manner following, *viz.*

I.	100 . 104 :: 1 :: (1.04
II.	100 . 104 :: 1.04 :: (1.0816
III.	100 . 104 :: 1.0816 :: (1.12486

That is to say, First if 100 *l.* put forth at interest for a year be augmented to 104 *l.* at the years end, what will 1 *l.* be then augmented unto at the same rate? *Answ.* 1.040 *l.* (that is 1 *l.* 0 *s.* 9 *d.* 2 *f.* and somewhat more) which 1.04 (or 1.04000, the cyphers after the 4 being of no value in decimals) is the first number in the second column belonging to 4 *per cent.* and is placed right against 1 year in the first column.

Secondly, say if 100 *l.* lent for a year be augmented to 104 *l.* at the years end, what will 1.04 *l.* be then augmented unto at the same rate? *Answ.* 1,0816 *l.* (that is 1 *l.* 1 *s.* 7 *d.* 2 *f.*+) which 1.0816 is the second number in the said column of 4 *per cent.* and is placed right against 2 years in the first column.

Thirdly, as 100 is to 104, so is 1.0816 to 1.124864 (or 1 *l.* 2 *s.* 5 *d.* 2 *f.*+) which 1.12486 is the third number in the column of 4 *per cent.* and is placed right against 3 years in the first column. Hence it appears, that 1 *l.* at 4 *per cent. per annum* compound interest, will at the end of 3 years be augmented unto 1.124864 *l.* (that is, 1 *l.* 2 *s.* 5 *d.* 2 *f.* and somewhat more.)

After the same manner the rest of the numbers in the second column, as also in the other columns are found out (*mutatis mutandis.*)

The use of the preceding third Table.

Quest. 1. What will 136 *l.* 15 *s.* 6 *d.* be augmented unto, being forborn 20 years, interest upon interest being computed at the rate of 6 *per cent. per annum*? *Answ.* 438 *l.* 13 *s.* 1 *d.* very near, which is thus found out.

First, looking into the fourth column of the said third *Table*, to wit, that column which hath the figure 6 placed at the head of it, I find right against 20 years the number 3.20713, which shews that 1*l.* being continued 20 years at 6 *per cent. per annum*, compound interest, and all forborn until the end of the said term will be augmented unto 3.20713 *l.* (that is 3*l.* 4*s.* 1*d.* 2*f.* and somewhat more) therefore after the 15 *s.* 6 *d.* in the question is reduced to the decimal .775 (by the sixteenth rule of the 23 Chapter of the preceding Book) I multiply the said tabular number 3.20713 by 136.775 (the sum propounded in the question) according to the second rule of the 26th Chapter, so the product is

found to be 438.665, *&c.* that is, 438 *l.* 13 *s.* 1 *d.* for the answer of the question. View the operation here following.

```
1 . 3.20713 :: 136.775 . (438.665 +
    136.775
    -------
     1603565
    2244991
   2244991
  1924278
 962139
320713
-----------
438|65520575
```

Quest. 2. If 320 *l.* be forborn 11 years, at interest upon interest at 5 *per centum, per annum,* what will be due at the end of those eleven years for principal and interest? *Answer,* 547 *l.* 6 *s.* 1 *d.* +. For in the third column of the third Table, under the figure 5 at the head of the column and right against 11 years you will find this number 1.71033, which shews that 1 *l.* at the end of 11 years will at five *per centum, per annum*, compound interest, be augmented to 1.71033 (that is 1 *l.* 14 *s.* 2 *d.* 1 *f.* and somewhat more) wherefore by multiplying the said 1.71033 by 320 the number of pounds propounded in the question) the product will be 547.305, *&c.* that is 547 *l.* 6 *s.* 1 *d.* + for the answer of the question. See the following operation:


```
1 . 1.71033 :: 320 . (547305 +
    320
    -------
    3420660
   513099
   --------
547|30560
```

After the same manner the numbers belonging to any of the other rates of interest mentioned in the third Table are to be used.

The manner of summing up Annuities in arrear with allowances of interest upon interest.

XIII. When an Annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount unto, compound interest being computed for each particular Annuity from the time it became due until the end of the term of years, the work will be as in the following example; *viz.* Suppose an Annuity of 300 *l.* payable at yearly payments be forborn, and all unpaid until the end of four years, the question is, what will then be due, compound interest being computed at the rate of 6 *per centum, per annum,* for each yearly payment from the time it becomes due to the end of the said term of four years? *Answer,* 1312 *l.* 7 *s.* 8 *d.* very near.

It is evident by the question, that there must be computed what 300 *l.* due at the third years end will be augmented unto in one year (to wit, the fourth year) at 6 *per centum*; Also what 300 *l.* due at the second years end will be augmented unto in two years (to wit, the third and fourth years;) like-

wise what 300*l.* due at the first years end, will be augmented unto, in the three following years (to wit the second, third and fourth years) all which sums being added to 300*l.* (the payment due at the end of the fourth year, which is incapable of any improvement) the aggregate or sum will be the total money in Arrear at the end of the fourth year, to wit, 1312, $\frac{3848}{10000}$*l.* as may appear by the following operation, *viz.*

	l.
The last payment of the Annuity due at the end of the fourth year is	300.
Again, the 300*l.* due at the third years end, will in one year after the rate of 6 *per centum*, be augmented unto	318.
Also 300*l.* due at the second years end, will in two years at the rate of 6 *per centum*, *per annum*, compound interest, be augmented unto (as appears by the first example of the twelfth Rule aforegoing.)	337.08
In like manner; 300*l.* due at the first years end, will in three years be augmented unto	357.3048
The summ due at the four years end	1312.3848

The invention of the numbers before-mentioned being well examined, it will appear, that if an Annuity or Rent payable at yearly payments be improved to the utmost as interest upon interest, and all forborn or respited unto the end of certain years, the total then due will be the sum of a rank of continual proportionals as many in number as there are yearly payments, the first of which proportionals is the first (or any one) years rent, and the second proportional proceeds from the first in the same rate as 106 proceeds from 100, if the rate of interest be 6 *per centum*, (or as 108 proceeds from 100, if the rate of interest be 8 *per centum*, &c.) and so likewise the third from the second, the fourth from the third, &c. (after the manner of the operation in the first example of the twelfth Rule of this Chapter.)

Otherwise.

Find a principal which may have such proportion to 300 as 100 hath to 6, and say by the *Rule of Three*,

6 . 100 :: 300 . 5000.

That is to say, as 6*l.* interest hath 100*l.* for a principal, so 300*l.* interest hath 5000*l.* for a principal; then seek what 5000*l.* will be augmented unto, being forborn four years at 6 *per centum*, *per annum*, compound interest (after the manner of the first example of the twelfth rule aforegoing;) so will you find 6312.3848, from which subtracting the said principal 5000*l.* the remainder (as before) is 1312.3848*l.* being the sum which 300*l.* Annuity will be augmented unto at the end of four years, according to the said rate of interest, the Annuity being payable at yearly payments.

The reaſon of the latter Rule.

If a principal be put forth at intereſt upon intereſt payable by yearly payments, and all be forborn until the end of certan years, the total then due is equal to the aggregate or ſum of theſe three numbers, to wit, the ſaid principal firſt put forth; the ſum of the annual ſimple intereſts of that principal; and the utmoſt improvement of thoſe ſimple intereſts by computing intereſt upon intereſt; wherefore if from the ſaid aggregate the firſt principal be ſubtracted, the remainder muſt neceſſaarily conſiſt of the ſum of the annual ſimple intereſts, (which are in the nature of an Annuity) and the utmoſt improvement of thoſe ſimple intereſt (or Annuity) by computing intereſt upon intereſt.

The Conſtruction of the following Table IV.

Upon the aforeſaid grounds, the following *Table* IV. is calculated, to ſhew what one pound Annuity, payable at yearly payments, and forborn any number of years under 31, will amount unto by computing intereſt upon intereſt at any of the rates expreſt at the head of the ſaid Table.

But the ſame *Table* may be more eaſily compoſed by the addition of the numbers in the preceding *Table* III. in this manner, *viz.* the firſt number in each of thoſe columns in the following *Table* IV. at the head whereof are placed the numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, ſignifying rates of intereſt *per centum*, is 1 or unity, the ſecond number in each of theſe columns in the latter *Table* is compos'd of 1 or unity, and the firſt number in the reſpective columns of the ſaid preceding *Table* III.

Alſo the third number in each of the ſaid columns of this *latter Table* is compos'd of 1, and the ſum of the firſt and ſecond number of the reſpective columns or the *former Table*, and in that order the reſt are found out; or more eaſily thus: the third number in the *latter Table* is compos'd of the ſecond number in the *latter*, and of the ſecond in the *former*; the fourth number in the *latter* is compos'd of the third in the *latter*, and of the third in the *former*, &c. But you are to obſerve that according to either of theſe ways of compoſing the fourth *Table* by Addition, the numbers in the preceding *Table* III. ought to be continued to more places than are there expreſt to prevent error which may happen by adding of defective decimal fractions.

TABLE IV.

Which sheweth what one Pound Annuity, payable by yearly payments, and forborn any number of years under 31, will amount unto, at the end of the term, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years	4	5	6	7	8	9	10	11	12
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.09000	1.00000	1.00000	1.00000
2	2.04000	2.50000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	3.24646	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100	4.70973	4.77932
5	5.41632	5.52563	5.63709	5.75072	5.86660	5.08471	6.10510	6.22780	6.35284
6	6.63297	6.80190	6.97531	7.15329	7.33592	7.52333	7.71561	7.91285	8.11518
7	7.89829	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21422	9.64900	9.89746	10.25980	10.63662	11.02847	11.43588	11.85943	12.29969
9	10.58279	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947	14.16397	14.77565
10	12.00610	12.57789	13.18079	13.81644	14.48656	15.19292	15.93742	16.72200	17.54873
11	13.48635	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116	19.56142	20.65458
12	15.02580	15.91712	16.86994	17.88845	18.97712	20.14071	21.38428	22.71318	24.13313
13	16.62683	17.71298	18.88213	20.14064	21.49529	22.95338	24.52271	26.21163	28.02910
14	18.29191	19.59863	21.01506	22.55048	24.21492	26.01918	27.97498	30.09491	32.39260
15	20.02358	21.57856	22.27596	25.12902	27.15211	29.36091	31.77248	34.40535	37.27971

A continuation of the preceding Table IV.

Years	4	5	6	7	8	9	10	11	12
16	21.82453	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972	39.18993	42.75328
17	23.69751	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917	50.39593	55.74971
19	27.67120	30.53900	33.75999	37.37896	41.44626	46.01845	51.15009	56.93948	63.43968
20	29.77807	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499	64.20283	72.05244
21	31.96920	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249	72.26514	81.69873
22	34.24796	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274	81.21430	92.50258
23	36.61788	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302	91.14788	104.60289
24	39.08260	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732	102.17415	118.15524
25	41.64590	47.72709	54.86451	63.24903	73.10599	84.70087	98.34705	114.41330	133.33387
26	44.31174	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176	127.99877	150.33393
27	47.08421	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994	143.07863	169.37400
28	49.96758	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993	159.81728	190.69888
29	52.96628	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092	178.39718	214.58275
30	56.08493	664.3884	79.05818	94.46078	113.28321	136.30753	164.49402	199.02087	241.33268

The use of the preceding Table IV.

The use of the said fourth *Table* will be manifest by the manner of solving this Question, *viz.* if an Annuity of 20 *l.* payable by yearly payments for 15 years; be all forborn or unpaid untill the end of the said term, what will it then amount unto, upon a computation of interest upon interest, at the rate of 6 *per centum, per annum*? *Answ.* 465*l.* 10*s.* 4*d.* 2*f.* very near, as by the following operation is evident; For in the column belonging to 6 *per centum* (to wit, that column which hath the figure 6 placed at the head of it) right against 15 years, you will find 23.27596, which shews that an Annuity of 1*l.* payable at yearly payments for 15 years, will at the end of the said term (compound interest being computed at 6 *per cent. per annum*) amount unto 23.27596 *l.* (or 23 *l.* 5 *s.* 6 *d.*+) Therefore multiplying the said tabular number 23.27596 by 20. (20 because the Annuity propounded is 20 *l.*) the product will be 465.519 +, that is 465 *l.* 10 *s.* 4 *d.* 2 *f.* which is the answer of the question; view the following operation.

$$1 \;.\; 23.27596 \;::\; 20 \;.\; (465,519+$$

$$\begin{array}{r} 23.27596 \\ 20 \\ \hline 465|51920 \end{array}$$

In the same manner the numbers in the other column are to be used.

Of rebate at Compound interest.

XIV. When a sum of money is due at a time to come, and it is required to know what it is worth in ready money, rebate being made at a given rate of compound interest, the work will not be much different from the 12 Rule of this Chapter, *viz.* there must be found a series or rank of continual proportionals more in number by one, than is the number of years in the question; of which proportionals, the first is the money propounded to be rebated, the second must decrease or lessen from the first, the third from the second, &c. in such manner or rate as 100 decreaseth from 106 (or as 100 from 108, if the rate of interest be 8 *per cent.*) then will the last proportional be the answer of the question: So if 378 $\frac{743088}{1000000}$ *l.* be due at the end of four years wholly to come, it will be found to be worth in ready money 300 *l.* rebate being made at compound interest at 6 *per cent.* as by the four following *Rules of Three* is manifest, which may be proved by the preceding twelfth rule, where it will appear that 300 *l.* being forborn four years, will at the said rate of compound interest be augmented unto 378.743088 *l.*

$$106 \;.\; 100 \;::\; \begin{cases} 378.743088 & .\; 357.3048 \\ 357.3048 & .\; 337.08 \\ 337.08 & .\; 318. \\ 318. & .\; 300. \end{cases}$$

(columns headed *l.* and *l.*)

Upon this ground the following *Table* V. is calculated, to shew what one pound due at the end of any number of years to come, is worth in present money, rebate being made at the rates of compound interest, mentioned in the said *Table*; by the help whereof and of *Multiplication*, questions of rebate for any sum propounded may be performed without considerable error.

TABLE

TABLE V.

Which sheweth what one pound, payable at the end of any term of years to come under 31, is worth in ready mony, discompt or rebate being yearly computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per ann. *compound interest.*

Years.	4	5	6	7	8	9	10	11	12
1	.961538	.952381	.943396	.934579	.925925	.917431	.909090	.900900	.892857
2	.924556	.907029	.889996	.873438	.857338	.841680	.826446	.811622	.797193
3	.888996	.863837	.839619	.816297	.793832	.772183	.751314	.731191	.711780
4	.854802	.822702	.792093	.762895	.735029	.708425	.683013	.658731	.635518
5	.821927	.783526	.747258	.712986	.680583	.649931	.620921	.593451	.567426
6	.790314	.740215	.704960	.666342	.630169	.596267	.546447	.534640	.506631
7	.759917	.710681	.665057	.622749	.583490	.547034	.513158	.481658	.452349
8	.730690	.676839	.627412	.582009	.540268	.501866	.466507	.433926	.403883
9	.702586	.644608	.591898	.543933	.500248	.460427	.424097	.390924	.360610
10	.675564	.613913	.558391	.508349	.463193	.422410	.385543	.352184	.321973
11	.649580	.584679	.526787	.475092	.428882	.387532	.350494	.317283	.287476
12	.624596	.556837	.496989	.444012	.397113	.355534	.318630	.285840	.256675
13	.600573	.530321	.468839	.414964	.367697	.326178	.289664	.257514	.229174
14	.577474	.505067	.442300	.387817	.340461	.299246	.263331	.231994	.204619
15	.555264	.481017	.417265	.362446	.315241	.274538	.239392	.209004	.182696

A continuation of the preceding Table V.

Years	4	5	6	7	8	9	10	11	12
16	.533908	.458111	.393646	.338734	.291860	.251869	.217629	.188292	.163121
17	.513373	.436296	.371364	.316574	.270269	.231073	.197844	.169632	.145644
18	.493628	.415520	.350343	.295864	.250249	.211993	.179858	.152822	.130039
19	.474642	.395733	.330512	.276508	.231712	.194489	.163508	.137677	.116106
20	.456386	.376889	.311804	.258419	.214548	.178430	.148643	.124034	.103666
21	.438833	.358942	.294155	.241513	.198655	.163698	.135130	.111742	.092559
22	.421955	.341849	.277505	.225713	.183940	.150181	.122840	.100668	.082642
23	.405726	.325571	.261797	.210947	.710315	.137781	.111678	.090692	.073787
24	.390121	.310067	.246978	.197146	.157699	.126405	.101525	.081705	.065882
25	.375116	.295302	.232998	.184249	.146018	.115967	.092296	.073608	.058823
26	.360689	.281240	.219810	.172195	.135201	.106392	.083905	.066313	.052520
27	.346816	.267848	.207367	.160930	.125186	.097607	.076277	.059742	.046893
28	.333477	.255093	.195630	.150402	.115913	.089548	.069343	.053821	.041869
29	.320651	.242946	.184556	.140562	.107327	.082154	.063039	.048487	.037383
30	.308318	.231377	.174110	.131367	.099377	.075371	.057308	.043682	.033377

The Construction of the preceding Table V.

The numbers 1,2,3,4, *&c.* to 30, in the first column on the left hand, signifie years; the numbers 4, 5, 6, 7, 8, 9, 10, 11 and 12, placed at the head of the rest of the columns signifie rates of interest for 100 *l.* lent for a year, and the numbers placed in the several columns underneath those rates of interest are found out by the *Rule of Three* in decimals, in manner following, *viz.*

I.	104.100 :: 1 . (,9615384615, *&c.*
II.	104.100 :: ,9615384615+ (,9245562, *&c.*
III.	104.100 :: ,9245562, *&c.* (,888996 +

That is to say, First, if 104 decrease to 100, or if 104 *l.* payable at the end of a year to come be worth 100 *l.* ready money, what ready money, is 1 *l.* due at the end of a year to come worth? *Answer*, .9615384615 + (or 19 *s.* 2 *d.* 3 *f.* very near) So that .961538 is the first decimal in the second column belonging to 4 *per centum*, in *Table* V. and is placed right against 1 year in the first column.

Secondly, say in like manner if 104 decrease to 100, what will .9615384615, *&c.* (the decimal found by the first Rule of Three) decrease unto? *Answ.* 9245562, *&c.* the first 6 places whereof, to wit, 924556 are the second decimal in the said column of 4 *per cent.* which is placed right against two years.

Thirdly,

Thirdly, as 104 is to 100; so is, 9245562, *&c.* (the decimal found by the second *Rule of three)* to .888996 + (or 17 *s.* 9 *d.* 1 *f.* +) which is the third decimal in the column of 4 *per centum*. Hence it appears, that 1 *l.* due at the end of 3 years to come is worth .888996 + (or 17 *s.* 9 *d.* 1 *f.* and somewhat more) in ready money, rebate being made at the rate of 4 *per centum, per annum,)* compound interest.

After the same manner the rest of the decimal fractions in the said second column, as also in the other columns are found out (*mutatis mutandis.*)

The use of the preceding Table V.

To exemplifie the said fifth Table, let it be required to find out how much ready money will discharge a debt of 356 *l.* payable at the end of seven years to come, by rebating at the rate of 7 *per centum, per annum*, compound interest? *Answ.* 221 *l.* 13 *s.* 11 *d.* 3 *f.* very near. For in the fifth colum, at the head whereof is placed 7, signifying 7 *per centum*, right against 7 years, I find .622749, which shews that 1 *l.* due at the end of 7 years to come is worth in present money .622749 decimal parts of a pound, rebate being made at the said rate of compound interest. Therefore multiplying the said tabular number .622749 by the said 356 *l.* (the debt propounded) the product (according to the second rule of the 26th Chapter) will be 221.698, *&c.* that is, 221 *l.* 13 *s.* 11 *d.* 3 *f.* which is the Answer of the question. See the subsequent operation.

I.

1 : ,622749 :: 356 . (221,698 +

```
      356
   -------
   3736494
  3113745
 1868247
 ---------
 221|698644
```

In the same manner the numbers in the other columns are to be used.

To find the present worth of Annuities by a computation of compound interest.

XV. The finding out the present worth of an Annuity is grounded upon this foundation, to wit, if the present money which is paid for the purchase of Annuity, to continue any term of years, be put forth at any rate of compound interest, and all forborn until the end of the said term, and that the total money then due be put into one Scale: also if the total sum of the utmost improvements of the annual payments of the Annuity, put forth at the same rate of compound interest, from the time those annual payments become due until the end of the term, be put into the other Scale, the Scales must be even, *viz.* the said two total sums of money must be equal one to the other.

Now to find out such a present worth of an Annuity, there are divers ways, some of which I shall here explain by examples:

First therefore let it be required to find the present worth of an Annuity of 378.73088 *l.* to continue three years compound interest being computed at 6 *per cent. per ann.* *Answer*, 1012.3848 *l.*

It

It is evident by the question, that there must be computed (after the manner of the Example upon the fourteenth Rule aforegoing) the present worth of 378 $\frac{743088}{1000000}$ *l.* due at the first years end, also the present worth of the like sum due at the second years end, and in like manner for the third year; all which particular present values being added together, the aggregate or sum will be the total present worth of the Annuity propounded, *viz.*

	l.
378.743088 *l.* payable at the end of one year is worth in ready money (as is evident by the fourteenth Rule aforegoing.)	357.3048
Also the like sum payable at the end of 2 years to come is worth in ready money	337.08
Again, the like sum payable at the end of three years to come, is worth in ready money	318.
Therefore the total present worth of an Annuity of 378.74088 *l.* to continue 3 years is	10112.3848

Otherwise.

Find a principal which may be in such proportion to the propounded Annuity 378.743088 *l.* as 100 is to 6. Which will be exactly 9312.3848 *l.* for

6 . 100 :: 378.743088 . 6312, 3848

Then supposing this principal so found to be a sum due at the end of three years to come, find what it will be worth in ready money, by diminishing it according to the fourteenth Rule of this Chapter, so will you find 5300 *l.* for the ready money equivalent to the said 6312.3848 *l.* due at the

end

end of three years, which ready money 5300 *l.* being subtracted from the said 6312.3848*l.* leaves (as before) 1012.3848*l.* for the present worth of the said Annuity of 378.743088*l.* to continue three years, compound interest being allowed at 6 *per centum per annum.*

The Reason of the latter Rule.

It will not be difficult to apprehend, that if 6312.3848*l.* ready money be put forth as a Principal at interest upon interest, it will at three years end be augmented unto an Aggragate or sum compos'd of these three numbers, to wit, the said Principal 6312.3848; the sum of the annual sinple interests of that principal, and the utmost improvement of those simple interests by interest upon interest: And because (by the operation aforegoing) 5300*l.* ready money (part of the said ready money 6312.3848.) will at three years end be augmented unto 6312.3848*l.* part of the said Aggregate, therefore 1012.3848 *l.* the complement or remaining part of the said ready money 6312.3848*l.* must necessarily be augmented unto the complement or remaining part of the said Aggregate, which remaining part last mentioned is composed of the sum of the aforesaid simple interests, and of their utmost improvement at interest upon interest, that is, the said remainder is the utmost improvement of an Annuity of 378.743088*l.* to continue three years, compound interest being allowed at 6 *per centum, per annum.*

The

The Construction of the following Table VI.

Upon the aforesaid grounds the following Table VI. is calculated to shew how much ready money an Annuity of one pound to continue any number of years under 31. and payable at yearly payments, is worth, upon a computation of compound interest at any of the rates *per centum,* mentioned at the head of the said Table. But the said Table VI. may more easily be compos'd by the help of the preceding Table V. in this manner, *viz.* the first number in every of the Columns (except the Column of years) in the following Table VI. is the same with the first number in the like Columns respectively in the preceding Table V. the second number in each of the said Columns of the sixth Table is the sum of the first and second numbers in the respective Columns of the sixth Table; the third number in the said Columns of the fifth Table is the sum of the first, second and third numbers in the respective Columns of the fifth Table: Or yet more easily thus, the third number in the sixth Table, is composed of the third in the fifth Table and of the second in the sixth; the fourth number in the sixth Table is composed of the fourth in the fifth and of the third in the sixth; the like is to be understood of the rest. But you are to observe that according to this way of composing the sixth Table by Addition, the numbers of the fifth Table must be continued to more places than are there exprest, to prevent error arising by the addition of defective Decimal fractions.

 TABLE

TABLE VI.

Which sheweth the present worth of one Pound Annuity, to continue any term of years under 31, and payable by yearly payments, compound interest being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years	4	5	6	7	8	9	10	11	12
1	.96153	.95238	.94339	.93457	.92592	.91734	.90909	.90090	.89285
2	1.88609	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553	1.71252	1.69005
3	2.77509	2.72324	2.67301	2.62431	2.57709	2.53129	2.48685	2.44371	2.40183
4	3.62989	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986	3.10244	3.03734
5	4.45182	4.32947	4.21236	4.10019	3.99270	3.88965	3.79078	3.69589	3.60477
6	5.24213	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526	4.23053	4.11140
7	6.00205	5.78637	5.58233	5.38928	5.20636	5.03295	4.86841	4.71219	4.56375
8	6.73274	6.46321	6.20979	5.97129	5.74663	5.53481	5.34492	5.14612	4.96763
9	7.43533	7.10782	6.80169	6.51523	6.24688	5.99524	5.57901	5.53704	5.32824
10	8.11089	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456	5.88923	5.65022
11	8.76047	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20651	5.93769
12	9.38507	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369	6.49235	6.19437
13	9.98964	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335	6.74987	6.42354
14	10.56312	9.89864	9.29498	8.74546	8.24423	7.78614	7.36668	6.98186	6.62816
15	11.11838	10.37965	9.71224	9.10791	8.55947	8.06068	7.60608	7.19087	6.81086

A continuation of the preceding Table VI.

Years	4	5	6	7	8	9	10	11	12
16	11.65229	10.83776	10.10589	9.44664	8.85136	8.31255	7.82371	7.37916	6.97398
17	12.16560	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155	7.54879	7.11962
18	12.65929	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141	7.70161	7.24966
19	13.13393	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492	7.83929	7.46577
20	13.59032	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356	7.96332	7.46944
21	14.02915	12.82115	11.76407	10.83557	10.01680	9.29224	8.64869	8.07507	7.56200
22	14.45111	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154	8.17574	7.64464
23	14.85683	13.48857	12.30337	11.27218	10.37105	9.58020	8.88322	8.26643	7.71843
24	15.24696	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474	8.34813	7.78431
25	15.62207	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704	8.42174	7.84313
26	15.98276	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094	8.48805	7.89505
27	16.32958	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722	8.54780	7.94255
28	16.66305	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656	8.60162	7.98442
29	16.98371	15.14107	13.59071	12.27767	11.584	10.19828	9.36960	8.65011	8.02180
30	17.29292	15.39244	13.76482	12.40004	11.2577	10.27365	9.42691	8.69379	8.05518

The use of the preceding Table VI.

The use of the said sixth Table will appear by the manner of solving these two subsequent questions, *viz.*

Quest. 1. What is the present worth of an Annuity or rent of 56 *l. per annum* payable by yearly payments for 21 years, accompting interest upon interest at the rate of 6 *per centum, per annum*? *Answer*, 658 *l.* 15 *s.* 9 *d.* very near, thus found out; In the fourth Column of the preceding *Table* VI. under the figure 6 at the head, and right against 21 years, I find 11.76407, which shews that an Annuity of 1 *l.* payable by yearly payment for 21 years, is worth in present money 11.76407 *l.* (or 11 *l.* 15 *s.* 3 *d.* 1 *f.* and somewhat more) interest upon interest being computed on both sides at the rate of 6 *per centum, per annum*; therefore multiplying the said tabular number 11.76407 by 56, (56 because the Annuity propounded is 56 pound) the product (according to the second rule of the 26th Chapter of the preceding Book) will be found to be 658.787, &c. that is, 658 *l.* 15 *s.* 9 *d.* very near; Wherefore I conclude that the Answer of the question is 658 *l.* 15 *s.* 9 *d.* View the following operation.

1 . 11.76407 : : 56 . (658,787 +
56

7058442
5882035

658|78792

Quest. 2. What is the present worth of an annual rent of 45 *l.* payable by yearly payments for 21 years, interest upon interest being computed at 10 *per centum, per annum*? *Answ.* 389 *l.* 3 *s.* 10 *d.* very near; for in the Column of 10 *per centum*, in the said sixth *Table*, right against 21 years, and under 10, at the head I find this number 8.64869; which shews that at 10 *per centum*, compound interest, an Annuity or rent of 1 *l.* payable by yearly payments for 21 years, is worth in ready money 8.64869 *l.* that is 8 *l.* 12 *s.* 11 *d.* 3 *f.* therefore multiplying the said tabular number 8.64869, by 45 (the rent propounded) the product will be 389.191 +, that is 389 *l.* 3 *s.* 10 *d.* very near, which is the Answer of the Question.

1 . 8.64869 : : 45 . (389.191 +
45

4324345
3459476

389|19105

In the same manner the numbers in the other Columns of *Table* VI. are to be used.

Moreover the numbers in the said sixth *Table* will at first sight shew how many years purchase an Annuity to continue any number of years under 31 is worth, to be sold for present money, compound interest being computed on both sides, at any of the said rates 4, 5, 6, 7, 8, 9, 10, 11 and 12 *per centum*; so if you desire to know how many years purchase an Annuity issuing out of Lands for 21 years, to begin presently, is worth, if it were to be sold for ready money, when the current rate of interest is 6 *per centum*; Seek in the first Column of *Table* VI. for 21 years, and carry your eye from thence equidistant to the head-line of the *Table* till you come under 6, which (as before hath been said) signifies 6 *per centum*. So in the fourth Column you will find 11,76407, whereof you need only consider 11.76, which shews that the said Annuity is worth 11 years purchase, (or 11 times one years rent whatever it be) and 76 parts of one years purchase divided into 100 parts, or a 11 $\frac{3}{4}$ years purchase and a little more. The same annuity when money was at 8 *per centum* was worth 10 years purchase and about $\frac{1}{100}$ part of a years purchase more, as the number in the Column of 10 *per centum* right against 21 years will discover.

To find how many years purchase an Annuity or a Lease for years is worth.

In like manner supposing 10 *per centum* to be a fit rate to be allowed in the valuation of Leases of houses, the Lease of a house for 21 years will be found by the said *Table* to be worth 8 years purchase and $\frac{64}{100}$ parts of a years purchase, or 8 years purchase

purchase and an half, and half a quarter of a years purchase, and somewhat more; But note here, that in valuing of Leases, the rate *per centum* is to be set higher or lower according to the goodness of the thing leased, and the certainty or uncertainty of the rent.

XVI. When a sum of Money is propounded, and it is required to know what Annuity (to continue any number of years, and according to any given rate) that sum will buy, you may presuppose at pleasure an Annuity for the term of years propounded, and find the value of that Annuity in ready money (according to the fifteenth Rule aforegoing) at the rate assigned; then will the proportion be as followeth.

Of the purchase of Annuities at compound interest.

As the value found is in proportion to the supposed Annuity; so is the sum of money propounded, to the Annuity required.

So if it be required to find what Annuity to begin presently, and to continue three years 500*l.* in present money will purchase, compound interest being computed at 6 *per centum, per annum*: The *Answer* will be 187 *l.* 1 *s.* 1 *d.* very near.

For presupposing an Annuity at pleasure, to wit, 378,743088 *l.* payable yearly for 3 years, the value thereof in present money will (by the fifteenth Rule of this Chapter) be found to be 1112.3848 *l.* Therefore by the *Rule of proportion* say,

1012.3848 . 378,743088 :: 500 . 187,054.

That is to say, if 1012.3848 *l.* in ready money will buy an Annuity of 378.743088 *l.* (to continue three years) then 500*l.* in present money will purchase an Annuity (to continue the same term of years, and at the same rate of interest) of 187.054, *&c.* that is, 187 *l.* 1 *s.* 1 *d.* very near.

The Construction of the following Table VII.

Upon this ground the following *Table* VII. is calculated to shew what Annuity (to continue any term of years under 31, and at any rate of interest mentioned at the head of that *Table*) one pound will purchase, by which *Table*, and by the help of *Multiplication*, questions concerning the *purchace of Annuities, Rents* or *Pensions*, by any sum of ready money propounded, may be resolved without considerable error. But a more ready way to make the said *Table* VII. may be this following, *viz.*

Forasmuch as it is evident by the construction of the third *Table* aforegoing, that one pound ready money is equivalent unto 1.06*l.* payable at the end of a year to come, at the rate of 6 *per centum, per annum*; therefore this 1.06 is to be the first number in the Column intitled 6 *per centum* in the subsequent *Table* VII. Again, the present value of one pound Annuity to continue two years at the said rate will be found by the preceding *Table* VI. to be near 1.83339*l.* Therefore by the Rule of Proportion, say,

1.83339

1.83339 : 1 :: 1. 54543, *&c.*

That is, if 1.83339 *l.* ready money will purchase an Annuity of 1 *l.* (to continue two years; what Annuity to continue the same time will 1 *l.* in present money purchase? *Answer*, an Annuity of .54543 *l.* that is 10 *s.* 11 *d.* very near, to continue two years; therefore the said Decimal .54543 *l.* is to be placed as the second number in the fourth Column of the subsequent *Table* VII. Hence it follows, that if 1 or unity be divided by every one of the numbers in all the Columns of *Table* VI. except the first Column of years, the quotients will give the respective numbers to be placed in the like Columns of the following *Table* VII. in which operation it will be requisite, that the numbers in the preceding *Table* VI. be continued to more places than are there exprest, to prevent error that will arise by adding of defective decimals.

TABLE

TABLE VII.

Which sheweth what Annuity, payable by yearly payments, to continue any term of years under 31, one pound will purchase, compound interest, being computed at any of these rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per centum, per annum.

Years	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	.53019	.53780	.54543	.55309	.56076	.56846	.57619	.58393	.59169
3	.36034	.36720	.37411	.38105	.38803	.39505	.40211	.40921	.41634
4	.27549	.28209	.28859	.29519	.30192	.30866	.31547	.32232	.32923
5	.22462	.23097	.23739	.24389	.25045	.25709	.26379	.27057	.27740
6	.19076	.19701	.20336	.20979	.21631	.22291	.22960	.23637	.24322
7	.16660	.17281	.17913	.18555	.19207	.19869	.20545	.21221	.21911
8	.14852	.15472	.16103	.16746	.17401	.18067	.18744	.19432	.20130
9	.13449	.14069	.14702	.15348	.16007	.16679	.17364	.18060	.18767
10	.12329	.12950	.13586	.14237	.14902	.15582	.16274	.16980	.17698
11	.11414	.12039	.12679	.13335	.14007	.14694	.15396	.16112	.16841
12	.10655	.11282	.11927	.12590	.13269	.13965	.14676	.15402	.16143
13	.10010	.10645	.10296	.11965	.12652	.13356	.14077	.14815	.15567
14	.09466	.10102	.10758	.11434	.12129	.12843	.13574	.14322	.15087
15	.08994	.09634	.10296	.10979	.11682	.12405	.13147	.13906	.14682

A continuation of the preceding Table VII.

Years	4	5	6	7	8	9	10	11	12
16	.08581	.09226	.09895	.10585	.11298	.10209	.12781	.13551	.14339
17	.08219	.08869	.09544	.10242	.10962	.11704	.12466	.13247	.14056
18	.07899	.08554	.09235	.09941	.10670	.11421	.12192	.12984	.13793
19	.07613	.08274	.08962	.09675	.10412	.11173	.11954	.12756	.13576
20	.07358	.08024	.08718	.09439	.10184	.10954	.11745	.12557	.13387
21	.07128	.07799	.08500	.09228	.09983	.10761	.11562	.12383	.13224
22	.06919	.07597	.08304	.09040	.09803	.10590	.11400	.12231	.13081
23	.06739	.07413	.08127	.08871	.09642	.10438	.11257	.12097	.12955
24	.06655	.07247	.07967	.08718	.09497	.10302	.11126	.11978	.12846
25	.06401	.07095	.07822	.08581	.09367	.10180	.11016	.11874	.12749
26	.06256	.06956	.07690	.08456	.09250	.10071	.10915	.11781	.12665
27	.06123	.06829	.07569	.08342	.09144	.09973	.10825	.11698	.12590
28	.06001	.06712	.07459	.08239	.09048	.09885	.10745	.11625	.12524
29	.05887	.06604	.07357	.08144	.08961	.09805	.10672	.11565	.12465
30	.05783	.06496	.07264	.08058	.08882	.09733	.10607	.11502	.12414

The uſe of the preceding Table VII.

Queſt. 1. What Annuity or yearly rent iſſuing out of Lands, to begin preſently, and to continue 14 years, will 320 *l.* purchaſe, compound intereſt being reckoned on both ſides, at the rate of 6 *per centum, per annum?* *Anſw.* 34*l.* 8*s.* 6*d.* very near, which is thus found out, *viz.* In the fourth Column of the preceding *Table* VII. under 6 at the head of that Column, and right againſt 14 years, you will find this decimal .10758, which ſhews that 1 *l.* ready money will purchaſe an Annuity of .10758 *l.* (that is 2 *s.* 1 *d.* 2 *f.* +) therefore multiplying the ſaid decimal .10758 by the ſaid 320; the product (according to the ſecond Rule of the 26th Chapter of the preceding Book) will be found to be 34.425, *&c.* that is 34 *l.* 8 *s.* 6 *d.* very near, which is the *Anſwer* of the queſtion.

```
1 . ,10758 : : 320 . (34.425 +
          320
    ———————————
       215160
       32274
    ———————————
     34|42560
```

In like manner, if 10 *per centum* be thought a fit rate of intereſt to be allowed in purchaſing Leaſes of houſes, 500 *l.* will buy a preſent yearly rent of 63 *l.* 18 *s.* 1 *d.* payable for 16 years out of a houſe. For underneath 10 at the head of the 8th Column, and right againſt 16 years (in the preceding *Table* VII.) you will find this decimal .12781, which being multiplied by 500, (the number of pounds propounded to purchaſe the Leaſe (the product will be found to be 63.90500, that is, 63 *l.* 18 *s.* 1 *d.* + as by the ſubſequent operation is manifeſt.


```
1 . ,12781 : : 500 . (63.905
          500
     ——————————
      63|90500
```

XVII. Upon the ſame foundations which have been laid in the 12, 13, 14, 15, and 16 Rules of this Chapter, for the making of Tables which reſpect yearly payments; Tables may be made for half yearly and quarterly payments, the intereſt of 100 *l.* for $\frac{1}{2}$ year, and likewiſe for $\frac{1}{4}$ year being firſt agreed upon: For if we ſuppoſe that at the rate of 6 *l.* for 100 *l.* for a year, the intereſt of 100 *l.* for $\frac{1}{2}$ year is 3 *l.* the numbers 100 and 103 are to be uſed in the ſame manner to calculate *Tables* for half yearly payments, as the numbers 100 and 106 have been before uſed to form *Tables* for yearly payments. But if at the rate of 6 *per centum, per annum*, the intereſt of 100 *l.* for $\frac{1}{2}$ year ought to be ſuch, that being added to the ſaid principal 100 *l.* and the whole put forth at intereſt for the next half year, at the ſaid rate, the ſum then due (to wit, at the years end) muſt exactly amount unto 106 *l.* In this caſe a Geometrical mean proportional number between the extreams 100 and 106 muſt be ſought, which mean will (by the following 18th Rule) be found to be near 102.956301+, and then the numbers 100 and 102.956301, *&c.* are to be uſed inſtead of the num-

The making of Tables for half yearly and quarterly payments.

numbers 100 and 106 in manner aforesaid. In like manner, if it be supposed that at the rate of 6 *per centum, per annum*, the interest of 100*l.* for $\frac{1}{4}$ year is 1*l.* 10*s.* or 1.5*l.* the numbers 100 and 101.5 are to be used for the calculating of Tables for quarterly payments, in the same manner as the numbers 100 and 106 for yearly payments. But if at the rate of 6 *per centum, per annum*, the interest of 100*l.* for $\frac{1}{4}$ year ought to be such, that being added to the said 100*l.* and the whole put forth at the same rate of interest for the next $\frac{1}{4}$ year, and in that manner for the third and fourth quarters, and that the sum due at the years end must exactly amount unto 106*l.* In this case a series or rank of five numbers in Geometrical proportion continued must be considered, *viz.* the principal 100*l.* (which is the lesser of the two extream proportionals;) the three sums (composed of principal and interests) due at the end of the first, second and third quarters of the year, (which are the three mean proportionals) and 106*l.* due at the years end (which is the greater of the two extream proportionals;) now between the said extreams 100 and 106, the first (to wit the least) of the said three mean proportionals is to be sought, which (by the following 20th Rule of this Chapter) will be found to be near 101.4673+. And then the numbers 100 and 101.4673, *&c.* are to be used instead of the numbers 100 and 106 in manner aforesaid.

To find a Geometrical mean proportional number between two numbers given.

XVIII. Two numbers being given to find a Geometrical mean proportional between them; multiply the two given numbers one by the other and extract the square root of the product, so is such square root the mean proportional sought: for example, if 8 and 18 are two numbers given, and it is required to find a mean number Geometrically proportional between them, multiply 18 by 8, so is the product 144, whose square root is 12 for the mean proportional sought, so that 8, 12 and 18, are three numbers in Geometrical proportion continued, *viz.* As 8 is in proportion to 12, so is 12 to 18. In like manner a Geometrical mean proportional between the extreams 100 and 106 will be found near 102.956301+.

To find the first of the Geometrical mean proportional numbers between two extream numbers given.

XIX. Two numbers being given, to find the first of two Geometrical mean proportional numbers between the extreams given, multiply the square of the lesser extream by the greater, and extract the cube root of the product, so is such cube root the lesser of the two mean proportionals required: for example, if 8 and 27 are assigned for two extreams, the lesser mean will be found 12; for according to the rule, the square of 8 the lesser extream is 64, which being multiplied by 27 (the greater extream) produceth 1728, whose cube root is 12 the lesser mean sought, then may the greater mean be found more easily the *Rule of Three*, for 8 . 12 : : 12 . 18, so that 12 and 18 are two means Geometrically proportional between the extreams 8 and 27, *viz.* these four numbers are in Geometrical proportion continued, to wit, 8. 12. 18 and 27.

XX. Two

To find the first of three Geometrical mean proportionals between two extream numbers given.

XX. Two numbers being given to find the first of three Geometrical mean proportionals between the extreams given, multiply the cube of the lesser extream by the greater, and extract the Biquadrate root of the product, so is such Biquadrate root the first (to wit, the least) of the three mean proportionals required: for example, if 2 and 32 are two extreams given, the first and least of three Geometrical mean proportionals will be found to be 4, for (according to the *Rule*) the cube of 2 (the lesser extream given) is 8, which being multiplied by 32 (the *greater extream*) produceth 256, the *Biquadrate root* whereof being extracted (according to the 29 *Rule* of the 33 *Chapter* of the preceding *Treatise*) gives 4 for the first and least of the three means sought, the other means may be easily found by the *Rule of Three*; for,

2 . 4 :: 4 . 8 :: 8 . 16 :: 16 . 32.

So that these five numbers will appear to be in Geometrical proportion continued, to wit,

2 . 4 . 8 . 16 . 32.

In like manner the first and least of three *Geometrical mean proportionals* between the extreams 100 and 106, will be found to be near 101.4673, *&c.* Thus have I shewed the most easie ways raised from clear grounds) to make *Tables* for the resolution of the usual questions, which depend upon the computation of interest, by the help of *Multiplication* only.

Questions

Questions to exercise the precedent Tables, with their use in solving Questions of the same nature, when the number of years exceeds 30.

Quest. 1. If the Lease of an house be worth 153 *l.* *Fine*, and 16 *l. yearly rent*, payable yearly for 21 years, and the *Lessee* be desirous to bring down the *Fine* to 50 *l.* and so to pay the more *Rent*, the question is, what rent the Tenant shall pay, accompting compound interest at the rate of 10 *per centum, per annum*? *Answ.* 27 *l.* 18 *s.* 1 $\frac{3}{4}$ *d.* near.

First, find the difference between the *Fines*, which is 103 *l.* Then after the manner of the examples of the use of the preceding *Table* VII. seek what Annuity or rent to continue 21 years, 103 *l.* ready money will purchase at 10 *per cent.* so will you find 11 *l.* 18 *s.* 1 $\frac{3}{4}$ *d.* which being added to the old rent 16 *l.* gives 27 *l.* 18 *s.* 1 $\frac{3}{4}$ *d.* which the Tenant must pay to the end that the *Fine* may be diminished unto 50 *l.*

Quest. 2. There is a *Lease* of certain Lands to be let for 14 years for 250 *l. Fine*, and 44 *l. Rent per annum*, payable yearly, but the *Tenant* is desirous to pay less *Rent*, *viz.* 20 pounds *per annum*, and to give a greater *Fine*; the question is what *Fine* ought to be paid to bring down the *Rent* to 20 *l. per annum*, accompting compound interest, at the rate of 6 *per cent. per annum*? *Answ.* 473 *l.* 1 *s.* 7 *d.*

First find the difference between the *Rents*, which will be 24 pounds *per ann.* Then by the help of the preceding *Table* VI. seek what Annuity or Rent of 24 *l. per ann.* to continue 14 years, is worth in ready money at 6 *per centum, per annum*, so will you

find

find 223 *l.* 1 *s.* 7 *d.* which being added to the first *Fine* 250 pounds, gives 473 *l.* 1 *s.* 7 *d.* which the *Tenant* must pay, to the end the *rent* may be brought down to 20 *l. per annum.*

Quest. 3. There is a *Lease* of certain Lands worth 32 *l. per annum*, more than the rent paid to the Lord for it of which *Lease* seven years are yet in being, and the *Lessee* is desirous to take a *Lease* in *reversion* for 21 years, to begin when his old *Lease* is expired, the question is what sum of money is to be paid for this *Lease* in *reversion*, accompting compound interest at the rate of 6 *per centum, per annum*. *Answ.* 250 *l.* 7 *s.* 2 *d.* +

First, by adding the 7 years of the *Lease* in being to the 21 years you would have in reversion after those seven are expired, the sum is 28. Then by the preceding *Table* VI.*

The present worth of 1 *l.* Annuity for 28 years at 6 *per centum* compound interest, is ———	13.40616
Likewise the present worth of 1 *l* Annuity for 7 years is———	5.58233
Therefore the difference of those present worths, shall be the present value of 1 *l.* Annuity for 21 years in reversion after 7 years ———	7.82383
Which multiplied by 32 (the yearly rent propounded) gives the *Answer* of the question———	250.36256.

Otherwise

Otherwise thus.

First, By the help of the said *Table* VI. find out how much 32 *l.* yearly rent for 21 years is worth in ready money, as if the 21 years were to begin presently, at the rate of 6 *per centum*, which ready money will be found 376.45024 *l.* Then by *Table* V. find what 376.45024 *l.* due at the end of 7 years to come, is worth in ready money, so will it be 250 *l.* 7 *s.* 2 *d.* which agrees with the *Answer* before found.

Quest. 4. One would bestow 630 *l.* to purchase a present yearly rent or Annuity of 60 *l.* to be paid by yearly payments, the question is to know how many years the said Annuity must continue, compound interest at 6 *per centum, per annum*, being allow'd on both sides. *Answ.* 17 years, and 23 days, very near.

First, I divide 630 by 60, the quotients is 10.5, which shews that 10 years purchase and an half are given for the Annuity; then searching for 10.5 in *Table* VI. in the Column of 6 *per cent.* I find it not exactly, but the nearest less than it, is 10.47725, standing right against 17 years, and the next greater than 10.5 is 10.82760 which is placed against 18 years. Whence I infer that the Annuity must continue 17 years and more, yet less than 18 years. Now the proportional part of a year to be added to 17 years, may be found out near enough for use, thus, *viz.* subtract the said lesser tabular number 10.47725 from the greater 10.82760, so the remainder will be found .35035: Also subtracting the said 10.47725 from 10.5 (the quo-

 tient

tient first found) the remainder will be .02275; then say by the rule of three in decimals, as .35035 the greater remainder is to .02275 the lesser; so is 1 year (the difference between 17 and 18 years) to .0649 parts of a year, or 23 days + (as will appear by the fourth *Rule* of the 26 *Chapter* of the preceding *Book*;) therefore the number of years sought by the question is 17 years, 23 days.

Quest. 5. If an Annuity of 96 *l.* payable by yearly payments for 14 years be sold for 826 *l.* what rate of interest *per centum*, is implied in that bargain? *Answ.* 7 *l.* 5 *s.* 7 ½ *d.* near.

First, dividing 826 by 96, the quotient is 8.60146, which shews how many years purchase was given for the Annuity; then searching for 8.60416 in *Table* VI. in a right line passing from 14 years, equi-distant to the head line of the *Table*, I find it not exactly, but the nearest less than it is 8.24423 (which stands in the Column of 8 *per cent.*) and the nearest greater is 8.74546 (which stands in the Column of 7 *per cent.*) whice I infer, that the rate of interest required is between 7 and 8 *per cent.* and the proportional part of 1 *l.* to be added to 7 *l.* may be found out near enough for practice thus, *viz.* subtract the said lesser tabular number 8.24423 from the greater 8.74546, the remainder will be .50123. Also subtract 8.60416 (the quotient first found, which falls between the said tabular numbers from the said greater tabular number 8.74546, the remainder will be 14130; then say by the *rule of three* in decimals, as 50123 the greater remainder (or difference between the two tabular numbers) is to 14130 the lesser remainder; so is 1 *l.* (the difference between 7 *per cent.* and 8 *per cent.*) to .2819, &c.

&c. or 5 *s.* 7 *d.* 2 *f.* which added to 7 *l.* gives 7 *l.* 5 *s.* 7 *d.* 2 *f.* which is near the rate of interest *p. c.* required.

Quest. 6. If a years rent (or one years purchase) be paid as a *Fine*, for renewing or adding 7 years to 14 years yet to come of an old *Lease* for 21 years, and accordingly a new *Lease* to be taken for 21 years, to begin presently (which proportion is ordinarily observed by *Bishops*, *Deans*, and *Chapters*, *Heads* and *Fellows* of *Colleges* in letting *Leases* of their Lands) what rate of interest *per centum* is implied in that Agreement? *Answ.* 11 *l.* 11 *s* 8 *d.* 1 *f.* and somewhat more.

To solve this Question, first I search in the preceding *Table* VI. to find out two numbers so seated in some one Column of Interest, that one of them may stand right against 14 years, and the other against 21 years; and so qualified that the difference between them may be exactly 1 or unity; but not finding any two numbers precisely answering those conditions, I take those numbers that come nearest, which will be found in the Columns of 11 and 12 *per cent.* for the difference between the numbers 6.98186 and 8.07507, which stand in the Column of 11 *per centum*, right against 14 years and 21 years, is 1.09321, which exceeds 1 (that is 1 years purchase) by .09321; Also the difference between 6.62816 and 7.56200, which stand in the Column of 12 *per cent.* right against 14 years and 21 years, is .93384, which wants .06616 of 1; therefore I divide 1 *l.* (the difference between 11 *l.* and 12 *l.* *per cent.*) into two parts, in such proportion one to the other, as the said decimals .09321 and .06616 are one to the other; so I find the said part of 1 *l.* to be near .5848 and .4151; or 11 *s.* 8 *d.* 1 *f.* + and 8 *s.* 3 *d.*

3 *d.* 2 *f.*+; the former of which being added to 11 *per centum*, or the latter being ſubtracted from 12 *l. per cent.* gives 11.5848 *l.* or 11 *l.* 11 *s.* 8 *d.* 1 *f.* +, which is very near the rate of intereſt required by the queſtion.

Queſt. 7. What is the preſent worth of 1 *l. per ann.* payable yearly for 10 years, compound intereſt being computed at the rate of 11.5848 *l. per cent. An.* 5 *l.* 15 *s.* 0 *d.* very near, which is found out by the help of the preceding *Table* VI. in this manner, *viz.*

The tabular number for 10 years at 11 *l. per centum* is——————— 5.88923
The tabular number for 10 years at 12 *per centum* is——————— 5.65022

Their difference is——————————0.23901

Then ſay by the *Rule of Three* in decimals, as 1 *l.* (the difference between 11 and 12 *per cent.*) is to 5848 *l.* (to wit, the decimal by which the given rate in the queſtion exceeds 11 *per cent.*) ſo is .23901 (the difference found out as above) to .13977+, which being ſubtracted from 5.88923 (the greater of the two tabular numbers above mentioned) there will remain 5.74946, or 5 *l.* 15 *s.* 0 *d.* which is near the preſent worth of one pound yearly rent to continue 10 years, at the propoſed rate of 11.5848 *l. per centum.*

After the ſame manner the preſent worth of 1 *l.* yearly rent payable for 21 years, at the ſame rate of intereſt, will be found to be 7.77503 *l.* or 7 *l.* 15 *s.* 6 *d.* very near, from which if you ſubtract 5.74946 (being the aforementioned preſent worth of 1 *l.* yearly rent for 10 years) there will remain 2.02557 or

or 2 *l.* 0 *s.* 6 *d.* which is near the preſent worth of a *Leaſe* of 1 *l.* rent *per annum*, for 11 years in reverſion, to begin after 10 years yet to come in a *Leaſe* are expired; Hence it is evident, that if a *Tenant* to a College hath 10 years yet to come in a *Leaſe*, at 1 *l.* rent *per annum*, and deſires to have 11 years renewed, or added to thoſe 10, and ſo take a new *Leaſe* for 21 years, to begin preſently at the ſame rent, he muſt give 2 *l.* 0 *s.* 6 *d.* or two years purchaſe and $\frac{1}{40}$ part of a years purchaſe, very near (according to the fundamental proportion before aſſumed in the ſixth queſtion.) The like may be done for any other term of years under 30, by the help of the ſaid *Table* VI.

Concerning the renewing of a College Leaſe of Lands.

But yet by a *Table* calculated purpoſely for the ſaid rate of 11.5848 *l. per centum*, (according to the fifteenth *Rule* of this *Chapter*) queſtions of the ſame kind with the two laſt, may be more eaſily anſwered, and therefore (for that they come often in practice) I ſhall here inſert ſuch a *Table*, a I find it ready calculated to my hand by Doctor *Newton*, in his Scale of Intereſt lately publiſh'd, which *Table* is to be uſed in every reſpect like to the preceding *Table* VI. and will be very ready and uſeful, for the proportioning of Fines, in the renewing of *Leaſes* held from Cathredral Churches and Colleges, as will be manifeſt by the manner of ſolving the two following queſtions.

Queſt. 8. If a College-Tenant hath 7 years yet to come or unſpent in a *Leaſe* of Lands for 21 years, at 1 *l.* yearly rent, and deſires to have 14 years renewed or added to thoſe ſeven years, and ſo to take a new *Leaſe* for 21 years to begin preſently, what muſt he pay for a Fine? *Anſw.* 3 *l.* 3 *s.* 0 *d.*

The rule for finding out the anſwer of the queſtion propoſed, and ſuch like, is this, *viz.*

From 7.77507 (being the number which anſwers to 21 years in this *Table* VIII) ſubtract always the tabular number which belongs to the number of years to come or unſpent in the old *Leaſe*, ſo the remainder will ſhew what Fine muſt be paid for the years to be renewed or added, to make thoſe unſpent years in the old *Leaſe* to be 21 years compleat again, at 1 *l.* yearly rent.

So to ſolve the queſtion propoſed.

TABLE VIII.

Shewing the preſent worth of one pound Annuity for any number of years under 22, at the rate of 11 l. 11 s. 8 d $1\frac{3}{10}$ f. per cent. compound intereſt.

Years.	*preſent worth*
1	0.90034
2	1 69938
3	2.41922
4	3.06438
5	3.64262
6	4.16088
7	4.62540
8	5.04176
9	5.41496
10	5.74948
11	6.04934
12	6.31819
13	6.55907
14	6.77507
15	6.96868
16	7.14226
17	7.29786
18	7.43737
19	7.56243
20	7.67455
21	7.77507

From

From the preſent worth of 1 *l.* yearly rent for 21 years, which is—	7 77507
Subtract the preſent worth of the ſame rent for 7 years (that were unſpent in the old *Leaſe*.	4.62540
And there will remain the Fine ſought, to wit———————	3.14967

That is to ſay, 3.14967 *l.* or 3 *l.* 3 *s.* 0 *d.* (very near) muſt be paid as a Fine, for renewing or adding 14 years to 7 years, that were unſpent in the old *Leaſe*, the yearly rent being 1 *l.* Alſo the ſaid 3.14967 ſhews, that ſuch a renewal is worth 3 years purchaſe and near $\frac{15}{100}$ parts of a years purchaſe (whatever the rent be.)

Queſt. 9. If a *Tenant* that hath 17 years yet to come in a Leaſe of Lands held of a College for 21 years, at 50 *l.* yearly rent, be deſirous to renew 4 years, and ſo make thoſe 17 years to be 21 years compleat again at the ſame rent, what muſt he give for a fine? *Anſw.* 23 *l.* 17 *s.* 2 *d.* 1 *f.* For according to the rule before given,

From the preſent worth of 1 *l.* yearly rent for 21 years———	7.77507
Subtract the preſent worth of the ſame rent for 17 years (that were unſpent in the old *Leaſe*.)	7.29786
And there will remain—————	0.47721
Which multiplied by the rent———	50
The product will be the Fine ſought, to wit, 23 *l.* 17 *s.* 2 *d.* 1 *f.*	23|86050

Queſtions

Queſtions of this nature may be readily ſolved without the loſs of one ſixteenth part of a years Purchaſe by the help of the following *Table* IX, which I have drawn from the foregoing *Table* VIII. for the benefit of ſuch as underſtand not Decimal fractions: for example, if a College-Tenant deſireth to have 10 years added to 11 years that are to come or unſpent in a Leaſe of Lands that he may have a new Leaſe for the term of 21 years to begin preſently, the following *Table* IX, ſhews that he muſt give for a Fine 1 years Purchaſe, and 2 quarters of a years Purchaſe, and 3 quarters of a quarter of a years Purchaſe, *viz.* one years rent, and half a years rent, and three quarters of a quarter of a years rent: Suppoſing then the rent to be 48 *l. per annum*, the Fine may be computed thus.

	l.	*s.*	*d.*
One years rent is	48	00	00
Half a years rent is	24	00	00
Three quarters of a quarter of a years rent is	9	00	00
The ſum is the Fine reqired	81	00	00

Whence it appears that the Tenant muſt give 81 *l.* as a Fine, for adding of 10 years to 11 years that were unexpired in his old Leaſe, to the end he may have a new Leaſe for 21 years in being.

In like manner the following *Table* IX. ſhews that the Fine for renewing or adding 7 years to 14 years that are unſpent in a Leaſe of Lands, to the end there may be a new Leaſe for 21 years in being, is valued at 1 years Purchaſe preciſely, which is the fundamental proportion aſſumed in calculating the foregoing *Table* VIII, as before was ſaid.

TABLE IX.

TABLE IX.

The Fine for renewing or adding

Years		*Years*		*Years purchaſe.*	*Quarters of a year.*	*Quarters of a quarter.*
1	to	20	is valued at	0	0	1
2	to	19		0	0	3
3	to	18		0	1	1
4	to	17		0	1	3
5	to	16		0	2	2
6	to	15	is valued at	0	3	0
7	to	14		1	0	0
8	to	13		1	0	3
9	to	12		1	1	3
10	to	11		1	2	3
11	to	10	is valued at	2	0	0
12	to	9		2	1	1
13	to	8		2	2	3
14	to	7		3	0	2
15	to	6		3	2	1
16	to	5	is valued at	4	0	2
17	to	4		4	2	3
18	to	3		5	1	1
19	to	2		6	0	1
20	to	1		6	3	2

The

The like may be done for renewing any other term of years under 21, at any rent proposed.

Of finding out tabular numbers for any term of years above 30.

But because it may sometimes happen, that the number of years in questions belonging to the preceding 3, 4, 5, 6 and 7 *Tables* may exceed 30, I shall by the five following questions shew, how by the help of those *Tables* the answer to any question of that nature may be found out near the truth, when the term of years is above 30.

Quest. 10. If 340 *l.* be put forth at 4 *per centum*, compound interest, and both principal and interest be forborn until the end of 45 years, what will then be due? *Answer*, 1986 *l.* very near.

To resolve this question and the like, observe this rule, *viz.* First make choice of such numbers of years in *Table* III. that if they be added together will make the number of years proposed in the question, as 17 and 28, or 15 and 30, each of which pairs make 45, then looking into *Table* III. in the Column belonging to 4 *per centum*, you will find right against 17 and 28 years these numbers, 1.94790 and 2.99870, which being multiplied one by the other will produce 5.84116+. or 5 *l.* 16 *s.* 10 *d.* which shall be the increase of 1 *l.* forborn 45 years at 4 *per centum*, compound interest; therefore multiplying the said 5.84116 by 340, the product will give 1985.994, *&c.* or 1986 *l.* very near for the Answer of the question.

The reason of the said Rule will be manifest by this Theorem, *viz.* If there be a rank of numbers in Geometrical proportion continued, beginning with

with 1 or unity, as 1, 2, 4, 8, 16, 32, 64, 128, *&c.* Also if the first term 1 be cast away, and over or under all the rest of the terms there be placed another rank of numbers, beginning at 1 and proceeding according to the natural order of numbers, as 1, 2, 3, 4, 5, 6, 7, &c. which may be called the *Indices* of those in the first rank, after the first term 1 is cast away; I say if any two of those remaining Geometrical proportionals be multiplied one by the other, the product shall be a proportional correspondent to that *Index*, which is equal to the sum of the *Indices* answering to the two proportionals that were multiplied one by the other.

Proport.	2 .	4 .	8 .	16 .	32 .	64	128
Indices.	1 .	2 .	3 .	4 .	5 .	6	7

So if 4 and 32, which are the second and fifth proportionals in the upper rank, be multiplied one by the other, the product is 128, which shall be the seventh proportional, because the sum of the *Indices* 2 and 5, which answer to the said 4 and 32, is 7. In like manner, because the sum of the *Indices* 3 and 4 is 7, therefore if the third and fourth proportionals, to wit, 8 and 16, be multiplied one by the other, the product shall also give the seventh proportional 128. Now forasmuch as the numbers in every one of the Columns, except the first Column of years in the preceding *Table* III. are continual proportionals whose first term is 1, but 'tis excluded out of the said Columns, as appears by the Construction of that *Table*, and for that the numbers of years 1, 2, 3, 4, 5, *&c.* are placed

placed as *Indices* ſhewing the order or ſeat of thoſe proportionals inſerted in the Columns, therefore the rule before given for continuing that Table to any number of years is manifeſt.

Queſt. 11. If one pound be due or payable 50 years hence, what is it worth in ready money, by rebating at 5 *per centum, per annum,* compound intereſt? *Anſw.* 08720, *&c.* or 1 *s.* 9 *d.* + which is found out by the help of *Table* V. in the ſame manner as the Anſwer to the laſt Queſtion; (reſpect being had to the ſecond and third rules of the 26th Chapter of the preceding Book concerning the multiplication of decimal fractions)

Queſt. 12. If an Annuity of one pound payable yearly for 40 years, be all forborn untill the end of that term, what will it then amount unto, compound intereſt being computed at 5 *per centum, per annum*? *Anſw.* 120 *l.* 16 *s.* 0 *d.* thus found out: Firſt, according to the ſecond way of calculating the fourth *Table* in the thirteenth *Section* of this *Chapter*, find out a Principal, which may have ſuch proportion to the propoſed Annuity 1 *l.* as 100 *l.* hath to 5, ſaying, if 5 *l.* intereſt hath 100 *l.* for a principal, what principal muſt 1 *l.* intereſt have? *Anſwer*, 20 *l.* Secondly, ſeek (after the manner of the preceding tenth queſtion) what 20 *l.* will be augmented unto being forborn 40 years, at the rate of 5 *per centum, per annum*, compound intereſt, ſo you will find 140.798 +, from which ſubtracting the ſaid principal 20 *l.* the remainder will be 120.798 +, or 120 *l.* 16 *s.* which is the anſwer of the queſtion.

Queſt. 13. If an Annuity of one pound payable yearly for 37 years, be to be ſold for preſent money,

what is it worth, compound intereſt being computed on both ſides at 6 *per centum, per annum*? *Anſwer*, 14 *l.* 14 *s.* 9 *d.* which is found out thus: Firſt, according to the ſecond way of calculating the ſixth *Table* in the fifteenth *Section* of this *Chapter*, find out a principal in ſuch proportion to one pound (the propoſed Annuity) as 100 is to 6, ſo will ſuch principal be found 16.66666 +, then after the manner of the preceding eleventh queſtion find out the ready money which is equivalent to 16.66666, due 37 years hence, ſo will ſuch ready money be found to be 1.92988 + (or 1 *l.* 18 *s.* 7 *d.*) which being ſubtracted from the ſaid principal 16.66666, the remainder will be 14.73678 +, or 14 *l.* 14 *s.* 9 *d.* which is the Anſwer of the Queſtion propounded.

Queſt. 14. What Annuity payable by yearly payments to continue 37 years will one pound Purchaſe at 6 *per centum, per annum*, compound intereſt? *Anſw.* 1 *s.* 4 *d.* near, which is found out thus; Firſt, find out the preſent worth of one pound Annuity to continue 37 years, which preſent worth (by the laſt queſtion) will be found 14.73678 *l.* Then ſay by the *Rule of Three*, if 14.73678 *l.* will purchaſe an Annuity of 1 pound, (to continue 37 years) what Annuity to continue the ſame term will 1 *l.* purchaſe? *Anſw.* .06785 +, or 1 *s.* 4 *d.* which is the Anſwer of the Queſtion propounded.

CHAP.

CHAP. VI.

A Demonstration of the Rule of Three, or Rule of Proportion.

I. FOur numbers are said to be proportionals, when the first containeth the second so often as the third containeth the fourth; likewise when the first is such part of the second, as the third is of the fourth: so these numbers following are called proportionals, *viz.*

$$4 \times 6 \,.\, 6 :: 4 \times 9 \,.\, 9$$
$$\tfrac{2}{3} \times 12 \,.\, 12 :: \tfrac{2}{3} \times 15 \,.\, 15$$

That is to say, 4 times 6 (or 24) is said to have such proportion to 6, as 4 times 9 (or 36) hath to 9. In like maner, $\frac{2}{3}$ of 12 (or 8) hath such proportion to 12; as $\frac{2}{3}$ of 15 (or 10) hath to 15.

II. When four numbers are proportionals, the product arising from the multiplication of the two extreams is equal to the product of the two means.

Demonstration.

By the preceding Definition in 1. these four numbers are proportionals, *viz.*

$$\begin{cases} 4 \times 6 \,.\, 6 :: 4 \times 9 \,.\, 9 \\ B \times C \,.\, C :: B \times D \,.\, D \end{cases}$$

The product of the two extreams is —— $\begin{cases} 4 \times 6 \times 9 \\ B \times C \times D \end{cases}$

The product of the two means is —— $\begin{cases} 6 \times 4 \times 9 \\ C \times B \times D \end{cases}$

But $\begin{cases} 4 \times 6 \times 9 \\ B \times C \times D \end{cases} = \begin{cases} 6 \times 4 \times 9 \\ C \times B \times D \end{cases}$

Therefore the Prop. is manifest.

Likewise.

By the preceding definition these four numbers are proportionals, *viz.*

$$\tfrac{2}{3} \times 12 \,.\, 12 :: \tfrac{2}{3} \times 15 \,.\, 15$$

The product of the two extreams is —— $\} \frac{2}{3} \times 12 \times 15$

The product of the two means is —— $\} 12 \times \frac{2}{3} \times 15$

But $\frac{2}{3} \times 12 \times 15 = 12 \times \frac{2}{3} \times 15$

Wherefore the Proposition is every way proved.

III. From the last Proposition ariseth the *Rule of Proportion* commonly called the *Rule of Three*, or *Golden Rule*, which teacheth by three numbers given to find a fourth proportional number in this manner, *viz.* *Multiply the second and third numbers mutually one by the other, and divide the product by the first number; so the quotient shall be the fourth proportional number sought, in a direct proportion.* This Rule hath been fully exemplified in the 8th Chapter of the preceding Book, and the truth of the said

ſaid Rule may be thus demonſtrated, *viz.* Let there be three numbers given to find a fourth in direct proportion, *viz.* if 24 gives 6, what ſhall 36 give? Or as 24 is in proportion to 6, ſo is 36 to a fourth proportional number ſought, which fourth proportional (whatſoever it be) we may ſuppoſe to be Q, and then theſe four numbers will be proportionals, *viz.*

$$24 . 6 :: 36 . Q$$

Therefore by the ſecond Propoſition of this Chapter.

$$24 \times Q = 6 \times 36$$

And becauſe if equal plain numbers be ſeverally divided by one and the ſame number, the quotients will neceſſarily be equal between themſelves, therefore

$$Q = \frac{6 \times 36}{24}$$

Whereby it is manifeſt that the fourth proportional number is equal to the quotient that ariſeth by dividing the product of the multiplication of the ſecond and third proportionals by the firſt, which was to be proved.

Note, That every *Rule of three inverſe* may be made a *Rule of Three direct*, by making the third term the firſt, and by proceeding forward to the other two terms; therefore one and the ſame demonſtration ſerveth for both rules.

CHAP.

CHAP. VII.

A Demonſtration of the Double Rule of Fellowſhip.

THe *Double Rule of Fellowſhip* (commonly called the *Rule of Fellowſhip with time*) preſuppoſeth two things, *viz.* 1. That the particular Stocks of Merchants in company, have continued unequal ſpaces of time in the common Stock. 2. That at the end of their Partnerſhip, the total gain or loſs is to be divided amongſt them, in ſuch manner, that their ſhares ſhall have ſuch proportion between themſelves, as thoſe ſums of intereſt money have one to another, which at any rate *per centum*, ſimple intereſt only being computed, might be gained by the particular Stocks, within the reſpective times of their continuance in the common Stock: Now for the effecting of ſuch a proportional partition, the ſaid *Double Rule of Fellowſhip* gives this direction, *viz.* Divide the total gain or loſs into ſuch parts, which ſhall have the ſame proportion one to the other, as is between the products ariſing out of the multiplication of each particular Stock by its correſpondent time.

For example, Suppoſe two Merchants *A* and *B* to be Partners in Traffick, for a certain time firſt

agreed on between them, and that A doth permit his Stock of 100 *l.* to be implied in their joint Traffick three months, and that B forbears his Stock of 50 *l.* eight months; I say (according to the said *Rule of Fellowship* with time) whatever the total gain or loss be, that part thereof which belongs to A must have such proportion to the gain or loss of B, as 100×3 (or 300) hath to 50×8 (or 400.) This rule hath been fully exemplified in the 13 Chapter of the preceding Book, and the truth thereof, taking the two premised Suppositions for granted may be thus demonstrated

1. Supposing 100 *l.* (the Stock of A) to gain in 3 months any certain sum of money, as two pounds; I seek how much 50 *l.* (the Stock of B) will gain in the same time, and at the said rate: so I find $\frac{2 \times 50}{100}$ *l.* for,

$$100 . 2 :: 50 . \frac{2 \times 50}{100}$$

2. Having found what 50 *l.* will gain in three months, I seek how much the said 50 *l.* will gain in 8 months, at the same rate, and so I find $\frac{2 \times 50 \times 8}{100 \times 3}$ *l.*

for,

$$\frac{3}{1} . \frac{2 \times 50}{100} :: \frac{8}{1} \frac{2 \times 50 \times 8}{100 \times 3}$$

3. Thus it appears, that if 100 *l.* in 3 months doth gain 2 *l.* then 50 *l.* in 8 months will gain at the same rate $\frac{2 \times 50 \times 8;}{100 \times 3}$ so that the proportion of the gain of A to the gain of B is,

$$\text{As } 2 \text{ is to } \frac{2 \times 50 \times 8}{100 \times 3}$$

4. If both the terms (to wit, the *Antecedent* and *Consequent*) of the said proportion be severally multiplied by the said *Denominator* 100×3, the products will be in the same proportion with the numbers of terms multiplied, (by 17 è 7. *Euclid.*) *viz.* the gain of A will be to the gain of B,

As $2 \times 100 \times 3$ is to $2 \times 50 \times 8$

5. Lastly, Because 2 (the suppositions gain first assumed) is a Multiplicator as well in the *Antecedent* as in the *Consequent* of the last mentioned proportion, it may be expung'd out of both, and so the gain of A will be to the gain of B in this proportion (which was to be proved) to wit,

As 100×3 is to 50×8

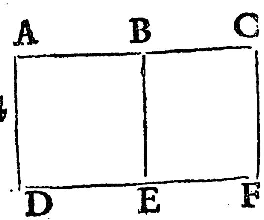
CHAP. VIII.

A Demonstration of the Rule of Alligation alternate, and the use of the said Rule in the Composition of Medicines.

I. IN order to the Demonstration of the said Rule, I shall premise this *Lemma*, *viz.* if the difference of any two numbers given, be multiplied by a number assigned, the product will be equal to the difference between the products which arise from the multiplication of those two numbers severally by the number assigned.

Suppositions.

Two lines or numbers given. } AC = 10
BC = 4
Their difference. AB = 10—4
A multiplicator assigned. } AD = 5



Which suppositions, and the *Diagram* being well viewed, the truth of the said *Lemma* will be evident, *viz.*

$$\overline{AB} \times \overline{AD} = \overline{AC} \times \overline{AD}, - \overline{BC} \times \overline{BE}\ (AD)$$

$$10-4 \times 5 = 10 \times 5, - 4 \times 5$$

II. To

II. To add the more light to the following *Demonstration* of the rule of *Alligation alternate*, I shall propound a question which properly belongs to the said rule, *viz.* Suppose a *Vintner* having *French Wines* at 5*d.* the *quart*, and at 10*d.* the *quart*, would make a mixture of them in such manner, that he might sell the mixt quantity at 7*d.* the *quart*, and so make as much money of the mixture, as if he should sell each quantity of *wine* at its own price; the question is to know what proportion the quantities of both sorts of *wine* in the mixture must bear one to another: Here according to the *Rule of Alligation alternate*, I take the difference between the mean price assigned for the mixture, and the two other given prices, and place those differences alternately, *viz.* the difference between 7 and 10 being 3, I write 3 against 5; likewise 2 being the difference between 7 and 5, I write 2 against 10; so I conclude, that the quantity to be taken of that sort of *wine* of 10*d.* the *quart*, must have such proportion to the quantity of 5*d.* the *quart*, as 2 to 3.

$$7\left\{\begin{array}{r|l} 10 & 2 \\ 5 & 3 \\ \hline & 5 \end{array}\right.$$

That is to say, if 2 *quarts* at 10*d.* the *quart* be mixed with 3 *quarts* at 5*d.* the *quart*, the total mixture 5 *quarts* being sold at 7*d.* the *quart*, will yield as much money as the said 3 *quarts* at 5*d.* the *quart*, together with the said 2 *quarts* at 10*d.* the *quart*; as is evident by the subsequent work.

 quarts

	quarts	*pence*		*quarts*	*pence*
I.	1 .	5	::	3 .	15
II.	1 .	10	::	2 .	20
III.	15+20		=	7×5=35	

From the premises it appears, that when two things are given to be mixt in such manner as the *Rule of Alligation alternate* requires, the proposition to be demonstrated will be this, namely,

Three numbers A. B. C. being given in such sort that A. is less than B. but greater than C. if the difference between A. and B. be multiplied by C. and the difference between A. and C. be multiplied by B. the sum of those products will be equal to the product arising from the multiplication of A. by the sum of the said differences.

Demonstration.

mean price	*extream prices*	*differences alternate*	
A	B	A—C	BA—BC
	C	B—A	CB—CA
		B—C	BA—CA=B—C × A

The difference between B. and A. is B—A. which multiplied by C produceth (as is evident by the *Lemma*

Lemma aforegoing in the first Section of this Chapter) CB—CA. Also the difference between A and C is A—C. which multiplied by B produceth BA BC. Then the sum of those two products is BA—CA. (for + CB and —CB expunge one the other) which sum is manifestly the same with the product arising from the multiplication of A the mean price, by B—C the sum of the aforesaid differences (to wit, the sum of A—C and B—A) for + A and — A expunge one another.

When more than two things of different prices are given to be mixt as aforesaid, the *Demonstration* will not be otherwise: for if the sum of every two products arising from the multiplication of two alternate differences by their respective prices, be equal to the product of the mean price multiplied by the sum of the said differences; the sum of all the said products will also be equal to the product of the mean price multiplied by the sum of all the differences; as will clearly appear by view of the subsequent work.

	Products of alternate differences multiplied by their respective prices.		*Products of the mean price multiplied by the several sums of alternate differences.*
If	D + E	=	F × G
and	H + K	=	F × M
Then D + E +	H + K	=	F × G + M

More

Moreover, because if equal numbers be severally divided by one and the same number, the quotients will be equal between themselves, therefore from the premises this *Corollary* will arise.

COROLLARY.

In the *Rule of Alligation alternate*, if the aggregate of the products arising from the multiplication of the several alternate differences by their respective prices, be divided by the sum of the said differences, the quotient will be equal to the main price. This may be a proof of any example of the said *rule of Alligation.*

OF THE COMPOSITION OF MEDICINES.

See more of this in Mr. J. Dee his Mathematical Preface, also Tom. 2. *of* P. Herigon *and Master* More's *Arithmetick.*

I. Medicines and Simples in respect of their qualities are considered in some of these five ways, *viz.* either as they are hot or cold, moist or dry, or as they are temperate; so that such Simples or Medicines which work heat in our bodies are said to be hot, such cold which are cause of coldness.

II. The mean or middle between the extream qualities of *Heat* and *Coldness*, also between *Dryness* and *Moistre*, is called *Temperate* or the *Temperature*; from which each of the said qualities *hot*, *cold*, *moist* and *dry*, doth differ in four *degrees*, so that a Medicine or Simple is said to be either *temperate*, or else *hot*, *cold*, *moist* or *dry*, in the first, second, third, or fourth degree.

III. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed as you see from A to B, the differences between 5 (the middle number) and the superiour numbers 6, 7, 8, 9, will be 1, 2, 3, 4, which may represent the 4 degrees of the qualities hot and dry; likewise the differences between 5 and the inferiour numbers 4, 3, 2, 1, will be 1, 2, 3, 4, which may represent the 4 degrees of the qualities cold and moist, the temperature represented by 0. being the mean or middle from whence the said degrees do swerve.

9 4 6
3 . . 8 4
0 4 6

	Ind.	*Degr.*	
B	9	4	*Qualities hot and dry.*
	8	3	
	7	2	
	6	1	
	5	0	*Temperature.*
	4	1	*Qualities cold and moist.*
	3	2	
	2	3	
	1	4	
	Ind.	*Degr.*	

IV. Since the *Rule of Alligation alternate* requires that of two things miscible, the one must exceed the mean

mean propounded and the other be less, therefore the questions of *Alligation* in this kind are to be wrought with the numbers in the aforesaid Column AB, for by them the degrees and qualities are discovered, being placed as you see in the Column adjacent to AB, and for distinction sake, those numbers in the said Column AB, may be called the *Indices* or *Exponents* of the *degrees*, which *Indices* are to be used in the same manner as the prices of Merchandizes in the questions of *Alligation alternate* in Chapter 14 of the preceding Book, and therefore those examples may be compared with these.

Prop. I.

Having divers Simples whose qualities are known, to make a composition of mixture of them, in such manner that the quality of the medicine may be some mean amongst the qualities of the simples, and the quantity thereof any quantity assigned.

Example 1. An *Apothecary* hath four sorts of Simples, A, B, C, D, whose qualities are as followeth, *viz.* A is hot in the fourth degree, B is hot in the second, C is temperate, and D is cold in the third degree; the question is to know what quantities of each ought to be taken, to make a Medicine, whose quantity may be 12 ounces, and the quality in the first degree of heat? Seek in the aforesaid column AB, for the *Indices* or *Exponents* of the qualities of the Simples given, *viz.* for A which is hot in the fourth degree, take 9; for B which is hot in the second, take 7; for C which

which is temperate, take 5; and for D which is cold in the third degree, take 2; that done, rank those numbers in the same manner as the prices of Merchandizes in the questions of the 14 *Chapter*, *viz* descend from the highest degree of heat unto the temperature, and so proceed downwards to the degrees of cold, setting 6 the *Index* or *Exponent* of the mean quality propounded, which is 1 degree of heat, as common to them all; then by crooked lines or otherwise connect two such *Indices*, whereof one may be greater than the mean, and the other less, and proceding according to the *Rule* of the fourteenth *Chapter* you will find that to make a Medicine of 9 ounces, and the quality resulting to be in the first degree of heat, you must take 1 ounce of A (being that Simple which was hot in 4) 4 ounces of B, 3 ounces of C, and 1 ounce of D, as will be manifest by the proof,

	degr.	oun.	simp.	*The Proof.*
6	9	1	A	9 x 1 = 9
	7	4	B	7 x 4 = 28
	5	3	C	5 x 3 = 15
	2	1	D	2 x 1 = 2
		9		9) 54 (6

Lastly, by the *rule of Proportion* you may increase the Medicine to the quantity of 12 ounces, and yet the quality to continue in the first degree of heat, according to the following operation.

Oun.		Oun.		Oun.		Oun.	
9	.	1	::	12	.	$1\frac{1}{3}$	of A
9	.	4	::	12	.	$5\frac{1}{3}$	of B
9	.	3	::	12	.	4	of C
9	.	1	::	12	.	$1\frac{1}{3}$	of D

The quantity assigned 12 *ounces.*

By other connexions of the qualities, other quantities of each *Simple* would arise, but that hath been sufficiently manifested in the questions of the fourteenth Chapter.

Example 2. Suppose there are five *Simple*, A, B, C, D, E, whose qualities are as followeth, *viz.* A is hot in 3°. B is hot in 2°. C is hot in 1°. D is cold in 1°. E is cold in 3°. and it is required to mix mix 4 ounces of B, with such quantities of the rest that the quality of the *Medicine* may be temperate?

Degr.		Oun.	Simp.	*The proof.*
8	1	1	A	$8 \times 1 = 8$
7	3	3	B	$7 \times 3 = 21$
6	1	1	C	$6 \times 1 = 6$
4	3+1	4	D	$4 \times 4 = 16$
2	2	2	E	$2 \times 2 = 4$
		11		11) 55 (5

Proceed

Proceed as before, so will you find that to make a *Medicine* of 11 ounces, and the quality of the *Form* resulting to be temperate, you must take 1 ounce of A, 3 ounces of B, 1 ounce of C, 4 ounces of D, and 2 ounces of E; then since the quantity of B, in the composition propounded is limited, *viz.* 4 ounces, find numbers which may be in such proportion to 4 (the quantity of B assigned) as the numbers 1, 2, 4, 2 (the quantities of A, C, D, E, in the aforesaid Composition of 11 ounces) are unto 3 (the quantity of B in the said Composition) in manner following:

Oun.		Oun.		Oun.		Oun.		
3	.	1	::	4	.	$1\frac{1}{3}$	of A.	*to be mixed with* 4 *ounces of* B.
3	.	1	::	4	.	$1\frac{1}{3}$	of C.	
3	.	4	::	4	.	$5\frac{1}{3}$	of D.	
3	.	2	::	4	.	$2\frac{2}{3}$	of E.	

Prop. II.

A *Medicine* being compounded of divers *Simples* whose qualities and quantities are known, to find the degree of the *Form resulting*, *viz.* the exact temperament of the *Medicine.*

Example 1. Suppose a *Medicine* to be compounded of two *Simples*, *viz.* 6 ounces of B hot in 4°, and three ounces of C hot in 3°. and it is required to find the temperament of the *Medicine*, *viz.* the degree and quality resulting from such mixture? Seek in the aforesaid Column AB for the *Indices*

of

of the respective degrees and qualities of the *Simples* given, and dispose them orderly in ranks right against their respective quantities; then multiply each *Index* by its respective quantity, and divide the sum of the products by the sum of the quantities: so will the quotient be the *Index* of the degree and quality of the Medicine.

Ind.		*Oun.*		*Prod.*
9	x	6	=	54
8	x	3	=	24
		9		78 ($8\frac{2}{3}$

So in the said example the Quotient will be found $8\frac{2}{3}$, which is the *Index* of $3\frac{2}{3}$ degrees of heat, and therefore the said *Medicine* is hot in $3\frac{2}{3}$ degrees.

Forasmuch as any two quantities miscible according to the *Rule of Alligation alternate*, are in such proportion one to the other, as the respective alternate differences between the mean quality of the mixture and the qualities correspondent unto the said quantities, the demonstration of the aforesaid *rule* will be manifest by the *Corollary* aforegoing in this Chapter.

Example 2. Suppose a *Medicine* to be compounded of 4 *Simples*, whose qualities and quantities are known, *viz.* 2 ounces of A hot in 3°. 3 ounces of B hot in 2°. 4 ounces of C temperate, and 5 ounces of D cold in 4°. and let it be required to find

find the mean quality resulting from such mixture. According to the aforesaid *rule*, I multiply each *Index* by its respective quantity, and divide the sum of the products by the sum of the quantities, so the quotient is $4\frac{3}{9}$, which is the *Index* of $\frac{4}{7}$ degrees of cold (for the difference between 5 the *Index* of the temperature, and $4\frac{3}{7}$ the *Index* found, is $\frac{4}{7}$ degrees of cold) which is the quality of the said *Medicine*.

Ind.		*Oun.*		*Prod.*
8	x	2	=	16
7	x	3	=	21
5	x	4	=	20
1	x	5	=	5
		14)		62 ($4\frac{3}{7}$

Example 3. Suppose a Medicine to be compounded of several *Simples*, whose qualities and quantities are as followeth, *viz.* 4 ounces of a *Simple* which is cold in 2°. and moist in 1°. 5 ounces hot in 3°. and (in respect of dryness and moisture) temperate; 3 ounces hot in 2°. and dry in 2°. 6 ounces hot in 1°. and moist in 4°. 4 ounces cold in 3°. and moist in 2°. the question is to know the temper resulting?

In the resolution of this question there must be two distinct operations, each of them like to that in the last example, *viz.*

 1. Find

1. Find in the same manner as before, the degree and quality resulting from the commixture of the qualities hot and cold; so will you find $5\frac{7}{22}$ which is the Index of $\frac{7}{22}$ degrees of heat (for the difference between 5 the Index of the temperature and $5\frac{7}{22}$ the Index found, is $\frac{7}{22}$ degrees of heat.)

Ind.		Own.		Prod.
3	x	4	=	12
8	x	5	=	40
7	x	3	=	21
6	x	6	=	36
2	x	4	=	8

22) 117 ($5\frac{7}{22}$

Ind.		Own.		Prod.
4	x	4	=	16
5	x	5	=	25
7	x	3	=	21
1	x	6	=	6
3	x	4	=	12

22) 80 ($3\frac{7}{11}$

2. Find in the same manner, the temper resulting from the mixture of the qualities dry and moist; so will you find $3\frac{7}{11}$ which is the Index of $1\frac{4}{11}$ degree of moisture, so the quality of the said Medicine is $\frac{7}{22}$ degree of heat, and $1\frac{4}{11}$ degree of moisture, as by the operation is manifest.

Prop. III.

To augment or diminish a Medicine in quality according to any degree assigned.

Suppose a Medicine to be compounded as followeth, *viz.* 1 dram of a Simple hot in 4°. 2 drams hot in 3°. 2 drams hot in 2°. 1 dram hot in 1°. 1 dram cold in 1°. and 1 dram cold in 2°. Then will the quality of the said Medicine be in $1\frac{1}{2}$ degree of heat

heat (as will be manifest by the second Proposition.) Now let it be required to augment the said Medicine in quality, *viz.* to add such a quantity of some one of the Ingredients (or some other simple) which may arise the quality of the Medicine $\frac{1}{2}$ degree; so that the temperament of the Medicine after it is increased in quantity, may be in 2°. of heat. Make choice of such a simple, the Index of whose quality may exceed the Index of the quality assigned, *viz.* make choice of that simple which is hot in 3°. whose Index is 8, then proceed according to the 1 example of the first Proposition; so will you find that if 1 dram of the aforsaid Medicine be mixed with $\frac{1}{2}$ dram of that simple which is hot in 3°. the temper resulting from such mixture will be in 2° of heat.

Lastly, by the *Rule of Three*, say, if 1 dram require $\frac{1}{2}$ dram, what shall 8 drams (the quantity of the Medicine first given) require?

Answ. 4 drams: So that if 4 drams of a simple which is hot in 3°. be mixed with 8 drams of a Medicine which is hot in $1\frac{1}{2}$ degree, the temper resulting will be in 2°. of heat, as by the operation is manifest.

	Ind.	*Drams.*
7 {	$6\frac{1}{2}$	1
	8	$\frac{1}{2}$

$1 . \frac{1}{2} :: 8 . 4$

The Proof.

Ind.	*Dra.*	*Prod.*
$6\frac{1}{2}$ x	8 =	52
8 x	4 =	32

12.) 84 (7

If it be required to diminish a Medicine in quality, you are to make choice of such a Simple, the Index of whose quality may be less than the Index of the quality assigned, and then to proceed as before.

Here observe, that if in questions of this nature, the quantities of the Simples be exprest by weights of divers denominations, they are to be reduced to that weight which is of the lowest denomination in the question, according to the sixth *Rule* of the seventh *Chapter* of the preceding *Book.*

The augmenting or diminishing of a Medicine in respect of quantity; Also the finding of the value of any quantity of a Medicine, the prizes of the Ingredients being known, will be familiar to such as understand the Rule of Proportion, and therefore I shall not insist upon them.

CHAP.

CHAP. IX.

A Demonstration of the common Rule of False by two Positions.

I. WHat the ordinary *double Rule of False* is, and how to be used in resolving such questions which cannot readily be applied to any of the other rules of *Arithmetick*, hath been fully declared in the 15 and 31 Chapters of the preceding Book; it remaineth to shew what kind of operation is presupposed before the said Rule can be applied to the resolution of a question, and then to demonstrate the truth of the Rule it self.

II. In the said *Rule of False*, look what operation the question requires to be performed with the number sought and some given number or numbers, the same kind of operation in every respect is to be made with each of the two feigned numbers (commonly called Positions) and the said given number or numbers; which threefold process being finisht (whether it be by any one, or all of these *rules*, to wit, *Addition, Subtraction, Multiplication*, and *Division*) there will arise three remarkable numbers or results, to wit, one resulting from the true number sought, and two others resulting from

the two feigned numbers; then from these three results, the errors are collected, which are nothing else but the differences between the true result, and each of the two false results.

III. After the said errors or differences are discovered, the *Rule of False* will be of no force, unless this Analogy or proportionality doth arise, namely the first error must have the same proportion to the second, as the difference between the number sought and the first feigned number hath to the difference between the said number sought and the second feigned number; here therefore it may be demanded, what kind of operation will produce the said Analogy? To this I answer, when the question requires the number sought to be increased, lessened, multiplied or divided by some given number, or the number arising from such operation to be increased, lessened, multiplied or divided by some given number; in any of those cases, the aforesaid Analogy will necessarily arise, as I shall here manifest in all the said cases. First, therefore I say when unto each of three numbers (namely the number sought by the *Rule of False* and the two feigned numbers) one and the same number is added, the said Analogy will ensue, for in this case the difference between the first sum and the second will be equal to the difference between the first and second of the said three numbers; likewise the difference between the first sum and the third will be equal to the difference between the first number and the third which may be proved in manner following.

Suppositions

Suppositions.

Let there be three numbers, to wit,

$$\begin{array}{ccccc} A & . & B & . & C \\ 12 & . & 7 & . & 5 \end{array}$$

Suppose also that the first number A is greater than either of the numbers B and C,

Suppose also, some number as D (3) to be added to each of the said three numbers, so will the three sums be,

$$\begin{array}{c|c} A+D & 15 \\ B+D & 10 \\ C+D & 8 \end{array}$$

The Proposition to be demonstrated is, that the difference between the first sum and the second is equal to the difference between the first number and the second; also that the difference between the first sum and the third is equal to the difference between the first number and the third.

Demonstration.

The difference between the first number and the second is,

$$A - B$$

The difference between the first sum and the second is,

$$A+D-B-D$$

But

But the latter difference is manifeſtly equal to the former (for $+D$ and $-D$ expunge one the other;) to wit,

$$A+D-B-D=A-B$$

Therefore the firſt part of the propoſition is proved.

Again, the difference between the firſt number and the third is,

$$A-C$$

The difference between the firſt ſum and the third is,

$$A+D-C-D$$

But the latter difference is manifeſtly equal to the former, for $+D$ and $-D$ expunge one the other, *viz.*

$$A+D-C-D=A-C$$

Wherefore the propoſition is fully proved.

The like property might be proved after the ſame manner, when one and the ſame number is ſubtracted from three numbers ſeverally.

Secondly, when three numbers (namely the numbers ſought by the *rule of Falſe* and the two feigned numbers) are ſeverally multiplied by one and the ſame number; the afore-mentioned Analogy will likewiſe enſue, as may be thus proved.

Suppoſitions.

Let there be three numbers, to wit,

$$A \;.\; B \;.\; C.$$
$$3 \;.\; 5 \;.\; 8.$$

Sup-

Suppoſe alſo that the firſt number A is leſs than either of the numbers B and C,

Suppoſe alſo, each of thoſe three numbers to be multiplied by one and the ſame number as D (4) and the three products to be theſe,

DA	12
DB	20
DC	32

The Propoſition to be demonſtrated is, that the difference betwen the firſt product and the ſecond hath ſuch proportion to the difference between the firſt product and the third, as the difference between the firſt number and the ſecond hath to the difference between the firſt number and the third, *viz.*

$$DB-DA \;.\; DC-DA :: B-A \;.\; C-A$$
$$8 \;.\; 20 :: 2 \;.\; 5$$

Demonſtration.

Foraſmuch as (by the 17th Prop. of the ſeventh Book of *Euclids Elem.*) if a number (D) multiplying two numbers ($B-A$ and $C-A$) produceth other numbers ($DB-DA$ and $DC-DA$) the numbers produced by the multiplication ſhall be in the ſame proportion as the numbers multiplied are, therefore

$$DB-DA \;.\; DC-DA :: B-A \;.\; C-A$$

which was to be demonſtrated.

Likewiſe when 3 numbers are divided by one and the ſame number, the *demonſtration* will not be

otherwiſe

otherwise; and because by the second *Section* of this *Chapter*, the errors in the *Rule of False* are the differences between the true result and the two false results, therefore from the precedent *demonstrations* it is evident, that the aforementioned Analogy or proportionality (namely, when the first error hath such proportion to the second, as the difference between the number sought and the first feigned number hath the difference between the said number sought and the second feigned number) will succeed from such operation, as is before declared in the beginning of the third *Section* of this Chapter.

To know whether a question be resolvable by the Rule of False or not.

IV. Now to discern what kind of operation will not produce the said Analogy, observe this note, *viz.* when a question requires some given number to be divided by the number sought or any part thereof, also when the number sought or some part thereof is to be squared, cubed, &c. likewise when some parts of the number sought are to be multiplied one by the other; I say from such operations the aforementioned Analogy will not arise, and in those cases, the *ordinary Rule of False* will be useless; as may partly appear by the two following examples, *viz. What number is that, by which if 360 be divided the quotient will be* 24? Here if two positions or feigned numbers be taken, and 350 be divided by each of them, the errors will not be in the same proportion with the differences between the true number sought and the 2 feigned numbers, and therefore the *rule of False* will be used in vain: yet if it be asked what number is that, which being multiplied

by 24, the product will be 360, the *Answer* to this latter question is the same with the answer to the former, and may be found by the *Rule of False*; but such kind of interpretations and inferences are not always obvious, and therefore since the preparative work of the *Rule of False* (after the number is taken by guess for the number sought) proceeds gradually from one condition in the question to another, it will for the most part be easie to determine whether the ordinary *Rule of False* will take place or not, by comparing the conditions of a question with the note before given.

Another Example; a certain person being demanded what number of years he had lived, answered, if $\frac{1}{10}$ of that number were multiplied by $\frac{1}{4}$ of the same number, the product would shew the number, or his age: here it will be in vain to search the number sought (which is 40) by the *rule of False*; for the aforementioned Analogy or proportionality will not succeed, and the question cannot easily be resolved without *Algebra*.

Now from this supposition, that after the preparative work of the *rule of False* is finisht, the errors will be in such proportion as aforesaid, I shall make it manifest that the *rule of False* will discover the number sought.

V. In the Rule of two false Positions there are 3 cases, *viz.* the errors are either both excesses and noted with +, or else both defects and noted with —, or lastly, one of the errors is noted with +, and the other with —.

In the two first cases the Rule is this, Multiply the Positions or feigned numbers by the altern errors, *viz.* the first Position by the second error, the

the second Position by the first error, and reserve those products; then dividing the difference of the said products by the difference of the said errors, the quotient shall be the number sought by the question.

The demonstration of the said Rule here followeth.

Case I. *When the errors are both excesses and noted with* +.

Suppositions.

1. Let some number unknown and sought by the *rule of False* be represented by . . . . A
2. Let the first Position (or feigned number) be . . . . B
3. And the second feigned number C
4. Suppose also that B is greater than C, and each of them greater than A.
5. Moreover suppose the error of the first Position to be . . . . F
6. And the error of the second Position to be. G
7. Suppose also that this Analogy will be found in the said numbers, *viz.*

$$B - A \;.\; C - A :: F \;.\; G$$

8. The Proposition to be demonstrated.

$$A = \frac{FC - GB}{F - G}$$

Demon-

Demonstration.

9. Forasmuch as by supposition in 7°.

$$B - A \,.\, C - A :: F \,.\, G$$

10. Therefore by comparing the rectangle of the extreams to the rectangle of the means.

$$GB - GA = FC - FA$$

11. And by equal addition of FA.

$$FA + GB - GA = FC$$

12. Again; forasmuch as by supposition in 4°.

$$B > C$$

13. And consequently out of 4°. and 12°.

$$B - A > C - A$$

14. Therefore out of 9°. and 13°.

$$F > G$$

15. Therefore

$$FA > GA$$

16. Therefore

$$FA - GA > 0$$

17. There-

17. Therefore by equal subtraction of GB from the equation in 11°.

$$FA-GA=FC-GB$$

18. Wherefore by dividing both parts of the last equation by F—G, equal quotients will arise, *viz.*

$$A=\frac{FC-GB}{F-G}$$

which was to be demonstrated.

Case II. *When the errors are both defects, and noted with*—

Suppositions.

1. Let some number unknown and sought by the *rule of False* be represented by . . . . } A
2. Let the first position (or feigned number) be . . } B
3. Suppose also that B is less than C, and each of them less than A.
5. Moreover, suppose the error of the first Position to be . . . . . } F
6. And the error of the second Position . . G
7. Suppose also that this Analogy will be found in the said numbers, *viz.*

$$A-B \,.\, A-C :: F \,.\, G$$

8 The Proposition to be demonstrated.

$$A=\frac{FC-GB}{F-G}$$

Demonstration.

9. Forasmuch as by supposition in 7°.

$$A-B \,.\, A-C :: F \,.\, G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extreams.

$$FA-FC=GA-GB$$

11. Any by equal addition of FC

$$FA=FC+GA-GB$$

12. Again, forasmuch as by supposition in 4°.

$$B > C$$

13. And consequently out of 4°. and 12°.

$$A-B > AC$$

14. Therefore out of 9°. and 13°.

$$F > G$$

15. Therefore

$$FA > GA$$

16. There-

16. Therefore

$$FA - GA > 0$$

17. Therefore by equal subtraction of GA from the equation in 11°.

$$FA - GA = FC - GB$$

18. Wherefore by dividing both parts of the last equation by F—G, equal quotients will arise, *viz.*

$$A = \frac{FC - GB}{F - G}$$

which was to be demonstrated.

Case **III.** *When one of the errors is an excess (to wit, noted by +) and the other a defect (noted by—)*

In this third Case the *Rule of False* is this, *viz.* Multiply the Positions by the altern errors, to wit, the first Position by the second error, also the second Position by the first error, and reserve those products; then dividing the sum of the said products by the sum of the said errors, the quotients shall be the number sought by the question.

The Demonstration of this latter Rule here followeth.

Suppositions.

1. Let some number unknown and sought by the *Rule of False* be represented by . . . . . A
2. Let the first Position be . . . . B

3. And

3. And the second Position C
4. Suppose also that B is greater than C, and also greater than A, and that C is less than A.
5. Moreover, suppose the error of the first Position to be . . . . . F
6. And the error of the second Position to be . G
7. Suppose also that this Analogy will be found in the said numbers, *viz.*

$$B - A \,.\, A - C :: F \,.\, G$$

8. The Proposition to be demonstrated.

$$A = \frac{GB + FC}{F + G}$$

Demonstration.

9. Forasmuch as by supposition in 7°.

$$B - A \,.\, A - C :: F \,.\, G$$

10. Therefore by comparing the rectangle of the means to the rectangle of the extreams.

$$FA - FC = GB - GA$$

11. And by equal addition of FC and GA to the last equation, this will arise,

$$FA + GA = GB + FC$$

12. Wherefore by dividing both parts of the last

equation

equation by $F \times G$, equal quotients will ariſe, *viz.*

$$A = \frac{GB + FC}{F + G}$$

which was to be demonſtrated.

The learned *Herigonius* (in *cap.* 13 *Tom.* 2. of his *Curſus Mathematicus*) hath delivered another way of reſolving the *rule of Falſe*, namely by the two following rules, *viz.*

When the ſigns of the errors are unlike.

Rule I. As the ſum of the errors is to the firſt error, ſo is the difference of the ſuppoſed numbers to a fourth proportional, which being added to the firſt ſuppoſed number, when the ſaid firſt ſuppoſition is leſs than the ſecond, or ſubtracted from it when it exceeds the ſecond; the ſum or remainder will be true number ſought.

When the ſigns of the errors are unlike.

Rule II. As the difference of the errors is to the firſt error, ſo is the difference of the ſuppoſed numbers to a fourth proportional, which being added to the firſt ſuppoſed number when the ſigns are — or ſubtracted from it when the ſigns are +; the ſum or remainder will be the number ſought.

Both which rules the ſaid *Herigonius* demonſtrateth geometrically by lines, upon a ſuppoſition of the *Analogy* or *Proportionality* before mentioned in the third Section of this Chapter, and the ſame may likewiſe be eaſily demonſtrated according to precedent method by letters.

CHAP.

CHAP. X.

A Collection of pleaſant and ſubtil Queſtions, to exerciſe all the parts of Vulgar Arithmetick. To which are alſo added various practical Queſtions about the menſuration of Superficial Figures and Solids.

Examples of the Rule of Three mixtly uſed with other Rules.

Queſt. 1. If a wedge of Gold weighing $17\frac{3}{7}$ *lb.* of *Troy weight* be worth $679\frac{5}{7}$ *lb. ſterling*, what is the value of $1\frac{3}{13}$ grain of that Gold?

Anſw. 2 pence.

I. $1\frac{3}{13}$ (or $\frac{16}{13}$) of $\frac{1}{24}$ of $\frac{1}{20}$ of $\frac{1}{12} = \frac{1}{4680}$

II. $\frac{122}{7} . \frac{4758}{7} :: \frac{1}{4680} . \frac{1}{120}$

Queſt. 2. A man dying gave to his eldeſt Son $\frac{2}{3}$ of $\frac{1}{4}$ of his eſtate, to his ſecond Son $\frac{1}{3}$ of $\frac{1}{2}$ of his eſtate, and when they had counted their Portions, the one had 40*l.* more than the other; the remainder of the eſtate was given to the wife and younger children. The queſtion is, what was the portion of the eldeſt Son, alſo of the ſecond, and how much did belong to the wife and younger children?

Anſw. The eldeſt Sons portion 100 *l.* the ſecond Sons portion 60 *l.* and 440*l.* for the wife and younger children.

The fractions being reduced, it will be manifeſt that the eldeſt Son had $\frac{1}{6}$, *and the ſecond* $\frac{1}{10}$ *alſo the* dif-

difference of the ſaid fraction is $\frac{1}{15}$*, then ſay,*

$$\frac{1}{15} \,.\, \frac{40}{1} :: \frac{1}{10} \,.\, \frac{60}{1}$$

	l.
The ſecond Sons portion	60
The difference of their portions . . . :	40
The eldeſt Sons portion	100

$$\frac{1}{15} \,.\, \frac{40}{1} :: \frac{1}{1} \,.\, \frac{600}{1}$$

Laſtly, 600—160=440 *for the Wife and younger Children.*

Queſt. 3. A young man received $66\frac{2}{3}$ *l.* which was $\frac{2}{3}$ of $\frac{1}{2}$ of his eldeſt brothers portion, and $3\frac{1}{2}$ times of his eldeſt brothers portion was $1\frac{1}{4}$ times of his fathers eſtate, the queſtion is, what was the fathers eſtate? *Anſw.* 560 *l.*

$\frac{1}{3} \,.\, 66\frac{2}{3} :: 1 \,.\, 200$

$200 \times 3\frac{1}{2} = 700$

$1\frac{1}{4} \,.\, 700 :: 1 \,.\, 560$

Queſt. 4. If A can finiſh a work in 20 days, and B in 30 days; in what time will the work be finiſhed by A and B working together? *Anſwer,* 12 days.

Firſt find what quantity of the work will be done by each workman in one and the ſame time; then it will be, as the ſum of thoſe quantities is in proportion to the ſaid time, ſo is 1 or the whole work to the time wherein ſuch work will be finiſhed by both workmen working together.

days		*work*		*days*		*work*
30	.	1	::	20	.	$\frac{2}{3}$
					add	1
					ſum	$1\frac{2}{3}$

Hence it appears that A and B working together 20 days will finiſh that work once, together with $\frac{2}{3}$ of the ſame work; therefore ſay again by the *Rule of Three.*

work		*days*		*work*		*days*
$1\frac{2}{3}$	.	20	::	1	.	12

Queſt. 5.

Æreus adſto leo, tubuli mihi lumina bina,
Oſque etiam, dextri ſic quoque planta pedis.
Binis dextro oculo, ternis lacus iſte diebus
Impletur lævo, ſed pede bis geminis.
Ori ſufficiunt ſex horæ. Dic ſimul ergo,
Quo ſpatio os, oculi, peſque replere valent?

The ſence is this. A brazen Lion being placed in an artificial fountain, conveyeth water into a Ciſtern by two ſtreams iſſuing from his eyes, alſo by one from his mouth, and by another at the bottom of his right foot. Now the Pipes through which theſe ſtreams paſs, are of different capacities, in ſuch ſort, that by the right eye ſet open alone the reſt of the ſtreams being ſtopt, the Ciſtern will be filled in two days (the length of a day being ſuppoſed to be 12 hours; by the left eye alone in three days; by the foot alone in four days; and

Gg 3

by the mouth alone in six hours. The question is, to find in what time the Cistern will be filled, if all those streams be set open at once?

Answer, $\frac{12}{37}$ days,

days		*Cist.*		*days*		*Cist.*
2	.	1	::	3	.	$1\frac{1}{2}$
4	.	1	::	3	.	$0\frac{3}{4}$
$\frac{1}{2}$	.	1	::	3	.	6
					add	1

The sum is $9\frac{1}{4}$ *Cisterns that will be filled in three days by all the four streams running together: Then say by the rule of Three.*

Cist.		*Days*		*Cist.*		*day*
$9\frac{1}{4}$	.	3	::	1	.	$\frac{12}{37}$

Quest. 6. A Cistern in a certain Conduit is supplied with water by one pipe of such bigness, that if the cock *A* at the end of the pipe be set open the Cistern will be filled in $\frac{1}{2}$ hour; moreover at the bottom of the Cistern two other cocks *B* and *C* are placed, whose capacities are such, that by the cock *B* set open alone (all the rest being stopt, the Cistern supposed to be full) will be emptied in $1\frac{3}{7}$ hour; also by the cock *C* set open alone the Cistern will be emptied in $2\frac{1}{3}$ hour: now because more water will be infused by the cock *A*, than can be expelled by both the cocks *B* and *C* in one and the same time; the question is to find in what time the Cistern will be filled if the said three cocks be set open at once? *Answ.* $1\frac{9}{61}$ hour.

After the manner of the fourth question of this Chapter, find how many times the Cistern will be emptied in one and the same space of time, by the cocks of *B* and *C* running together; also how much of the Cistern will be filled by *A* in the same time; then will the difference shew how much of the Cistern is gained by the filling cock in the said time: Lastly, as the Cisterns or parts gained are in proportion to the correspondent time; so is the whole Cistern, to the time wherein it will be gained or filled.

	hou.		*cist.*		*hou.*		*cist.*	
I.	$2\frac{1}{3}$	.	1	::	$1\frac{3}{7}$	.	$\frac{30}{49}$	*emptied by* C
					add		1	*emptied by* B
					sum		$1\frac{30}{49}$	*emptied by* B & C, *in* $1\frac{3}{7}$ *hou.*
	hou.		*cist.*		*hou.*			
II.	$\frac{1}{2}$	:	1	::	$1\frac{3}{7}$	:	$2\frac{6}{7}$	*filled by A*
							$1\frac{12}{49}$	*gained by A*

	cist.	*hou.*		*cist.*		*hou.*
III.	$1\frac{12}{49}$	$1\frac{3}{7}$	::	1	::	$1\frac{9}{61}$

Quest. 7. Suppose a Dog, a Wolf and a Lion, were to devour a Sheep, and that the Dog could eat up the Sheep in an hour, the Wolf in $\frac{3}{4}$ hour, and the Lion in $\frac{1}{2}$ hour; now if the Lion begin to eat $\frac{1}{8}$ hour before the other two, and afterwards all three eat together, the question is, in what time the Sheep would be devoured? *Answ.* $\frac{31}{104}$ hour.

		hou.		*sh.*		*hou.*		*sh.*
I.	If	$\frac{1}{2}$	.	1	::	$\frac{1}{8}$	.	$\frac{1}{4}$

 Thus

Thus it appears that $\frac{1}{4}$ of the Sheep would be eaten by the Lion, before the Dog and Wolf began to eat.

II. Proceed according to the fourth queſtion, ſo will you find the remaining $\frac{3}{4}$ to be eaten by them all in $\frac{9}{32}$ hour, which added to $\frac{1}{8}$ gives $\frac{31}{104}$ hour, in which time the Sheep would be devoured.

Queſt. 8. If $120\frac{1}{3}$ *l.* be to be diſtributed amongſt three perſons A, B, C, in ſuch ſort, that as often as A takes 5, B ſhall take 4, and as often as B takes 3, C ſhall take 2; what ſhall be the ſhare of each?

Anſw. A $51\frac{4}{7}$ *l.* B $41\frac{9}{35}$ *l.* C $27\frac{53}{105}$ *l.*

Find three Numbers which may expreſs the proportions of their ſhares, by the *Rule of Three,* or (to avoid fractions) thus,

$$\begin{array}{ccc} 5 & \cdots\cdots & 4 \\ 3 & \cdots\cdots & 2 \\ \hline \end{array}$$

$$\underbrace{15 \;.\; 12 \;.\; 8}_{\textit{thus found}}$$

$$5 \times 3 = 15$$
$$3 \times 4 = 12$$
$$4 \times 2 = 8$$

$$35 \;.\; 120\tfrac{1}{3} \;::\; \begin{cases} 15 \;.\; 51\frac{4}{7} \\ 12 \;.\; 41\frac{9}{35} \\ 8 \;.\; 27\frac{53}{105} \end{cases}$$

Queſt. 9. A Governour of a certain Garriſon, being deſirous to know how much money the Port or Paſſage of the Garriſon did amount unto in certain months, made choice of a loyal ſervant, giving him order to receive of every Coachman paſſing with a Coach 4*d.* of every Horſeman 2*d.* and of every Footman $\frac{1}{2}$*d.* Now at the years end, the ſervant making his accompt to the Governour, giveth him 94*l.* 15*s.* 10*d.* and lets him know that as often as 5 paſſed with Coaches, 9 paſſed on Horſeback; and as often as 6 paſſed on Horſeback, 10 paſſed on foot; the queſtion is how many Coaches, Horſemen, and Footmen paſſed? *Anſwer,* 2500 Coaches, 4500 Horſemen, 7500 Footmen.

Find three proportional numbers after the manner of the 8 queſtion, which will be 5, 9, 15, then proceed as followeth,

		d.
5	*Coaches* ..	20
9	*Horſemen*	18
15	*Footmen* .	$7\frac{1}{2}$

$$\text{If } 45\tfrac{1}{2} \;.\; 22750 \;::\; \begin{cases} 5 \;.\; 2500 \\ 9 \;.\; 4500 \\ 15 \;.\; 7500 \end{cases}$$

Queſt. 10. A Factor would exchange 780 *l. ſterling* for double Ducats, Dollars, and French Crowns, the Ducats at 7 *s.* 6 *d.* the piece, the Dollars at 4 *s.* 4 *d.* and the French Crowns at 6 *s.* the piece, to be in ſuch proportion, that $\frac{1}{2}$ of the number of Ducats may be equal to $\frac{1}{3}$ of the number of Dollars, and $\frac{1}{4}$ of the Dollars equal to $\frac{3}{16}$ of the Crowns, the queſtion is, how many pieces of each coin he ſhall receive for his 780 pounds.

Anſw. 600 Ducats, 900 Dollars, 1200 Crowns.

Find three proportional Numbers (after the man-

manner of the eighth question) which will be 6,4,3,

$$\tfrac{1}{2} \ldots\ldots\ldots \tfrac{1}{3}$$
$$\tfrac{1}{4} \ldots\ldots\ldots \tfrac{3}{16}$$
$$\overline{\tfrac{1}{8} \;.\; \tfrac{1}{12} \;.\; \tfrac{1}{16}}$$
$$6 \;.\; 4 \;.\; 3$$

Thus it appears that six times the number of Ducats must be equal to four times the number of Dollars, also equal unto three times the number of Crowns. Then make choice of three numbers to answer those proportions, such as are these, 2,3,4, (for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which numbers proceed as followeth,

$$\begin{array}{ll} & l. \\ 2 \textit{ ducats} \ldots & \tfrac{3}{4} \\ 3 \textit{ dollars} \;. & \tfrac{13}{20} \\ 4 \textit{ crowns} \;.\;. & 1\tfrac{1}{5} \end{array}$$

$$\text{say if} \ldots \overset{l.}{2\tfrac{3}{5}} \;.\; 780 \;::\; \left\{\begin{array}{ll} l. & l. \\ \tfrac{3}{4} & .\; 225 \\ \tfrac{13}{20} & .\; 195 \\ 1\tfrac{1}{5} & .\; 360 \end{array}\right.$$

$$\begin{array}{llll} l. & \textit{ducat} & l. & \\ \tfrac{3}{8} \;. & 1 \;::\; & 225 \;. & 600 \textit{ ducats.} \\ & \textit{doll.} & & \\ \tfrac{13}{60} \;. & 1 \;::\; & 195 \;. & 900 \textit{ dollars.} \\ & \textit{crown} & & \\ \tfrac{3}{10} \;. & 1 \;::\; & 360 \;. & 1200 \textit{ crowns.} \end{array}$$

Quest. 11. Twenty Knights, 30 Merchants, 24 Lawyers and 24 Citizens, spent at a dinner 64 pound, which was divided amongst them in such manner, that 4 Knights paid as much as 5 Merchants, 10 Merchants as much as 16 Lawyers; and 8 Law-

8 Lawyers as much as 12 Citizens; the question is, to know the sum of money paid by all the Knights, also by the Merchants, Lawyers and Citizens.

Answer, The 20 Knights paid 20 pounds, the 30 Merchants 24 pounds, the 24 Lawyers 12 pounds, and the 24 Citizens 8 pounds.

Find four numbers to express the proportions of their payments by the *Rule of Three*, or (to avoid fractions) in manner following, so will the proportional numbers be 4, 5, 8, 12, *viz.* 4 Knights paid as much as 5 Merchants, or 8 Lawyers, or 12 Citizens.

$$4 \ldots\ldots\ldots\ldots 5$$
$$10 \ldots\ldots\ldots\ldots 16$$
$$8 \ldots\ldots\ldots\ldots 12$$
$$\overline{320 \;.\; 400 \;.\; 640 \;.\; 960}$$
$$\underbrace{4 \;.\; 5}_{} \;.\; \underbrace{8 \;.\; 12}_{}$$

thus found,

$$4 \times 10 \times 8 = 320$$
$$10 \times 8 \times 5 = 400$$
$$8 \times 5 \times 16 = 640$$
$$5 \times 16 \times 12 = 960$$

Then presupposing that a Knight is to pay 4 *s.* proceed as followeth, *viz.*

20 *Knights*

	l.
20 *Knights*	4
30 *Merchants*	$4\frac{4}{5}$
24 *Lawyers*	$2\frac{2}{5}$
24 *Citizens*	$1\frac{3}{5}$

$$\text{say, if } 12\tfrac{4}{5} \,.\, 64 :: \begin{cases} 4 & .\ 20 \\ 4\frac{4}{5} & .\ 24 \\ 2\frac{2}{5} & .\ 12 \\ 1\frac{3}{5} & .\ 8 \end{cases}$$

l. — 64

Quest. 12. A certain man with his wife did usually drink out a Vessel of Beer in 12 days, and the husband found by often experience, that his wife being absent, he drank it out in 20 days; the question is, in how many days the wife alone could drink it out? *Answer,* 30 days.

Note, It is to be supposed that the husband in 12 of the 20 days where he drank alone, did drink as much as in the 12 days wherein he drank with his wife; hence it followeth, that in the remaining 8 of the said 20 days, he drank asmuch as his wife did in 12 days. Therefore by the *Rule of Three* say, If 8 give 12, what 20? *Answ.* 30. view the following form of the work.

From 20
Subtract 12

Then if 8 . 12 :: 20 . 30

Quest. 13. If a house be to be built by three Carpenters, A B, C, working in such sort, that A, alone will finish it in 30 days, B in 40 days and A, B, C, together in 15 days, in what time could C alone build the house? *Answ.* 120 days.

I. After the manner of the fourth question, (find in what time A and B working together will finish the house; *Answ.* $17\frac{1}{7}$ days.

days	*work*	*days*	*work*
40 .	1 .	30 .	$\frac{3}{4}$
		add	1
		sum	$1\frac{3}{4}$

work	*days*	*work*	*days*
$1\frac{3}{4}$.	30 ::	1 ...	$17\frac{1}{7}$

II. Supposing the work of A and B to be performed by one person, as D, the house will be built by D in $17\frac{1}{7}$ days, but by D and C together in 15 days; Then find (according to the 12th question) in what time C will build the same; *Answ.* 120. *days.*

From $17\frac{1}{7}$
Subtract 15

Then if $2\frac{1}{7}$. 15 :: $17\frac{1}{7}$. 120

The proof may be wrought according to the fourth or fifth questions.

Quest. 14. Two Travellers A and B perform a Journey to one and the same place in this manner, *viz.* A travels 14 miles every day, and had travelled 8 days before B began; upon the ninth day B sets forward, and travels 22 miles every day; the

the queſtion is, to find in what times B ſhall overtake A? *Anſw.* at the end of 14 days.

I. Find how many miles A had travelled before B ſet forward? *Anſw.* 112 miles; For

day	*miles*	*days*	*miles*
1 .	14 ::	8 .	112

II. Find how many miles B gains of A in a day; *Anſwer*, 8 miles; For,

22 — 14 = 8

	miles	*day*	*miles*	*days*
III. If	8 .	1 ::	112 .	14

Queſt. 15. There is an Iſland which is 36 miles in compaſs. Now if at the ſame time, and from the ſame place, two footmen A and B ſet forward to travel round about the ſaid Iſland, and follow one another in ſuch manner that A travelleth every day 9 miles, and B 7 miles; the queſtion is to find in what ſpace of time they will again meet, alſo how many miles, and how many times about the Iſland each footman will then have travelled?

Anſwer, They will meet at the end of 18 days from their firſt parting; and then A will have travelled 162 miles (or $4\frac{1}{2}$ times the compaſs of the Iſland) and B will have travelled 126 miles (or $3\frac{1}{2}$ times the compaſs of the Iſland.)

miles

	miles
From . . .	9
Subtract	7

	day	*miles*	*days*
2 .	1 ::	36 .	18

mult 18 | *mult.* 18.
by 9 | *by* 7

36) 162 ($4\frac{1}{2}$ | 36) 126)$3\frac{1}{2}$

Queſt. 16. Two footmen A and B depart at the ſame time from *London* towards *York*, travelling at this rate, *viz.* A goeth 8 miles every day, B goeth 1 mile the firſt day, 2 miles the ſecond day, 3 miles the third day, and in that progreſſion he goeth forward, travelling in every following day one mile more than in the preceding day; the queſtion is to know in how many days B will overtake A?

Anſwer, 15 *days*:

To reſolve this and ſuch like queſtions, double 8 (the number of miles which A travelleth daily) which make 16, from which ſubtract 1, the remainder is 15 the number of days ſought.

Queſt. 17. If *Exceter* be diſtant from *London* 140 miles, and that at the ſame time one footman A departed from *London* towards *Exceter*, travelling every day 8 miles, and another B from *Exceter* towards *London*, travelling every day 6 miles, the queſtion is in how many days they will meet one another, and how many miles each footman will have then travelled?

Anſwer,

Answer, They will meet at the end of 10 days, and then A will have travelled 80 miles, and B 60 miles.

add $\left\{\begin{array}{l}\text{8 miles travelled dally by A.}\\ \text{6 miles travelled daily by B.}\end{array}\right.$

sum 14 miles which A and B together did travel daily.

m. da. miles da.
14 . 1 :: 140 . 10 in which time A and B will meet each other.
10 x 8=80 miles travelled by A.
10 x 6=60 miles travelled by B.

Quest. 18. A certain footman A departed from *London* towards *Lincoln*, and at the same time another footman B departeth from *Lincoln* towards *London*; also A travelleth every day $2\frac{1}{2}$ miles more than B. Now supposing those two Cities to be 100 miles distant one from the other, and that those two footmen do meet one another at the end of 8 days after the beginning of their Journies; the question is, how many miles each will have then travelled, as also how many miles each travelled daily?

Answer, A 60 miles, B 40 miles. Also A travelled $7\frac{1}{2}$ miles every day, and B 5 miles.

day miles days miles
1 . $2\frac{1}{2}$ :: 8 . 20

Hence it appears that at the time of their meeting A had travelled 20 miles more than B, which

20 miles being subtracted from 100 miles leave 80 miles, whereof the half is 40 miles which B had travelled, therefore A had travelled 60 miles.

Now to find how many miles each travelled daily, say,

days miles day miles
8 . 40 :: 1 . 5

miles

Therefore $\left\{\begin{array}{l}A\\B\end{array}\right\}$ travelled $\left\{\begin{array}{l}7\frac{1}{2}\\5\end{array}\right\}$ daily.

Quest. 19, There is an Island which is 134 miles in compass; now at the same time, and from the same place, two footmen A and B begin a journey round about the said Island, but they travel towards contrary parts, at this rate, *viz.* A travelleth 11 miles in every 2 days, and B 17 miles in 3 days, the question is to find in what space of time A and B will meet one another; and how many miles each will then have travelled?

Answer, They will meet at the end of 12 days, and then A will have travelled 66 miles, and B 68 miles.

After the manner of the fourth Question of this Chapter, the time sought will be found 12 days.

days miles days miles
2 . 11 :: 3 . $16\frac{1}{2}$
add 17

days miles days
$33\frac{1}{2}$. 3 :: 134 . 12

The miles travelled by each will be found in this manner.

days	*miles*	*days*	
2 .	11 ::	12 .	66 miles travelled by *A*.
3 .	17 ::	12 .	68 miles travelled by *B*.

Quest. 20. If a Clock hath two Indices (or hands) one of which (to wit *A*) is carried twice round the whole circumference of the Dyal in one day; and the other (*B*) once in 30 days, and that both at once shewing the same point begin to be moved; the question is, in what time they will be again conjoined?

Answer, $\frac{30}{59}$ day or $\frac{12}{59}$ hours.

day	*circum.*		*days*	*circum.*
1 .	2	::	30 .	60
			subtract	1
				59

Hence it appears, that in 30 days *A* will have run through 60 circumferences, and *B*, one circumference only in the same time; therefore *A* gains of *B* 59 circumferences in 30 days, therefore say,

circum.	*days*		*circum.*	*day*
59 .	30	::	1 .	$\frac{30}{59}$

Quest. 21. If 6 *lb.* of Sugar be equal in value to 7*lb.* of Raisins; 5*lb.* of Raisins to 2*lb.* of Almonds; 3*lb.* of Almonds to 5*lb.* of Currants; 2*lb.* of Currants to 18*d.* how many pence are the value of 3*lb.* of Sugar? *Answ.* 21*d.*

$$\text{mult. these contin.}\left\{\begin{array}{rcl} 6S. &=& 7\ R. \\ 5R. &=& 2\ A. \\ 3A. &=& 5\ C. \\ 2C. &=& 18\ d. \\ ?\ d. &=& 3\ S. \end{array}\right\}\text{mult. these contin.}$$

180)3780(21

Quest. 22. If 3 dozen pair of Gloves be equal in value to 2 pieces of Ribbon; 3 pieces of Ribbon to 7 dozen of Points; 6 dozen of Points to 2 yards of Flanders-lace; and 3 yards of Flanders lace to 81 shillings; how many dozen pair of Gloves may be bought for 28 shillings?

Answ. 2 dozen pair of Gloves.

$$\text{mult. these contin.}\left\{\begin{array}{rcl} 3\ G. &=& 2\ R. \\ 3\ R. &=& 7\ P. \\ 6\ P. &=& 2\ L. \\ 3\ L. &=& 81\ S. \\ 28\ S. &=& ?\ G. \end{array}\right\}\text{mult. these contin.}\ 2268)\ 4536\ (2$$

4536 2268

Quest. 23. Suppose a *Greyhound* to be coursing a *Hare*, in such sort that the *Hare* takes five leaps for every four leaps of the *Greyhound*, and that the *Hare* is one hundred of her own leaps distant from the *Greyhound*; now if three of the *Greyhounds* leaps be equal to four leaps of the *Hares*; the question is to know how many leaps the *Greyhound* must take before he obtain his prey?

Answer, 1200 *leaps.*

I. If $3 \;.\; :: 4 \;.\; 5\frac{1}{3}$

Thus it appears, that 4 of the *Greyhounds* leaps are equal to $5\frac{1}{3}$ of the *Hares* leaps; and because by the question the *Greyhound* takes 4 leaps for every 5 of the *Hares*, therefore the *Greyhound* in every four of his leaps gains $\frac{1}{3}$ of one of the *Hares* leaps; therefore say by the *Rule of Three*,

II. If $\frac{1}{3} \;.\; 4 :: 100 \;.\; 1200$

Quest. 24. There is a certain room whose Basis is a long square, which is in circuit $50\frac{1}{2}$ feet, and the height of the walls or sides of the room is $8\frac{1}{4}$ feet; all which walls of the room except a space taken out for a window in the form of a long square, whose height is five feet, and breadth four feet, are to be furnished with Hangings of ell-broad stuff at 3 *s.* 4 *d.* the yard, the question is to know how much money the stuff will cost?

Answer, 5 *l.* 17 *s.* $6\frac{2}{9}$ *d.*

$50\frac{1}{2} \times 8\frac{1}{4} = 416\frac{5}{8}$ *square feet.*
$5 \times 4 = 20$ *subtract*

$396\frac{5}{8}$

$3\frac{3}{4} \times 3 = 11\frac{1}{4}$ *square feet in one yard of stuff.*

	feet	*d.*	*feet*	*d.*
If	$11\frac{1}{4}$.	40 ::	$396\frac{5}{8}$.	$1410\frac{2}{9}$

Quest. 25. There is a certain Walk which is a long square, whose length is 40 yards, and breadth 7 yards, to be paved with stones, each of which being in form of a long square is 28 inches in length, and 24 inches in breadth: the question is to know how many such stones will be requisite to pave the said Walk?

Answer, 540.

Inches		*Inches*	
1440	$\times$	252 = 362880	*square Inches.*
28	$\times$	4 = 672	*square Inches.*
672 .	1 ::	362880 . 540	*Stones.*

Quest. 26. Suppose a piece of *Tapestry* to be $5\frac{3}{8}$ yards *English* in length, and $3\frac{7}{8}$ yards in breadth, the question is, how many square ells *Flemish* are contained in that piece of *Tapestry*, when the length of 1 ell *Flemish* is equal to $\frac{3}{4}$ of a yard *English*?

Answer, $37\frac{1}{36}$ square ells *Flemish*.

$5\frac{3}{8} \times 3\frac{7}{8} = \frac{1333}{64}$ *square yards.*

Then because $\frac{9}{16}$ of a square yard is equal to 1 ell square of *Flemish* measure (for $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$) say,

If $\frac{9}{16} \;.\; 1 :: \frac{1333}{64} \;.\; 37\frac{1}{36}$

Quest. 27. A Workman hath performed a piece of Tiling bearing the form of a long square, whose length is 273 feet, 7 inches; and breadth 21 feet 5 inches; now when Tiles are sold at the rate of 11 *s.* $10\frac{3}{4}$ *d.* for 1000 Tiles, and every square of Tiling consisting of 10 feet as well in length as in breadth doth take up 1000 Tiles, what doth the said piece of Tiling amount unto?

Answer,

Answer, 34 *l.* 17 *s.* $0\frac{4001}{57600}$ *d.*

I. $273\frac{7}{12} \times 21\frac{5}{12} = \frac{843731}{144}$ *square feet.*

II. 100 . $142\frac{3}{4}$: : $\frac{843731}{144}$. $8364\frac{4001}{57600}$ (d. / d.)

Quest. 28. A Merchant would beſtow 220 *l.* in Cloves, Mace and Nutmegs, the Cloves being at 5*s.* the pound; the Mace at 11 *s.* the pound, and the Nutmegs at 6 *s.* the pound; now he would have of each ſort an equal quantity, the queſtion is how many pounds he may have of each ſort?

Anſwer, 200 *lb.*

s.
5
11
6

22 . 1 : : 4400 : 200

The Proof.

lb.	*s.*		*l.*
200 at	5	*amounts unto*	50
200 at	11	*amounts unto* ...	110
200 at	6	*amounts unto* ...	60
			220

Quest. 29. A Factor is to receive a ſum of money, and is offered Dollars at 4 *s* 4 *d.* which are worth but 4 *s.* 3 *d.* or French Crowns at 6 *s.* $1\frac{1}{2}$ *d.* which are

are worth but 6 *s.* the queſtion is by which coin he ſhall ſuſtain the leaſt loſs?

Anſwer, the Dollars.

d.		*d.*		*d.*		*d.*
52	:	1	: :	$73\frac{1}{2}$	:	$1\frac{43}{104}$

That is, in receiving the Dollars every 6 *s.* $1\frac{1}{2}$ *d.* loſeth $1\frac{43}{104}$ *d.* but in receiving the Crowns 6 *s.* $1\frac{1}{2}$ *d.* loſeth $1\frac{1}{2}$ *d.* which is a greater loſs than $1\frac{43}{104}$ *d.*

Queſt. 30. A Butcher agrees with a Graſier, for the feeding of 20 Oxen, during the ſpace of 12 equal months, but at 2 months end, the Butcher adds 5 Oxen more, and $6\frac{2}{3}$ months after that, he added 10 Oxen more, and then it is agreed between them that the Graſier ſhall feed them all, ſo long time as will be equivalent to the keeping of the firſt twenty during 12 months; the queſtion is how long time he ſhall feed them all, after the putting in of the laſt 10?

Anſwer, 1 month.

Conſider, that as he receives more Oxen to feed he ought to keep them all the leſs time; therefore work as the queſtion imports by the *Rule of Three Inverſe.*

	mon.	*Oxen.*		
	12	20		
Oxen	2	5	*mon.*	*Oxen*
If 20 :	10 : :	25 .	(8	25
			$6\frac{1}{3}$	10

If 25 .... $1\frac{2}{3}$ .. 35 (1 *mon.*

Queſt.

Examples of the Rule of Fellowship.

Quest. 31. Two Merchants, *viz.* A and B, have entred Company; A puts in 500 *l.* and at 4 months end takes out a certain ſum, leaving the remainder to continue 8 months longer, B puts in 250*l.* and at five months end puts in three hundred pounds more, and then his whole ſum continues ſeven months longer. Now at the making of their Accompt A findeth that he hath gained $106\frac{2}{3}$ pounds, and B gained $133\frac{1}{3}$ pounds; the queſtion is to know how much A took out of the bank at 4 months end?

Anſwer, 240 *l.*

$$250 \times 5 = 1250$$
$$\text{add } 300$$
$$550 \times 7 = 3850$$
$$5100$$
$$133\tfrac{1}{3} \;.\; 5100 :: 106\tfrac{2}{3} \;.\; 4080$$
$$500 \times 4 = 2000 \text{ (subtract}$$
$$8)2080(260$$

Laſtly, 500—260=240 *taken out by* A.

The Proof.

l.		*mon.*	
500	×	4	= 2000
Subtract 240			
260	×	8	= 2080
			4080

Queſt. 32. Five Merchants, *viz.* *A*, *B*, *C*, *D*, and *E* have gained 2025*l.* which they divide in ſuch ſort that $\frac{1}{2}$ of the ſhare of *A* is equal ſeverally to $\frac{1}{4}$ of the ſhare of *B*, $\frac{1}{5}$ of *C*, $\frac{1}{6}$ of *D*, $\frac{1}{8}$ of *E*. The queſtion is, what was the ſhare of each Merchant?

Anſwer, A 162*l.* *B* 324*l.* *C* 405*l.* *D* 486*l.* *E* 648*l.*

Divide a number at pleaſure into ſuch parts which may be in ſuch proportion as the ſhares required, and proceed according to the ſubſequent operation.

A 2
B 4
C 5
D 6
E 8

If 25 . 2025 :: { 2 (162 *for A; whereof* $\frac{1}{2}$ *is* 81; 4 (324 *for B, whereof* $\frac{1}{4}$ *is* 81; 5 (405 *for C, whereof* $\frac{1}{5}$ *is* 81; 6 (486 *for D, whereof* $\frac{1}{6}$ *is* 81; 8 (648 *for E, whereof* $\frac{1}{8}$ *is* 81 } (*l.*)

2025

Queſt. 33. Two Merchants A and B are in company, the ſum of their ſtocks is 300 *l.* the money of A continuing in company 9 months, the money of B 11 months, they gain 200*l.* which they divide equally, the queſtion is to know how much each Merchant did put in?

Anſwer, A 165 *l.* B 135 *l.*

Divide 300 into two ſuch parts which may be in proportion as 11 to 9, ſo will the greater part be the ſtock of A, and the leſſer the ſtock of B, which ſtocks being multiplied by their reſpective times, the products will be equal.

$$\begin{array}{l} 11 \\ \underline{9} \\ 20 \;.\; 300 :: \left\{ \begin{array}{l} 11 \;.\; 165 \text{ for A} \\ \\ 9 \;.\; 135 \text{ for B} \end{array} \right. \end{array}$$

Quest. 34 Two Merchants, *viz.* *A* and *B*, are in company, *A* did put in 325 *l.* more than *B*, and the ſtock of *A* continued in company $7\frac{1}{2}$ months, *B* put in a certain ſum which is unknown, and it continued in company $10\frac{3}{4}$ months: after a certain time they divided the gain equally; the queſtion is, what each Merchant did put in?

Anſwer, *B* 750 *l.* and *A* 1075 *l.*

Divide the product of the difference of their ſtocks multiplied by the time of *A*, by the difference of their times, ſo will the quotient be the ſtock of *B*. which added to 325 *l.* gives the ſtock of *A*.

$$\begin{array}{l} 325 \times 7\tfrac{1}{2} = 2437\tfrac{1}{2} \\ 3\tfrac{1}{4})\; 2437\tfrac{1}{2}\; (750 \textit{ ſtock of B} \\ \qquad\qquad \textit{add } \underline{325} \\ \qquad\qquad 1075 \textit{ ſtock of A} \end{array}$$

Queſt. 35. A Goldſmith hath ſome Gold of 24 Carects, others of 22 Carects, and another ſort of 18 Carects fine; he would ſo mix theſe together that the maſs mixed might be 60 *lb.* and that the whole mixture might bear 20 Carects fine. How much of each ſort muſt he take?

Examples of the Rule of Alligation, How the fineneſs of gold and ſilver is eſtimated, v. p. 111.

$$\textit{Anſwer}, \left\{ \begin{array}{l} \textit{lb.} \\ 12 \textit{ of } 24 \textit{ Carects.} \\ 12 \textit{ of } 22 \textit{ Carects.} \\ 36 \textit{ of } 18 \textit{ Carects.} \end{array} \right.$$

$$20 \left\{ \begin{array}{l} 24 \\ 22 \\ 18 \end{array} \right. \quad \begin{array}{c} 2 \\ 2 \\ \underline{4+2} \end{array} \left| \begin{array}{c} 2 \\ 2 \\ \underline{6} \end{array} \right.$$

$$10 \;.\; 60 :: \left\{ \begin{array}{l} 2 \;.\; 12 \\ 2 \;.\; 12 \\ 6 \;.\; 36 \end{array} \right.$$

Note, Some may think that queſtions of *Alligation* are capable onely of ſo many ſeveral anſwers as there are different ways to connect the mean rate or price with the extream rates or prices; yet it is moſt certain, that any ordinary queſtion of Alligation, where three or more things are propounded to be mixt in ſuch manner as that rule requires, is capable of infinite anſwers, if fractions be admitted, and ſometimes of many anſwers in whole numbers, which are not diſcoverable by the common rule of *Alligation*: ſo albeit to the laſt mentioned queſtion, the ſaid rule of *Alligation* can find but one anſwer onely, which is before given; yet there are eight other anſwers in whole numbers, which are theſe that follow (the invention whereof I have ſhewn in the 19th Queſtion of the Thirteenth Chapter of my ſecond Book of *The Elements of Algebra.*)

Of

Of 24 *Carects*	18	16	14	10
Of 22 *Carects*	3	6	9	15
Of 18 *Carects*	39	38	37	35

Of 24 *Carects*	8	6	4	2
Of 22 *Carects*	18	21	24	27
Of 18 *Carects*	34	33	32	31

See Chap. 8. of this Appendix.

Quest. 36. An Apothecary hath several Simples, *viz.* A hot in 3°. B hot in 2°. C temperate, D cold in 2°. and E cold in 4°. Now he desires to make a Medicine of those Simples in such sort that the temper thereof in respect of quality may be in 1°. of heat, and the quantity $8\frac{1}{2}$ Drams, the Demand is what quantity of each Simple he must take?

Answer, $4\frac{1}{2}$ Drams of A, $\frac{1}{2}$ Dram of B, $\frac{1}{2}$ Dram of C, 1 Dram of D, and 1 Dram of E.

	Indices		*Drams*	
6	8	1, 3, 5	9	A.
	7	1	1	B.
	5	2, 1	3	C.
	3	2	2	D.
	1	2	2	E.
			17	

Drams.

17 . $8\frac{1}{2}$ ::	9	. $4\frac{1}{2}$	A.
	1	. $0\frac{1}{2}$	B.
	3	. $1\frac{1}{2}$	C.
	2	. 1	D.
	2	. 1	E.
		$8\frac{1}{2}$	

Quest.

Quest. 37. A Merchant buyeth 2 sorts of Clothes, *viz.* of blacks and whites for 68*l.* 2*s.* after the rate of 21 *s.* the yard for the blacks, and 12 *s.* the yard for the white, and he taketh so much of each sort, that $\frac{5}{8}$ of the number of yards of the black, are equal to $\frac{7}{8}$ of the white; the demand is how many yards be bought of each sort?

Examples of the Rule of False Position

Answer, 42 yards of black, and 40 yards of white.

Quest. 38. A certain person A payeth unto the use of B for ever 2500 *l.* in present money, upon this condition, that B shall pay unto A an Annuity or yearly rent to be continued four years, the equality of their agreement being thus grounded, *viz.* the said 2500 *l.* is supposed to be put forth at interest for a year (to commence from the time of their agreement) at the rate of 8 *per centum, per annum.* Then from the sum of that principal and interest (arising due at the years end) the first payment of the Annuity being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the second year; then from the composed of this principal and interest (due at the second years end) the second payment of the Annuity being subtracted, the remainder is likewise supposed to be put forth at the same rate of interest for the third year; then from this principal and interest the third payment of the Annuity being subtracted, the remainder is in like manner supposed to be put forth at the same rate of interest for the Fourth year: lastly, from this principal and interest the fourth and last payment of the Annuity being subtracted, there must be nothing left: the question is, what sum of money must be

yearly

yearly paid to satisfie those conditions?

Answ. $754\frac{14117}{17602}$ *l.* as will be manifest by the subsequent proof.

I. $100 . 108 :: 2500 . 2700$

Subtract the first payment $754\frac{14117}{17602}$

$1945\frac{3485}{17602}$

II. $100 . 108 :: 1945\frac{3485}{17602} . 2100\frac{14325}{17602}$

Subtract the second payment $754\frac{14117}{17602}$

$1346\frac{208}{17602}$

III. $100 . 108 :: 1346\frac{108}{17602} . 1453\frac{12194}{17602}$

Subtract the third payment $754\frac{14117}{17602}$

$698\frac{15679}{17602}$

IV. $100 . 108 :: 698\frac{15679}{17602} . 754\frac{14117}{17602}$

Subtract the last payment .... $754\frac{14117}{17602}$

000

Quest 39.

Mulæ, Asinæque duos imponit servulus utres
Impletos vino; segnemque ut vidit Asellam
Pondere defessam vestigia figere tarda,
Mula rogat; quid chara parens cunctare, gemisque?
Unam ex utre tuo mensuram si mihi reddus,
Duplum oneris tunc ipsa feram; sed si tibi tradam
Unam mensuram, fient æqualia utrique
Pondera, mensuras dic docte Geometer istas?

The sence is this. A *Mule* and an *Ass* carried two unequal quantities of Wine, each consisting of a certain

certain number of measures, in such sort, that if the *Ass* imparted one of her measures to the *Mule*, then the *Mules* number of measures so increased would be the double of those which the *Ass* had remaining; but if the *Mule* gave one measure to the *Ass*, then the *Asses* measures with that increase would be equal to the *Mules* remaining measures. The question is, how many measures each carried?

Answer, the *Mule* 7 and the *Ass* 5.

Quest. 40.

Æs ferrum, stamnum miscens, aurique metallum,
Sexaginta minas pensantem finge coronam.
Æs aurumque duos simul efficiunto trientes.
Ternos quadrantes stanno mixtum impleat aurum.
At totidem quintas auri vis addita ferro.
Ergo age dic fulvi quantum tibi conjicis auri
Miscendum: dic quantum æris stannique requiras:
Dic quoque sufficiant duri quot pondera ferri:
Præscriptam ut valeas rite efformare coronam.

The sense is this, Suppose a Crown that shall weigh 60 *l.* is to be made of Gold, Brass, Iron, and Tin, mixed together in such proportion, that the weight of the Gold and of the Brass together may be 40 *l.* the joint weight of the Gold and of the Tin 45*lb.* and the joint weight of the Gold and of the Iron, 36*lb.* The question is how much of every one of those four metals must be taken?

Answer, *l.*
- $30\frac{1}{2}$ *of Gold*
- $9\frac{1}{2}$ *of Brass.*
- $5\frac{1}{2}$ *of Iron,*
- $14\frac{1}{2}$ *of Tin.*

Quest.

Queſt. 41. One being demanded what was the preſent hour of the day, anſwered, that the time then paſt from noon was equal to $\frac{1}{5}$ of $\frac{3}{8}$ of the time remaining untill midnight. The queſtion is, what a clock it was? (ſuppoſing the time between noon and midnight to be divided into twelve equal parts or hours.)

Anſwer, $\frac{36}{43}$ hour after noon.

Queſt. 42. A Factor delivers 6 French Crowns and 2 Dollars for 45 ſhillings ſterling; alſo at another time he delivers 9 French Crowns and 5 Dollars (at the ſame rate with the former) for 76 ſhillings. The queſtion is to know the value of a French Crown, alſo of a Dollar?

Anſwer, A Crown was valued at 6 *s.* 1 *d.* and a Dollar at 4 *s.* 3 *d.*

Queſt. 43. A certain Uſurer received 36 Dollars for the ſimple intereſt of 186 *l.* lent for a certain time unknown; alſo he received 90 Dollars for the gain of 360 *l.* at the ſame rate of intereſt for a certain time unknown; now the ſum of the months wherein both the ſaid numbers of Dollars were gained was twenty months. The queſtion is to know in what time as well the 36 Dollars as the 90 Dollers were gained?

Anſwer, The 36 Dollars were gained in $8\frac{8}{11}$ months, and the 90 Dollars in $11\frac{3}{11}$ months, as may be proved by the *Double Rule of Three.*

Which anſwer may be diſcovered by the following Canon found out by the *Algebraick Art.*

Multiply the Dollars firſt gained, the latter Principal, and the given time, according to the rule of continual Multiplication, for a dividend; then multiply the firſt principal by the Dollars laſt gained; alſo multiply the latter principal by the Dollars firſt gained, and reſerve the Sum of theſe two laſt products for a Diviſor: laſtly, divide the Dividend firſt found by the ſaid Diviſor, ſo ſhall the quotient be the time wherein the firſt number of Dollars was gained, which ſubtracted from the time given in the queſtion diſcovers the time wherein the latter number of Dollars was gained.

$$\frac{36 \times 360 \times 20 = 259200.}{186 \times 90, + 300 \times 36, = 29700} = 8\tfrac{3}{11}$$

And conſequently .... $20 - 8\frac{8}{11} = 11\frac{3}{11}$

Queſt. 44. 3481 Soldiers are to be placed in a ſquare Battel, how many are to be ſet in Rank or in File? *Examples of the Extraction of Roots.*

Anſw. 59 (for the ſquare root of 3481 is 59.)

Queſt. 45 If 4050 Soldiers are to be ſet in battle in a figure, which beareth the form of a long ſquare in ſuch manner, that the number in File may be to the number in rank, as 1 to 2, how many Soldiers are to be placed in Rank, and how many in File?

Anſw. 90 in Rank and 45 in File (found by this Canon or general rule) *viz.*

As the greater term of the proportion given is to the leſſer, ſo is the number of Men to be placed in Battle to a fourth proportional, whoſe ſquare root is the leſſer number ſought (whether it be for the Rank or File: alſo as the leſſer term of the given proportion is to the greater; ſo is the number of Men to be ſet in battle to a fourth proportional, whoſe

whose square root is the greater number sought (whether it be for the Rank or File.)

I.	2.	1 :: 4050 . 2025
II.	√q.	2025 — 45 (*Men in File.*
III.	1.	2 :: 4050 . 8100
IV.	√q.	8100—90 (*Men in Rank.*

The Proof.

45 x 98 = 4050
Also 45 . 90 :: 1 . 2

Or when one of the numbers sought (whether it be for the Rank or File) is found, the other may be discovered by *Division, viz.*

45) 4050 (90
90) 4050 (45

Quest. 46. Suppose the wall of a Garrison to be in height 21 feet, and the breadth of the Moat surrounding the said wall to be 28 feet; the question is, what length must a scaling ladder have to reach from the outermost side of the Moat to the top of the Wall?

Answer, 35. (to wit, the squre root of the sum of the squares of 21 and 28.)

21 x 21—441
28 x 28 784

√q. 1225 35

Quest.

Quest. 47. If 100 *l.* being put forth for interest at a certain rate, will at the end of two years be augmented unto 112 $\frac{36}{100}$ *l.* (compound interest, or interest upon interest being computed) what principal and interest will be due at the first years end?

Answer, 106 *l.* composed of 100 *l.* principal and 6 *l.* interest (which 106 is a mean Geometrically proportional between 100 and 112. 36 (and may be found by the eighteenth rule of the fifth Chapter of this Appendix.)

100 x 112.36=11236 (106

Quest. 48. If a 100 *l.* being put forth for interest at a certain rate, will at the end of 3 years be augmented unto 115.7625 *l.* (compound interest being computed) what principal and interest will be due at the first years end?

Answer, 105 *l.* composed of 100 *l.* Principal, and 5 *l.* interest) which 105 is the first of two mean proportional numbers between 100 and 115,7625 *l.* (See the nineteenth rule of the fifth Chapter of this Appendix.)

Various Practical Questions to exercise Decimal Arithmetick, in the mensuration of Superficial Figures and Solids.

Quest. 49. If the side of a square Superficies be 3 feet, what is the Area or content of that Superficies? Or (which is the same thing) how many squares, each of which is a Foot square, are contained in that Superficies?

See the second Section of the 23 Chapter of the preceding Book.

 Answer,

Anſwer, 9 ſquare feet, which content is found out by multiplying the given ſide 3 by it ſelf, *viz.* 3 multiplied by 3 produceth 9.

In like manner, if the ſide of a ſquare Pavement of ſtone be 15.7 Feet, the ſuperficial content of that pavement will be 246.49 feet, that is 246 feet and an half very near, (for 15.7 multiplied by it ſelf produceth 246. 49.)

Likewiſe, a ſquare piece of Wainſcot whoſe ſide is 3.24 yards, will be found to contain 10. 49 + yards, or 10 yards and an half almoſt; for 3. 24 multiplied by it ſelf, to wit, by 3.24 will produce 10. 49 +

Alſo if the ſide of a ſquare piece of Sand be 37.25 perches, the content in ſquare Perches (neglecting the fraction in the product) will be found 1387, which being reduced (according to the ſeventh *Table* in *Rule* 4, *Chapter* 7 of the preceding *Book*) will give 8 acres, 2 roods, and 27 perches for the content of that ſquare piece of Land.

Queſt. 50. If a long ſquare be 8 feet in length and 5 feet in breadth, what is the ſuperficial content?

Anſwer, 40 feet; which content is found out by multiplying the length by the breadth, *viz.* 8 multiplied by 5 produceth 40. So if one of the lights of a glaſs window ſuppoſed to be in the form of a long ſquare, had for its length 3.06 feet, and breadth 1.47 feet, the content of that glaſs will be 4.4982 feet, or 4 feet, and an half almoſt, (for 3.06 multiplied by 1. 47 produceth 4.4982)

In like manner if there be a piece of Wainſcot, Plaſtring, or any other ſuperficies in the form of a

a long ſquare, which is in length 6.325 yards and in breadth 3.214 yards; the ſuperficial content will be found 20.32 + yards, that is 20 yards, one quarter of a yard, and ſomewhat more, for, 6. 325 multiplied by 3.204 produceth 20.32 +.

Likewiſe a piece of Tiling in the form of a long ſquare whoſe length is 18.5 feet, and breadth 11.7 feet will be found to contain 216. 45 ſquare feet, which will be reduced to 2. 1645 ſquares of Tiling by allowing (according to cuſtom) 100 ſquare feet to one ſquare of Tiling.

Alſo if a piece of Sand in the Form of a long ſquare be 48.75 perches in length, and 36.25 in breadth, the Area or content in perches, will be found 1767.18 +, which 1767 perches being reduced will give 11 acres and 7 perches for the content of that piece of Ground.

Queſt. 51. If it be required to ſet forth in a Meadow one acre of graſs to lye in the faſhion of a long ſquare, and that the length thereof be limited or agreed to be 20 perches, what muſt the breadth be?

Anſwer, 8 perches, which breadth is found out by dividing 160 (the number of ſquare perches contained in an acre) by the given length 20. If two acres were required, then 320 (to wit, twice 160) muſt be divided by the given ſide, whether it be the length or breadth; ſo if 7.25 perches be preſcribed for the breadth of two acres, the length muſt be 44. 13 + perches.

In like manner, if the breadth of a Board be 1. 32 foot, and it be demanded how far one ought to meaſure along the ſide thereof to have a ſuperficial foot, or a foot ſquare of that Board; divide

 1 by

1 by the given breadth, ſo you will find in the quotient this decimal fraction .757+, which repreſents three quarters of a foot or nine inches and ſomewhat more, and ſo much in length ought to be meaſured along the ſide of that Board to make a ſuperficial foot. Likewiſe if the breadth of a board be given in inches, then 144 (the number of ſquare inches contained in a ſuperficial foot ſquare) being divided by the given breadth, the quotient will ſhew how many inches ought to be meaſured along the ſide of that board to make a ſuperficial foot; ſo the breadth of a board being 9 inches, the length forward to make a ſuperficial foot will be found 16 inches.

Queſt 52. If the three ſides of a piece of land that lies in the form of a triangle be 15 perches, 14 perches, and 13 perches, what is the Area or number of ſquare perches contained in that triangle?

Anſwer, 84 perches, or half an acre and four perches, which content is found out by this Rule, *viz.*

From half the ſum of the three ſides of any plain triangle ſubtract each of the three ſides ſeverally, and note the three remainders; then multiply the ſaid half ſum and thoſe three remainders one into the other (according to the rule of continual Multiplication;) that done, extract the ſquare root of the laſt product, ſo ſhall ſuch ſquare root be the Area or content of the triangle.

The

	Perches
The 3 ſides of a triangle	15 14 13
The ſum of the 3 ſides	42
The half of that ſum	21
The 3 remainders found out by ſubtracting each ſide from the half ſum	6 7 8
The product ariſing from the continual multiplication of the four laſt numbers	7056
The ſquare root of which product is the content required, to wit,	84

Another Example.

	Perches
The 3 ſides of a triangle	120 . 5 112 . 6 90 . 3
The ſum of the 3 ſides	323 . 4
The half of that ſum	161 . 7
The 3 remainders found by ſubtracting each ſide from the half ſum	41 . 2 49 . 1 71 . 4
The product ariſing from the continual multiplication of the four laſt numbers	23355380 . 1096
The ſquare root of that product	4832 . 7+

Wherefore

Wherefore I conclude that the content of a plain triangle, whose three sides are 120.5 perches 112.6 perches, and 90.3 perches, is 4832.7 + perches, which reduced give 30 acres and 32 perches (the fraction of a perch being neglected.)

Now forasmuch as every irregular piece of ground may be divided into triangles, for a four-sided field will be divided into two triangles by one imaginary streight line leading overthwart from corner to corner called a *Diagonal Line*; a five-sided field into three triangles by two *Diagonals*; a six-sided ground into four triangles by three *Diagonals*, &c. the rule before given will be of excellent use to find out the Contents of large fields, especially if the Land be of a dear value, as also when any controversie ariseth by the reason of the different admeasurements of Surveyors of land: for if the sides of those triangles be measured in the Fields, and their lengths be agreed on, all Artists to whom the reason of the rule before given is known, will agree in one and the same content. But yet this way of measuring presupposeth that there is no obstacle, as Water, Wood, or other impediment, to hinder the measuring of the sides of those Triangles into which the Field is divided as aforesaid.

Quest. 53. If the Diameter of a Circle be 28.25, what is the Circumference?

Answer, 88.749.+: for as 113 is in proportion to 355; or as 1 is to 3. 14159, so is the Diameter to the Circumference: Therefore multiplying always the diameter given by the said 3. 14159 the product shall be the Circumference required.

Quest. 54. If the diameter of a Circle be 28.25, what

what is the superficial content of that Circle?

Answer, 626.79 +: for as 1 is in proportion to .78539, so is the square of the Diameter to the superficial content. Therefore multiplying always the said decimal Fraction .78539 by the square of the given Diameter (which square is the product of the multiplication of the diameter by it self) the product shall be the superficial content required.

Quest. 55. If the Diameter of a Circle be 28.25. what is the side of a square which may be inscribed within the same Circle?

Answer, 19.975 + for the square root of half the square of the Diameter, or the square root of the double of the square of the Demidiamiter, shall be the side of the inscribed square sought. Otherwise, as 1 is to .707166, so is the diameter to the side required. Therefore if you multiply (always) the said .707106, by the diameter given, the product will be the side of the inscribed square required.

Quest. 56. If the Circumference of a Circle be 88.75 what is the Diameter?

Answer, 28.249 + for as 355 is to 113, or as 1 is to .318309, so is the Circumference to the Diameter. Therefore if .318309 be multiplied always by the given Circumference, the product shall be the diameter required.

Quest. 57. If the Circumference of a Circle be 88. 75, what is the superficial content of that Circle?

Answer, 626 801+; for as 1 is to .079578, so is the square of the Circumference to the superficial content. Therefore if .079578 be always multiplied by the square of the given Circumference, the product shall be the superficial content sought.

Quest.

Quest. 58. If the circumference of a Circle be 88.75. what is the side of a square that may be inscribed within the same Circle?

Answer, 19.975 ; for as 1 is to ·225078, so is the circumference to the side required. Therefore if .225078 be always multiplied by the circumference given, the product will be side of the inscribed square sought.

Quest. 59. If the superficial content of a Circle be 626.8, what is the diameter?

Answer, 28.25 + ; for as 1 is to 1.27324, so is the content to the square of the diameter. Therefore multiplying always 1.27324 by the given content, the square root of that product shall be the diameter required.

Quest. 60. If the superficial content of a Circle be 626.8, what is the cirumference?

Answer, 88.75 + for as 1 is to 12.5664, so is the content to the square of the circumference. Therefore if 12.5664 be always multiplied by the given content, the square root of the product shall be the circumference required.

Quest. 61. If the superficial content of a Circle be 626.8, what is the side of a square equal to the same Circle?

Answer, 25.035+, for the square root of the given content is the side of the square required.

Quest. 62. If the side of a Cube be 12 inches, how many cubical inches are contained in that Cube?

Answer, 1728, what a Cube is may be well represented by a Dye, which is a little cube it self being a rectangular or square solid, that hath an equal length, breadth and depth, and is comprehended

hended under six equal squares; now if the side of one of those equal squares (which is also the side of the Cube) be 12 inches, the superficial content of that square will be 144 square inches (for according to the preceding 49th question, 12 multiplied by 12 produceth 144) which multiplied by the depth 12 inches, produceth 1728 cubical inches, and such is the solid content of that Cube whose side is 12 inches: so that by one foot of timber or stone in whatsoever kind of solid it be found, is understood a Cube, containing 1828 cubical or dye-square inches, and consequently half a foot solid contains 864 cubick inches, and a quarter of a foot solid contains 432 cubick inches.

In like manner, if a side of a Cube of stone be 2.53 feet, the solid content of that Cube will be found 16.194+ feet, for 253 being multiplied by it self produceth 6.4009 superficial feet, which product being multiplied by the said 2.53 will produce 16.194 + solid feet.

Also if the side of a Cube of stone or wood be 6 inches, or .5 foot, the solid content will be found 216 cubick inches, or .125 parts of a foot solid (for 6 multiplied cubically produceth 216, likewise .5 multiplied cubically produceth .125;) whence it may be inferr'd, that 8 little cubes of stone or wood, each of which is half a foot or 6 inches square, are contained in a foot of stone or timber; for 8 times 216 produceth 1728 (being the number of cubick inches contained in a foot solid) likewise 8 times .125 produceth 1 (to wit, one entire foot solid.)

Quest. 63. If the breadth of a squared piece of timber, supposed to be streight and terminated at both

both ends by two equal ſquares, be 1. 55 foot, the depth alſo 1. 55 foot, and the length 17.33 feet, how many cubick feet are contained in that piece of Timber?

Anſwer, 41.635 feet, that is, 41 feet and an half, and about half a quarter of a Foot. Which ſolid content is found out by this rule, *viz.* multiply the breadth 1. 55 by the depth 1. 55 the product will be 2.4025 ſuperficial Feet, which is the content of the Baſs (that is, the Area of either of the two equal ſquares at the ends of the piece;) laſtly, multiplying the ſaid Baſs 2.4025 by the length 17.33 the product will be 41.635 +, which is the ſolid content required.

In like manner if the breadth of a ſquared piece of Timber, ſuppoſed to be ſtreight and terminated at both ends by two equal long ſquares (which are called the Baſes) be 2.34 feet, the depth 1. 61 foot, and the length 17. 58 feet, the ſolid content will be 66. 23 + feet; for (as before) multiplying the breadth by the depth, and that product by the length, the laſt product ſhall be the ſolid content required.

Queſt. 64. If the breadth, as alſo the depth of a ſquared piece of Timber having equal ſquare Baſes, be 1.55 foot, how far ought one to meaſure along the length of that piece of Timber to make a foot ſolid?

Anſwer, .416 parts of a foot, or 5 inches very near; which decimal is thus found, *viz.* Firſt find the ſuperficial content of the Baſs, which will be 2.4025 (for 1.55 multiplied by 1. 55 produceth 2.5025;) Then dividing 1 (to wit 1 ſolid foot) by the Baſe 2.4025 the quotient will be .416 + or $\frac{416}{1000}$ parts of a foot, or five inches almoſt, and ſo far ought to be meaſured along the length of the piece to make a foot ſolid. In like manner, if the breadth be 2. 34. feet, and the depth 1.61 feet, the length forward along the piece to make one ſolid foot will be found .265 parts of a Foot, or three inches and almoſt $\frac{1}{3}$ part of an inch.

Queſt. 65. If a ſtreight ſquared piece of timber be terminated by unequal Baſes, whereof one contains 1.92 ſuperficial Foot, the other .85 foot, and the length of that piece of timber be 17.4 feet; what is the ſolid content, or how wany cubical Feet are contained in that piece of timber?

Anſwer, 23.474 + feet (found out by one of Mr. *Oughtred*'s Rules for meaſuring a ſegment of a Pyramid in *Problem* 21. Chapter 19. of his *Clavis Mathemat.*) The Rule is this.

Multiply the greater baſe by the leſs, and extract the ſquare root of that product, then multiply the ſum of the two Baſes and that ſquare root by one third part of the length of the ſolid propounded, ſo ſhall the laſt product be the ſolid content required.

Example.

Example.

The greater Base	1 . 92
The lesser Base	0 . 85
The product of the multiplication of those two Bases	1 . 6320
The square root of that product	1 . 2774
The sum of that square root and the two Bases	4 . 0474
One third part of the length is	5 . 8
The product of the multiplication of the two last numbers is the solid content required	23 . 474+

Quest. 66. A Pyramid is a solid comprehended under plain surfaces, and from a triangular, quadrangular, or any multangular Base, diminisheth equally less and less till it finish in a point at the top; now if the superficial content of the Base of a Pyramid be 5.756 Feet, and the height thereof 14. 25 feet (which height is the length of the perpendicular line that falleth from the top of the Pyramid to the Base) what is the solid content of that Pyramid?

Answer, 27.341 + feet: for if the Area of the Base of a Pyramid, be multiplied by one third part of the height thereof, the product shall be the solid content of the Pyramid; therefore 5. 756 x 4.75 =27. 341 feet=, the solidity of the Pyramid propounded.

Note, If a Pyramid be cut into two segments by a Plane parallel to the Base, one of those segments will be a Pyramid, and the other will have two unequal Bases, for the measuring of which latter segment; a rule hath been already given in the sixty-fifth question, the Area of each Base being known.

Quest. 67. A Cone is a solid, which hath a Circle for its Base, from whence it grows equally less and less (like a round Steeple of a Church) till it finish in a point at the top; now if the Area of the Base of a Cone be 5. 756 feet, and the height thereof be 14. 25 feet, what is the solid content of that Cone?

Answ. 27.341 feet; for if the Area of the Base of a Cone be multiplied by one third part of the height thereof, the product shall be the solid content of the Cone.

Note, If a Cone be cut into two segments by a Plane parallel to the Base, one of those segments will be a Cone, and the other segment will have 2 unequal Bases which are Circles, the solidity of which latter segment may be found out by the rule before given in the 65 question, the Area of each Base (or Circle) being known.

Quest. 68. A Cylinder is a solid which may be well represented by a Stone-roll. such as are used in Gardens for the rolling of Walks. Now if the circumference of a Cylinder be 4. 57 feet, and the length 3. 25 feet, what is the solid content of that Cylinder?

Answer, 5.4 + Feet, thus found out: First by the help of the given Circumference 4.57, find out the superficial content of that Circle (being the Base of the Cylinder) which content (by the preceding 57th question) will be found 1.6619 + feet, then multiplying the said 1.6619 by the given length 3.25, the product will be 5.4008 which is the solid content required.

Quest.

Quest. 69. If the Base of a Cylinder be 1.6619 foot, how much in length of that Cylinder will make a foot solid?

Answer, .601 parts of a foot; for 1 (to wit, 1 solid foot) being divided by the base 1.6619, gives in the quotient the decimal .681 + for the length required.

Quest. 70. A Globe is a perfect round body contained under one Plane; in the middle of the Globe there is a point called the Center, from whence all streight lines drawn to the outside are of equal length, and called Semidiameters, the double of any one of which is equal to the Diameter of the Globe; now if the Diameter of a Globe of Stone be 1.75 feet, how many Feet solid are contained in that Globe?

Answer, 2.807 + feet, for as 21 is in proportion to 11, or as 1 is to .5238, so is the Cube of the Diameter to the solid content of the Globe: Therefore, multiplying always the Cube of the Diameter by the said decimal .5238, the product shall be the solid content required: So the Diameter 1.75 being first multiplied by it self, the product will be 3.0625, which multiplied by the said 1.75, gives in the product 5.359375, to wit, the Cube of the diameter, which being multiplied by .5238, the product thence arising will be 2.807 +, which is the solidity of the Globe propounded.

Quest. 71. What is the Diameter of a Globe of Stone, which cantains 4 cubical or solid Feet?

Answ. 1 96 + foot, for as 11 is in proportion to 21, or as 1 is to 1.9090909 so is 4 (the solid content given) to a fourth proportional, to wit, 7.636363 + whose cubick root is 1.96 + the diameter required.

Con-

Concerning the gauging of Vessels.

The easiest and aptest ways for practice in gaging, are those which are perform'd by the help of Tables, or Gaging-rods purposely compos'd: Nevertheless to give the Reader of this Treatise some light in this matter, I shall here insert one rule to find out the number of Gallons contained in a full Tun, Pipe, Hogshead, Barrel, or such like Vessel according to Mr. *Wingate's* way of ruducing a Vessel to a Cylinder. The Rule is this;

Having found the difference of the two diameters at the bung and head of the vessel, take $\frac{7}{10}$ of that difference and add it to the lesser diameter; then square that sum and reserve the product; that done, if the content be required in Wine gallons multiply the product reserved, this decimal fraction .0034, and the length of the vessel, one into the other (according to the *Rule of continual Multiplication*) so shall the last product be the number of Wine gallons required: but if the content be required in Ale gallons, multiply the product before reserved, this decimal fraction .0027, and the length of the vessel, one into the other continually, so shall the product be the content in Ale gallons: This Rule I shall first explain by two questions, and then shew how it is raised.

Quest. 72. If the diameter at the bung of a vessel be 32 inches, the diameter at the head 28.2 inches, and the length 39 inches (which dimensions

are

are ſaid to agree very near with thoſe of an Engliſh veſſel called a Pipe) what is the content of that veſſel in Wine gallons?

Anſwer, 126.278 Wine gallons, that is 126 Wine gallons and about a quart more (found out by the rule above given, as will be manifeſt by the following operation.

Explication.

The Diameter at the bung	32 . 0
The Diameter at the head	28 . 2
The difference	3 . 8
Which multiplied by $\frac{7}{10}$, that is	0 . 7
The product will be	2 . 66
Which added to the leſſer diameter gives the mean diameter	30 . 86
Which mean diameter being ſquared (that is, multiplied by it ſelf) produceth	952 . 3396
Which product multiplied by	0 . 0034
The product thence ariſing will be	3 . 2379+
Which multiplied by the length of the veſſel	39 . 0
The product is the number of Wine gallons ſought, *viz.*	126.278+

Queſt. 73. If the diameter at the bung of a barrel be 23 inches, the diameter at the head 19.9 inches, and the length 27.4 inches; what is the content of that barrel in Ale gallons?

Anſwer, 36.031 Ale gallons, that is 36 gallons and about a quarter of a Pint more (found out by the preceding Rule.)

Explication

Explication.

The diameter at the bung	23 . 0
The diameter at the head	19 . 9
Their difference	3 . 1
Which multiplied by $\frac{7}{10}$, that is	0 . 7
The product will be	2 . 17
Which added to the leſſer diameter gives the mean diameter	22 . 07
Which mean diameter being ſquared (that is, multiplied by it ſelf) produceth	487 . 0849
Which product multiplied by	0 . 0027
The product thence ariſing is	1 . 315+
Which multiplied by the length of the veſſel	27 . 4
The product is the number of Ale gallons ſought, to wit	36.031+

The reaſon of the Rule.

Two things are taken for granted in the ſaid Rule, *viz.* Firſt it is ſuppoſed that if $\frac{7}{10}$ of the difference of the two diameters at the bung and head, be added to the leſſer diameter, the ſum ſhall be an equated or mean diameter (near enough for practical uſe though it be not exact) *viz.* If there be a Cylinder whoſe diameter is equal to that mean diameter, and whoſe length is equal to the length of the veſſel, that Cylinder ſhall be equal to the capacity of the veſſel very near. Secondly

 the

the ſaid Rule preſuppoſeth that 231 cubick inches are equal to a Wine gallon, and 282 equal to an Ale gallon; concerning which equalities (eſpecially the latter) Artiſts differ ſomewhat in their experiments; but according to any equality which in that particular ſhall be agreed on, from this that follows a rule may be framed, and Tables thence calculated for gaging a full veſſel without conſiderable error.

Taking then thoſe two things above mentioned for granted, we may rightly inferr that if a Cylinder hath for its Baſe a Circle whoſe ſuperficial content is 231 inches, every inch in length of that Cylinder will contain 231 cubick inches, or one intire Wine gallon; now foraſmuch as all Circles are in ſuch proportion one to the other as the ſquares of their diameters, it ſhall be as 294.11844, (to wit, the ſquare of the diameter of that Circle whoſe ſuperficial content is 231) is to 1 (to wit, the ſuperficial content 231 conſidered as the Baſe of one Wine gallon;) or as 1 is to .0034; So is the ſquare of the equated (or any other) diameter, to the ſuperficial content of that Circle in Wine gallons and parts of a gallon, which content multiplied by the length of the veſſel will produce its ſolidity or capacity in Wine gallons. Therefore the firſt part of the preceding rule for finding of the number of Wine gallons contained in a full veſſel is manifeſt: And after the ſame manner, ſuppoſing as before 282 cubick inches are equal to an Ale gallon, the decimal .0027 preſcribed in the ſaid rule will be found out.

Upon thoſe grounds Mr. *Wingate* compos'd his Gaging rod; Mr. *Oughtred* alſo in his Circles of Propoſition

Proportion hath delivered another rule for Gaging, from whence his Gaging-rod is deduced; but the particular conſtructions of thoſe rods, and likewiſe the making of Tables for the ſame purpoſe, being handled by ſeveral Artiſts, I ſhall not inſiſt upon them.

Now if the induſtrious and more curious Arithmetician, after he is well exercis'd in vulgar Arithmetick, deſires further knowledge in finding out the Anſwer of ſubtil Queſtions about numbers, his beſt Guide will be the admirable *Algebraical* Art, which diſcovers rules for the ſolving of *Problems*, as well Arithmetical as Geometrical, that are above the reach of any of the rules of common Arithmetick, or practical Geometry, as may partly appear by the two rules in the aforegoing 52 and 65 Queſtions, as alſo by the two following Queſtions. with which I ſhall conclude this Chapter.

Queſt. 74. To find two numbers in a given proportion, ſuppoſe the leſſer to the greater as 2 to 3 and ſuch, that if the leſſer number be added to the ſquare of the greater, alſo if the greater number be added to the ſquare of the leſſer, the two ſums ſhall be ſquare numbers whoſe roots are expreſſible by rational or true numbers (fractions being admitted for numbers.)

Anſwer, $\frac{1}{10}$ and $\frac{3}{20}$.

The square of $\frac{3}{20}$ (the greater number) is ———— $\frac{9}{400}$

To which adding the lesser number ———— $\frac{1}{10}$

The sum in its least terms will be ———— $\frac{49}{400}$

Which is a square number, whose root is ———— $\frac{7}{20}$

Again, the square of $\frac{1}{10}$ (the lesser number) is ———— $\frac{1}{100}$

To which adding the greater number ———— $\frac{3}{20}$

The sum in its least terms will be ———— $\frac{4}{25}$

Which is a square number whose root is ———— $\frac{2}{5}$

Also the said numbers $\frac{1}{10}$ and $\frac{3}{20}$ are one to the other as 2 to 3, wherefore the question is solved. which numbers $\frac{1}{10}$ and $\frac{3}{20}$ are found out by this following

Theoreme.

If the fraction $\frac{1}{4}$ be divided into any two parts; either of those parts being increased with the square of the other part shall give a fraction having a rational square root.

Wherefore by dividing $\frac{1}{4}$ into the two fractions $\frac{1}{10}$ and $\frac{3}{20}$, which are in the prescribed proportions of 2 to 3, those fractions will satisfie the conditions in the question propounded.

Likewise these two fractions $\frac{722}{10080}$ and $\frac{1013}{10080}$ will answer the question, and are found out without extracting any root; but the manner of finding out the said Theorem and last mentioned fractions, I have shewn in the 24th question of my third Book of the Elements of *Algebra.*

Quest.

Quest. 75. To find 3 numbers, such that the square of any one of them being added to the other two numbers, the sum of such addition shall be a square number, whose root is a rational number.

Answer, 1, $\frac{8}{3}$, and $\frac{16}{3}$.

The Proof.

First, the square of the first number 1 is ———— 1

To which adding the second and third numbers $\frac{8}{3}$ and $\frac{16}{3}$, the sum will be ———— 9

Which is a square number whose root is ———— 3

Secondly, the square of the second number $\frac{8}{3}$ is ———— $\frac{64}{9}$

To which adding the first and third numbers 1 and $\frac{16}{3}$, the sum in its least terms will be ———— $\frac{121}{9}$

Which is a square number whose root is ———— $\frac{11}{3}$

Thirdly, the square of the third number $\frac{16}{3}$ is ———— $\frac{256}{9}$

To which adding the first and second numbers 1 and $\frac{8}{3}$, the sum in its least terms will be ———— $\frac{289}{9}$

Which is a square number whose root is ———— $\frac{17}{3}$

Wherefore it is manifest that the three numbers 1, $\frac{8}{3}$ and $\frac{16}{3}$ will satisfie the conditions in the question, which may be solved also by other numbers, but the manner of finding them out I have shewn in the 32 Question of my third Book of the Elements of *Algebra*.

CHAP. XI.

Of Sports and Pastimes.

Probl. I.

To discover a number which any one shall have in his mind, without requiring him to reveal any part of that or any number whatsoever.

AFter any one hath thought upon a number at pleasure, bid him double it, and to that double bid him add any such even number which you please to assign, then from the sum of that addition let him reject one half, and reserve the other half: Lastly, from this half bid him to subtract the number which he first thought upon; then may you boldly tell him what number remaineth in his mind after that subtraction is made, for it will always be half the number which you assigned him to add.

For example suppose he thought upon 6, the double thereof is 12, to which bid him add some even number at your pleasure, suppose 4, so will the sum be 16, whereof the half is 8, from which if he subtract 6 (the number first thought on) the remainder is 2 (to wit, half the number 4, which was by you assigned to be added;) which remainder you discover, notwithstanding all the operation was performed in his mind, without his making known of any number whatsoever. Note, that the adding of an even number as aforesaid is not of necessity, but only to avoid a fraction which will arise by taking the half of an odd number.

The reason of the Rule.

If to the double of any number (which number for distinction sake I call the first) a second number be added, the half of the sum must necessarily consist of the said first number, and half the second; therefore if from the said half sum the first number be subtracted, the remainder must of necessity be half of the second number which was added.

Probl. II.

Two numbers, the one even and the other odd, being propounded unto two persons, to the end they may (out of your sight) severally chuse one of those numbers; to discover which of these numbers each person shall have chosen.

Suppose you have propounded unto *Peter* and *John* two numbers, the one even and the other odd as 10 and 9, and that each of those persons is to chuse one of the said numbers unknown to you. Now to discover which number each person shall have chosen, you must take two numbers, the one even and the other odd, as 2 and 3; then bid *Peter* multiply that number which he shall have chosen by 2; and cause *John* to multiply that number which he shall have chosen by 3; that done, bid them add the two products together, and let them make known the sum to you, or else demand of them whether the said sum be even or odd, or by any other way more secret endeavour to discover it, by bidding them to take the half of the said sum,

for by knowing whether the said sum be even or odd, you do obtain the principal end to be aimed at, because if the said sum be an even number, then infallibly he that multiplied his number by your odd number (to wit, by 3) did chuse the even number (to wit, 10;) but if the said sum happen to be an odd number, then he whom you caused to multiply his number by your odd number (to wit, by 3) did infallibly chuse the odd number (to wit, 9.)

For example, if *Peter* had made choice of 10, and *John* 9, suppose you willed *Peter* to multiply his number 10 by 2, and *John*, to multiply his number 9 by 3; the products will be 20 and 27, whereof the sum is 47, which being an odd number, you may thence conclude that *John* whom you caused to multiply his number by 3, did chuse the odd number 9, and therefore *Peter* did chuse 10. But if you had willed *John* to have multiplied his number 9 by 2, and *Peter* to have multiplied his number 10 by 3, the products would have been 18 and 30, whereof the sum is 48, which is an even number, from whence you may infer, that he that multiplied his number by 3 did chuse the even number, and therefore *Peter* chose 10, and *John* 9.

Demonstration.

The reason of the said rule is very easie, and dependeth principally upon the 28 and 29 Propositions of the 9th Book of *Euclid*; for one may infer from the 21 of the same Book, that an even number multiplied by any number whatsoever produceth an even number, but an odd number is of a different nature, for if it be multiplied by an even number,

ber, the product is an even number (by the said 28 proposition;) and if it be multiplied by an odd number, the product is odd (by the said 29 proposition.) Therefore if in making this sport it happeneth that the even number be multiplied by your odd number, both the products shall be even, and consequently the sum shall be infallibly an even number) by the said 21 proposition.) But if it happen that you cause the odd number to be multiplied by your odd number, that product will be odd, and the other product even, therefore the sum of these two products shall be an odd number (as *Clavius* hath demonstrated upon the 23 of the 9th of *Euclid.*

Probl. 3.

A certain number of distinct things being propounded, to dispose them in such an order, that casting away always the ninth, or the tenth, or any other that shall be assigned, unto a certain number, those remaining may be such as were first intended to be left.

This Problem is usually propounded in this manner, *viz.* fifteen *Christians* and fifteen *Turks* being at Sea in one and the same Ship in a terrible Storm, and the Pilot declaring a necessity of casting the one half of those Persons into the Sea, that the rest might be saved; they all agreed that the persons to be cast away should be set out by lot after this manner, *viz.* the thirty persons should be placed in a round form like a Ring, and then beginning to count at one of the Passengers, and proceeding circularly, every ninth person should be cast into the Sea, until of the thirty persons there

there remained only fifteen. The question is, how those thirty persons ought to be placed, that the lot might infallibly fall upon the fifteen *Turks*, and not upon any of the fifteen *Christians*? For the more easie remembring of the rule to resolve this question, I shall presuppose the five vowels, *a*, *e*, *i*, *o*, *u*, to signifie five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, and (*u*) five; then will the rule it self be briefly comprehended in these two following verses.

From numbers, aid and art
Never will fame depart.

In which verses you are principally to observe the vowels, with their correspondent numbers before assigned, and then beginning with the *Christians*, the vowel *o* (in *from*) signifieth that four *Christians* are to be placed together; next unto them, the vowel *u* (in *um.*) signifieth that five *Turks* are to be placed; In like manner *e* (in *bers*) denoteth 2 *Christians*; *a* (in *aid*) 1 *Turk*, *i* (in *aid*) 3 *Christians*, *a* (in *and*) 1 *Turk*, *a* (in *art*) 1 *Christian*, *e* (in *ne*) 2 *Turks*, *e* (in *ver*) 2 *Christians*, *i* (in *will*) 3 *Turks*, *a* (in *fame*) 1 *Christian*, *e* (in *fame*) 2 *Turks*, *e* (in *de*) 2 *Christians*, *a* (in *part*) 1 *Turk*.

The invention of the said Rule, and such like, dependeth upon the subsequent demonstration, *viz.* if the number of persons be thirty, let thirty figures or cyphers be placed circularly, or else in a right line as you see,

oooooooooooooooooooooooooooooo

That done, begin to count from the first, and mark the ninth (or what other shall be assigned) by puting a point or cross over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to do the same, beginning again when you shall be at the end (if the cyphers are placed in a right line) and passing over those, which you shall have already marked, until you have marked the number required, as in the example propounded, until you have marked 15, for then all the cyphers marked shall be those which must be cast away, and the others those which shall remain. Hence it is evident, that if you observe how those cyphers which are marked, are disposed amongst those which are not marked, you will easily make a rule for any number whatsoever.

By this invention (as some do conjecture) the famous Historian *Josephus* the *Jew*, preserved his life very subtilly in the Cave, to which himself and forty of his Country men had fled from the furious and conquering *Romans* at the Siege of *Jotapata*: for his said Countrey-men having most wickedly resolved to kill one another, rather than yield to their enemies, he at length (when no arguments that he could use would disswade them from so horrid an act) prevailed with them to execute their tragical design by lot; and so by the help of the aforesaid artifice (as we may suppose) himself with one other person only remaining alive, after the rest were inhumanly murthered, they agreed to put an end to the lot, and thereby save their lives. This story you may see at large in the fourteenth Chapter of the Third Book of the History of *Josephus* of the Wars of the *Jews*.]

Probl.

Probl. 4.

Many numbers which proceed from 1 or unity in a progression, according to the natural order of numbers, (such as these, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a Ring; to discover which of those numbers any one shall have thought upon.

Let any multitude of numbers in the aforesaid progression, suppose these 10, to wit, 1. 2. 3. 4. 5. 6. 7. 8. 9. 10 be written upon 10 Ivory counters (or for want thereof upon ten small pieces of paper) which may be represented by these 10 letters, A. B.C.D.E.F.G.H.K.L. *viz.* suppose 1 to be written upon the counter A, 2 upon B, 3 upon C. *&c.* Then having placed those Counters circularly as you see (with their blank faces uppermost, and the figures underneath, that the subtilty of the sport

A
1
L 10 2 B
K 9 3 C
H 8 4 D
G 7 5 E
6
F

may the better be concealed) let any one think upon any number of unites which doth not exceed 10; that done bid him touch one of those Counters at pleasure, and to the number on the backside of the counter touched (which you cannot be ignorant of, having noted well the place of 1 or A) add secretly in your mind, the just number of all the counters, and reserve the sum; then bid him imagaine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, untill he shall have made up the aforesaid sum, which you reserved, so will his computation in fallibly end upon the counter upon which the number thought upon is written.

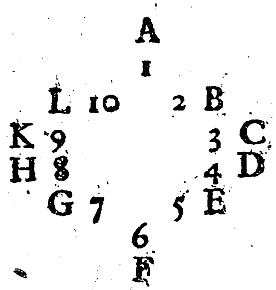
For example, suppose that he thought 7 or G, and that he touched B, to wit, 2. Add to 2 the number of all the counters, to wit, 10, so the sum will be 12; then bid him to count unto 12 beginning at B going backwards, and esteeming B to be the number thought, to wit 7, so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H, and lastly, 12 upon the counter G, which being returned up will shew 7 the number thought.

The reason of this rule is not difficult to be apprehended, two principles being presupposed, the one is this, to wit, many counters or things whatsoever being disposed orderly one after the other, in one continued line, whether it be right or circular, if you value or name the first counter to be some number of unites at pleasure, and continue to count forward according to their natural order of numbers untill another number be named which falleth upon the last counter; or if you imagine or name the last counter, to be the same number of unites as before you put upon the first, and continue to count backwards unto the first counter; I say, that the same number will be named at the end of both those computations: for example, in these 9 letters A. B. C. D. E. F. G. H. K. if the letter A be esteemed

esteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you esteem K to be 4, and count backwarks from K to A, the letter A will likwise fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principal is this, to wit, many counters being disposed in a round manner like a Ring, if you esteem any one of those contents to be some number at pleasure, and then from that counter if you count circularly, untill you end upon the counter where you began, the number last named will be equal to the sum of the number of all the counters, and of the number which you put upon the first counter; for example, If D be one of 10 Letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progression of numbers, till



you

you end with D where you began; the number 17 which is composed of 10 and 7 will necessarily fall upon D; for 9 (which is the number of letters in the circumference besides D) being added to 7 (which was first put upon D) makes 16, to which 1 being added (because D doth end as well as begin the circumference) the sum is 17.

Now these two principles being presupposed, it will not be difficult to apprehend the reason of the aforesaid rule in all cases that can happen; for imagine that one hath thought upon 7, or the counter G, then that counter which he shall touch must either be the same counter G or some other that proceedeth or followeth G.

First, therefore supposing the counter or number touched to be the same with the number thought, the truth of the rule will be then evident, for by the rule given, he shall begin to count from the same G unto 17, putting 7 upon G, therefore by the second presupposition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10, then according to the rule adding 10 (the multitude of all the counters placed circularly) unto 10; or L (the counter touched) bid him count backwards unto 20 by beginning at L, and esteem L to be 7. Now because by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the first presupposed principle, if we esteem L to be 7 and count backwards, the number 10 will infallibly fall upon G, and then the number 20 shall also fall upon the same G by the second presupposed principle.

Lastly,

Lastly, imagine he touched some number or
counter which precedeth 7 the number sought, as
B or 2; then adding 10 to 2, you are to bid him
count unto 12, he having first imagined B to be the
number thought 7, and going backwards to A, L,
K, *&c.* Now because by proceeding to count at B,
which is 2, and beginning to count forward to C,
D, *&c.* the number 7 falleth upon G; therefore if
one imagine that G is 2, and from thence count
backwards towards F, E, *&c.* the number 7 will
fall upon B (by the first presupposed principle;)
therefore when one assumeth B to be 7, and count-
eth towards A, L, *&c.* to any assigned number, it is
in effect as much as when one imagineth G to be
2, and counteth towards F, E, *&c.* unto the said as-
signed number, for each of those computations will
end in the same point; but it is manifest (by the
second presupposed principle) that esteeming G to
be 2, and counting towards F, E, D, *&c.* round the
whole circumference, the number 12 will fall upon
the same G. And because G being supposed to be
2, and counting on the same coast as before, the
number 7 falls upon B; therefore if the computa-
tion be continued on the same coast from B 7, unto
12, the number 12 will fall upon the same G. So
that the practice of this sport in all its cases is fully
demonstrated.

Note, that to the number of the counter touch-
ed you may not only add the number of all the
counters once (as the rule directs) but twice, thrice
or more times: for example, B being touched, you
may cause him to count unto 12, or unto 22, or to
32, 42, *&c.* the reason whereof is evident from the
second presupposed principle.

Probl.

Probl. 5.

Many numbers being shewed by pairs, to wit, two by two, unto any one, that he may think upon any one of those pairs at pleasure; to discover the pair that was thought upon.

Let 20 numbers, suppose these, 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. be written upon Ivory counters (or for want thereof upon small pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, *&c.* Then dispose them into pairs as you see, *viz.* Suppose 1 and 2 to be one pair, 3 and 4 to be another

1.	2
3.	4
5.	6
7.	8
9.	10
11.	12
13.	14
15.	16
17.	18
19.	20

pair, *&c.* and of these pairs let any one think upon which pair he pleaseth. That done, you are to distribute the said 20 numbers in ranks, into the form of a long square, until there be 5 numbers in length, and 4 in breadth, after this manner, *viz.*

Lay

Lay the three firſt numbers, 1, 2 and 3 in a rank (as you ſee in the ſecond figure) from A towards B; then place 4 underneath 1, and 5 after 3 (in the ſaid rank AB.) Again place 6 under 4, and 7 after 5 (in the ſaid rank AB.) Then place 8 under 6, alſo 9. 10. 11. on the right hand of 4 in the rank CD. Again place 12 under 9, and 13 on the right hand of 11 in the rank CD. and 14 under 12. Moreover place 15. 16. 17 on the right hand of 12 in the rank EF. Laſtly, place 18. 19. 20. on the right hand of 14 in the rank GH, ſo will all the numbers be ranked as you ſee in the Table. That done, you are to demand of him that thought upon two numbers as aforeſaid, in what rank or ranks the ſaid numbers do happen to be found, *viz.*

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

in which of the ranks AB, CD, EF, GH, or in which two of the ſaid ranks: now if he anſwer that the two numbers which he firſt thought upon are in the firſt rank AB, then 1 and 2 ſhall be the numbers thought upon; if in the ſecond CD, then 9 and 10 ſhall be the numbers thought; if in the third rank EF, then 15 and 16 ſhall be the numbers thought: if they are in the fourth rank GH, then 19 and 20 ſhall be the numbers thought; but if he ſhall ſay that the numbers thought are in different ranks, then you are heedfully to mark the ſaid numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which

which may be called the keys of the ſport, in regard they ſerve not onely to diſcover the two numbers thought, when they are both in one and the ſame rank (as aforeſaid) but alſo when they are in two different ranks, for in this latter caſe as ſoon as it hath been declared to you in which two ranks the two numbers thought are placed, you muſt take the key of the higheſt of thoſe two ranks, and deſcending in a down right line from the firſt number of that key unto the lower of the ſaid two ranks, you ſhall there find one of the two numbers thought, and upon the right hand of the ſecond number of the ſaid key, at the ſame diſtance ſidewiſe from the ſecond number of the key, as one of the numbers thought was diſtant from the firſt number of the key, you ſhall find the other number thought.

For example, ſuppoſe the two numbers thought are 7 and 8, and that it ſhall be declared unto you that they are in the firſt and fourth ranks; take then the key of the higheſt of theſe two ranks, to wit, of the firſt, which is 1 and 2, and deſcending down right from 1 unto the fourth rank, you ſhall there find 8 one of the numbers thought; then ſeek ſidewiſe on the right hand of 2, the ſecond number of the key, a number as far ſeparated from 2, as 8 is diſtant from 1, and you will find 7 the other number thought.

Again, ſuppoſe he ſaid that the numbers thought are in the ſecond and third ranks; take then the key of the ſecond rank which is 9 and 10, and deſcending downright from 9 to the third rank, you ſhall there find 12 which is one of the numbers thought; then ſeek ſidewiſe on the right hand

hand of 10 (the second number of the key) a number as far distant from 10 as 12 is from 9, and you shall find 11 which is the other number thought.

The reason of this will be apparent from a serious consideration of the placing of the numbers according to the rules before given, for it is thereby evident that of the two numbers coupled two by two, there can never be found more than one pair in one and the same rank, and of all the other pairs one number is always found in one rank, and the other number in another rank.

Note also, that this sport may be practised with divers persons at once, and not only with 20 numbers, but with any such multitude of numbers which is produced by the multiplication of any two numbers which differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42 which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is the placing of the numbers in ranks according to the directions before given: and for the more easie comprehending of that order, I have in the following Table ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illustration than can be exprest by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

Probl.

Probl. 6.

Three jealous husbands with their wives, being ready to pass by night over a river, do find at the river side a boat which can carry but two persons at once, and for want of a Boatman they are necessitated to row themselves over the river at several times: the question is how these 6 persons shall pass 2 by 2, so that none of the three wives may be found in the company of 1 or of 2 men unless her husband be present.

They must pass in this manner, *viz.* First two women pass, then one of them bringeth back the boat and repasseth with the third woman; that done, one of the three women bringeth back the boat, and sitting down upon the ground with her husband permitteth the other two men to pass over to find their wives; then one of the said men with his wife bringeth back the boat, and placing her upon the ground he taketh the other man, and repasseth with him; lastly, the woman which is found with the three men entereth into the boat, and at twice goeth to fetch over the other two women.

Probl. 7.

Two merry Companions are to have equal shares of 8 Gallons of Wine, which are in a vessel containing exactly 8 Gallons, now to make this equal partition they have only two other empty vessels, whereof one containeth 5 Gallons, and the other 3; the question is, how they shall exactly divide the wine by the help of those three vessels.

First, from the vessel which containeth 8 gallons and

and is full of wine, let 5 gallons be poured into the empty vessel of 5, and from this vessel so filled let 3 be poured into the empty vessel of three, so there will remain 2 gallons within the vessel of 5. Then let the three gallons which are within the vessel of 3 be poured into the vessel of 8, which will now have 6 gallons within it, that done let the 2 gallons which are in the vessel of 5, be put into the empty vessel of 3, then of the 6 gallons of wine which are within the vessel of 8 fill again the five, and from those 5 pour out 1 gallon into the vessel of 3, which wanted only 1 gallon to fill it, so there will remain exactly 4 gallons within the vessel of 5 and 4 gallons within the other two vessels. This question may be resolved in another way, but I leave that as an exercise to the wit of the ingenious Reader.

Now albeit at first sight it may be thought by some, that the two last mentioned *Problems* cannot be resolved by any certain Rule, but only by many trials, yet by infallible argumentation and discourse, the solution of those questions may be found out, or else the impossibility of them, if by chance they should have been propounded, impossible; as the most ingenious *Gasper Bachet* hath manifested in a little Book in the *French* Tongue, intituled *Problemes plaisans & delectables qui se font par les nombres*, from which Book I have extracted the Contents of this Chapter.

Soli Deo Gloria.

Thomas Jones 1707

Tho: Jones 1709

Tho: Jones
T S T Prm